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Supplementary materials for

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1 Extension of case study 2 of a fractional-HARX system with third-order nonlinearity

To further investigate the response of the proposed scheme by exploiting the knacks of EEFOA, the 3rd order nonlinearity has been taken into account. In the second case study, the proposed system comprises seven parameters, i.e., [k_1, k_2, j_1, j_2, r_1, r_2, r_3], while the value of the fractional order=0.4. The system performance is evaluated using multiple evaluation matrices for five different noise scenarios presented as u=0.0, u=0.00015, u=0.0015, u=0.015, and u=0.15.

The proposed third order fractional-HARX system is expressed as

$$K(z) = 1 + 0.45(z^{-1})^{0.4} + 0.3(z^{-2})^{0.4} \tag{S1}$$

$$J(z) = 0.16(z^{-1})^{0.4} + 1(z^{-1})^{0.4} \tag{S2}$$

$$\bar{m}(t) = 0.56m(t) + 0.36m(t)^2 + 0.8m(t)^3\,, \tag{S3}$$

where the parameter vector is given as

$$\begin{aligned}\omega &= [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7] \\ \omega &= [k_1, k_2, j_1, j_2, r_1, r_2, r_3] = [0.45, 0.29, 0.16, 1, 0.56, 0.36, 0.8]\end{aligned}.$$

The fitness convergence curves are shown in Fig. S1; the response of the proposed system was assessed using MSE. The system achieves maximum fitness values of 4.11E-12, 4.19625E-08, 7.09322E-07, 3.27075E-05, and 2.78103E-03 for the corresponding noise interferences, i.e., u=0.0, u=0.00015, u=0.0015, u=0.015, and u=0.15, as presented in Fig. S1. It has been observed that the change in the order of nonlinearity in the proposed scheme increases the complexity. The system convergence for the no-noise scenario shows that the system becomes stable while approaching 1200 iterations in achieving the best fitness value. For u=0.00015, the u=0.0015 system becomes stable over 800 iterations and no further convergence is recorded. For the noise scenarios u=0.015 and u=0.15, the system achieves stability over 200 iterations in approaching the corresponding best fitness values. Increasing the complexity by changing the order of nonlinearity affects the performance of the proposed scheme; still, EEFOA's performance is commendable. However, increasing the noise level decreases the accuracy in defining the natural behavior of the system.

The fitness results of 60 autonomous executions of the EEFOA for case study 2's fractional-HARX identification problem are presented in Fig. S2 The results indicate that the proposed EEFOA provides reasonable fitness values for all noise variations, i.e., u=0.0, 0.00015, 0.0015, 0.015, and 0.15. Further, it is seen that the fitness values decrease with increasing noise level but, even then, the proposed EEFOA attains a fitness value of around 10^{-2} for all 60 executions of the scheme. The results presented in Fig. S2 confirm the consistent outcome of the EEFOA for the fractional-HARX identification problem considered in case study 2. However, fluctuations are observed in the different autonomous executions; therefore, in order to further investigate EEFOA's performance, statistical values are also given in Table S1.

Table S1 Estimated parameters for all noise scenarios of case study 2

Noise levels	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
u=0	0.4500	0.2900	0.1600	1.0000	0.5600	0.3599	0.8000
u=0.00015	0.4502	0.2904	0.1601	0.9980	0.5585	0.3565	0.7982
u=0.0015	0.4500	0.2900	0.1588	0.9945	0.5574	0.3520	0.7955
u=0.015	0.4499	0.2882	0.1617	0.9764	0.5361	0.3210	0.7838
u=0.15	0.4271	0.2482	0.1253	0.9011	0.4543	0.2168	0.7443

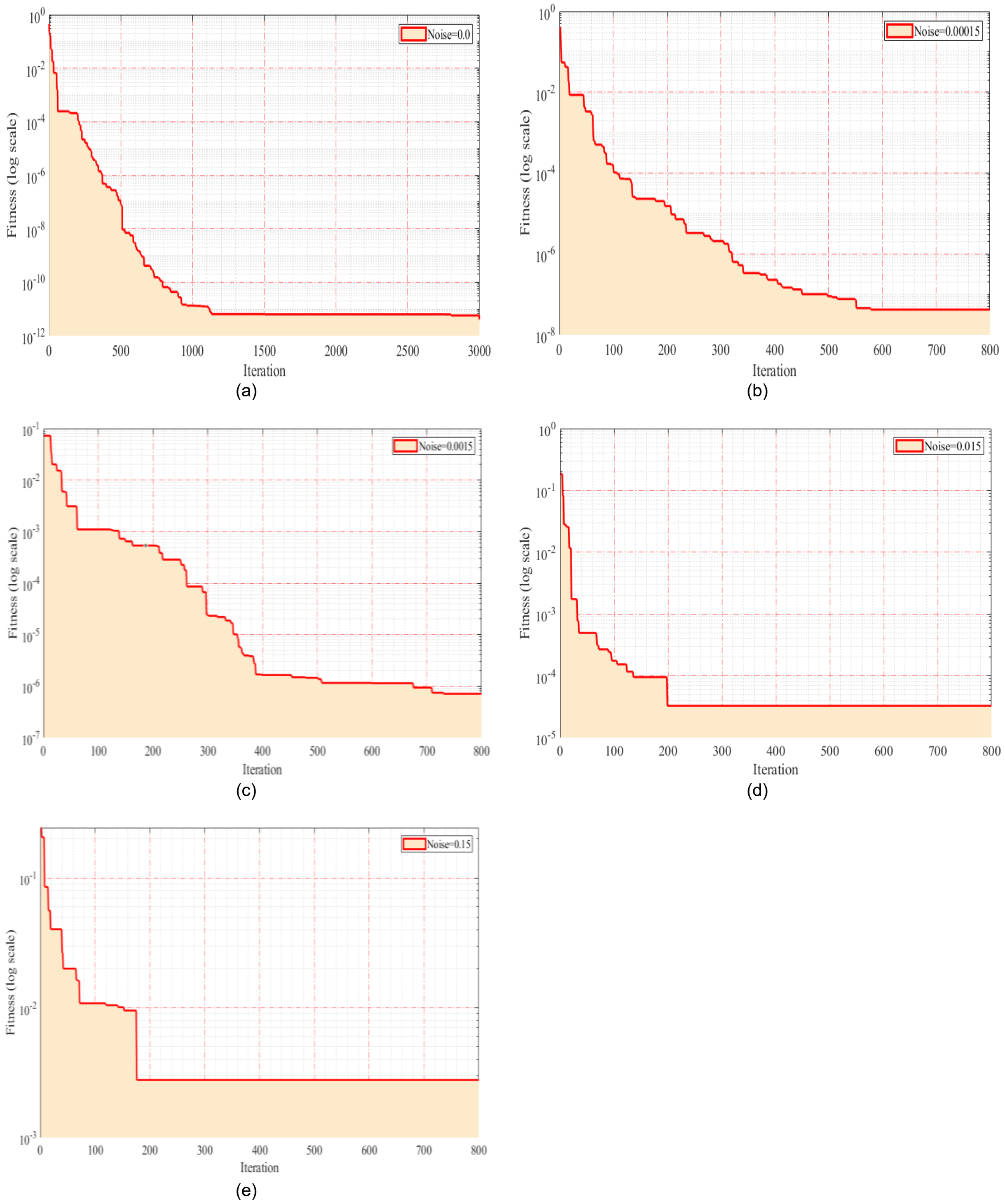


Fig. S1 Case study 2: iterative fitness curves of the best independent execution for all noise scenarios: (a) noise=0; (b) noise=0.00015; (c) noise = 0.0015; (d) noise=0.015; (e) noise=0.15

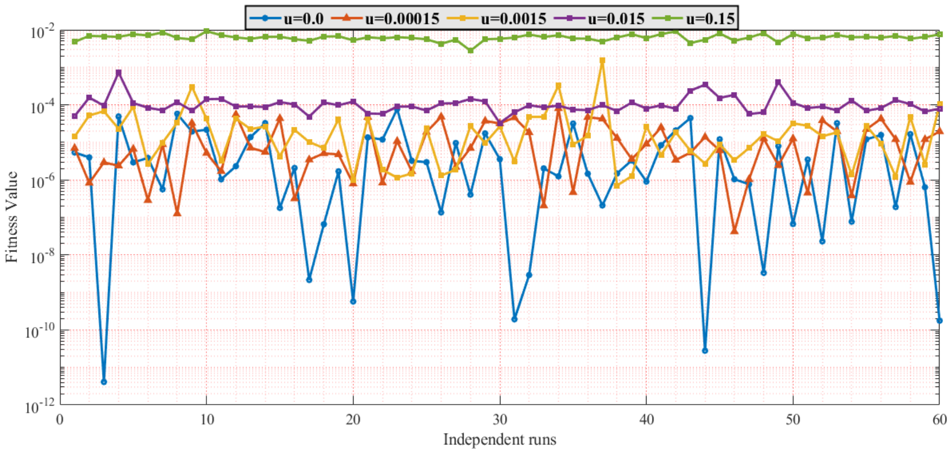


Fig. S2 Case study 2: statistical analysis over 60 independent runs

To evaluate EEFOA's performance in terms of accuracy, the estimated parameters have been plotted in Figs. S3–S7 for different noise interference scenarios. It has been observed that, in the no-noise scenario, the accuracy of the system parameters is perfect. This illustrates EEFOA's speed in estimating parameters and approaching the steady state. Similarly**,** when we increase the noise up to *u*=0.0015, the EEFOA still provides accurate parameter estimation to a very close value. In high noise scenarios at levels *u*=0.015 and *u*=0.15, the accuracy is disturbed as observed in Figs. S3–S7 and Table S3.

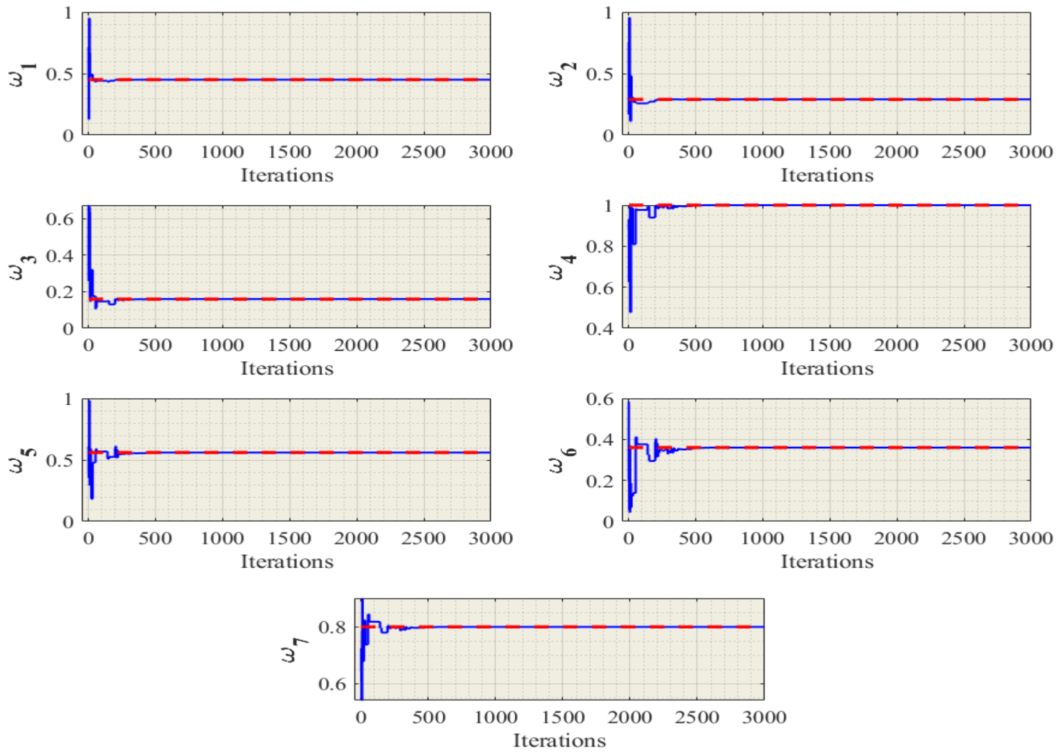


Fig. S3 Case study 2: parameter estimation curves for the best independent run for *u*=0

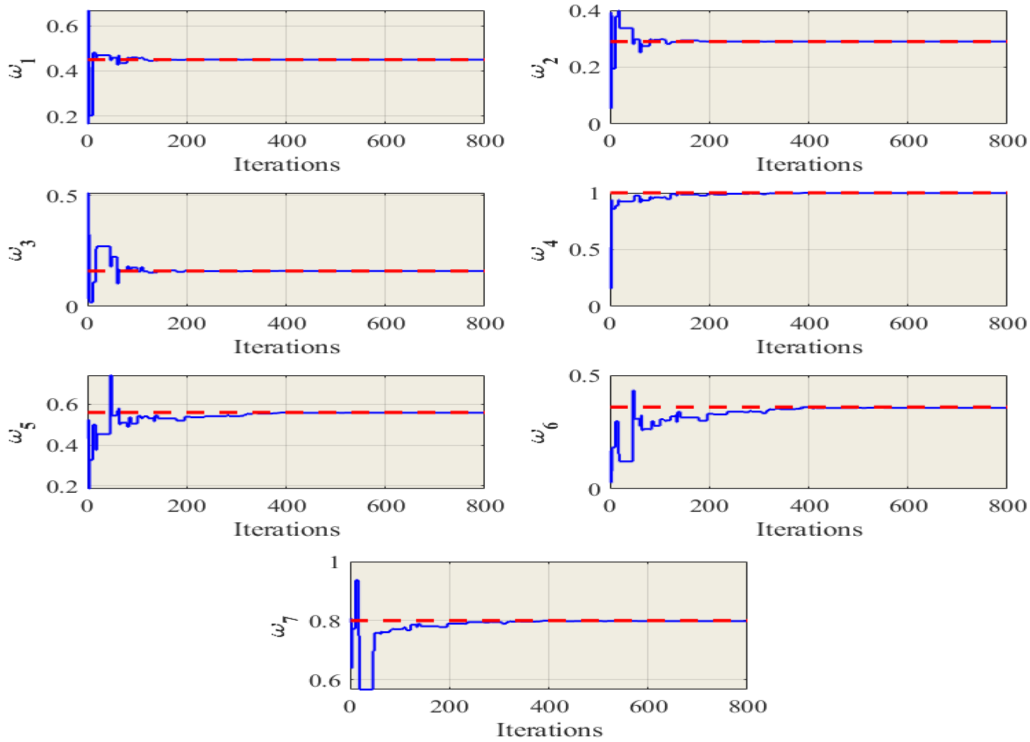


Fig. S4 Case study 2: parameter estimation curves for the best independent run for *u*=0.00015

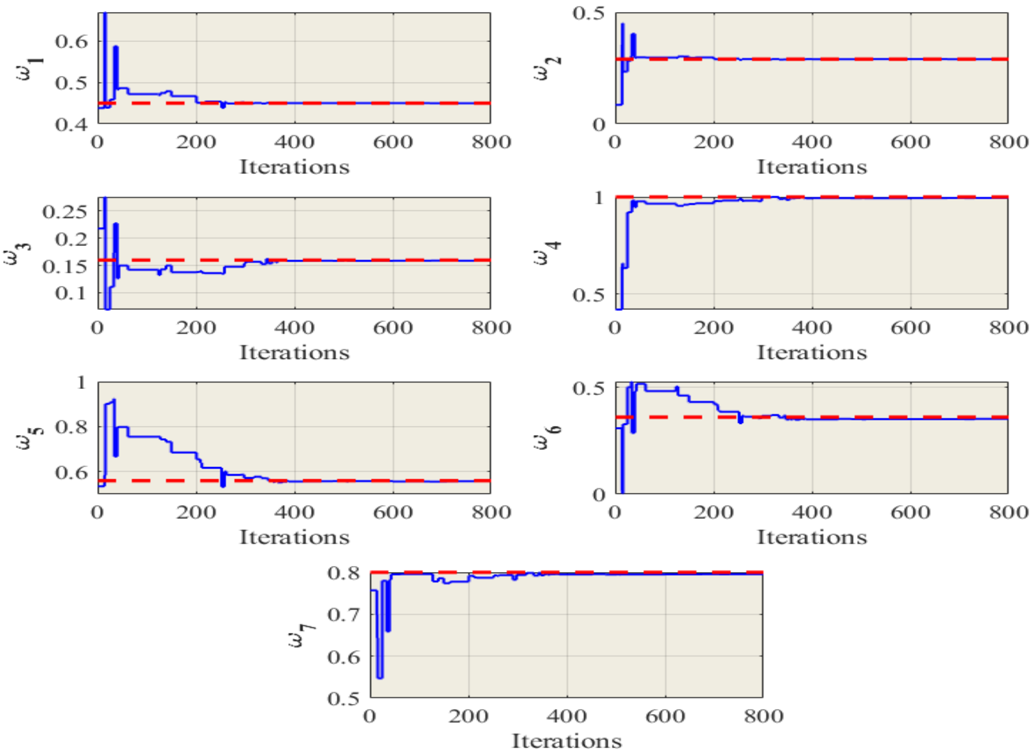


Fig. S5 Case study 2: parameter estimation curves for the best independent run for *u*=0.0015

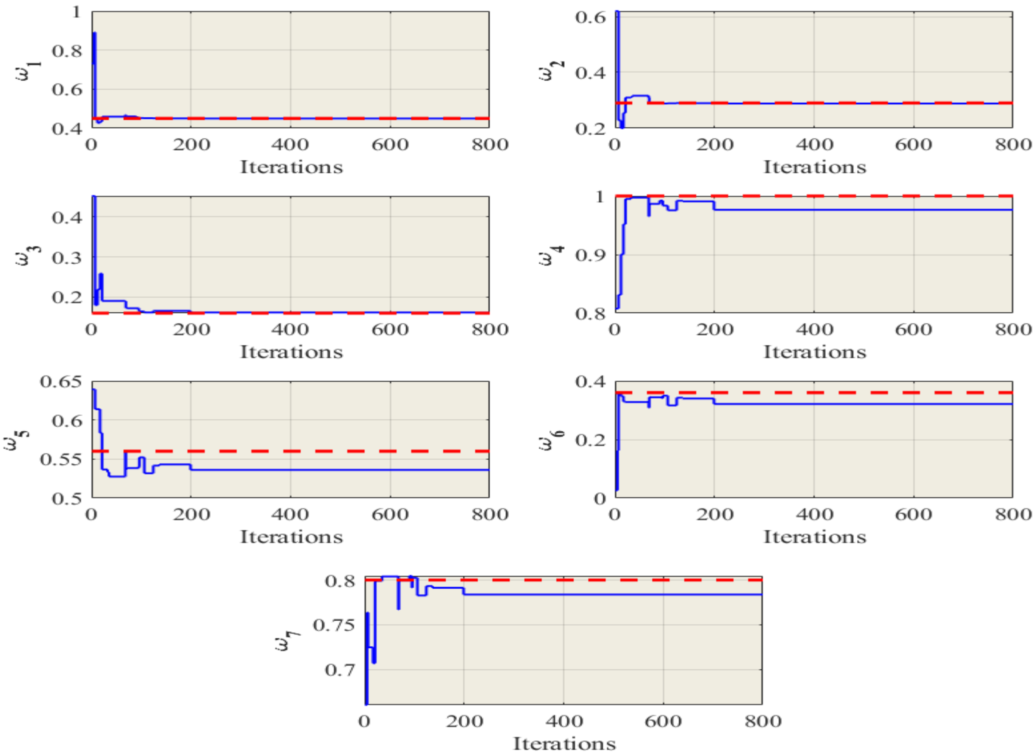


Fig. S6 Case study 2: parameter estimation curves for the best independent run for *u*=0.015

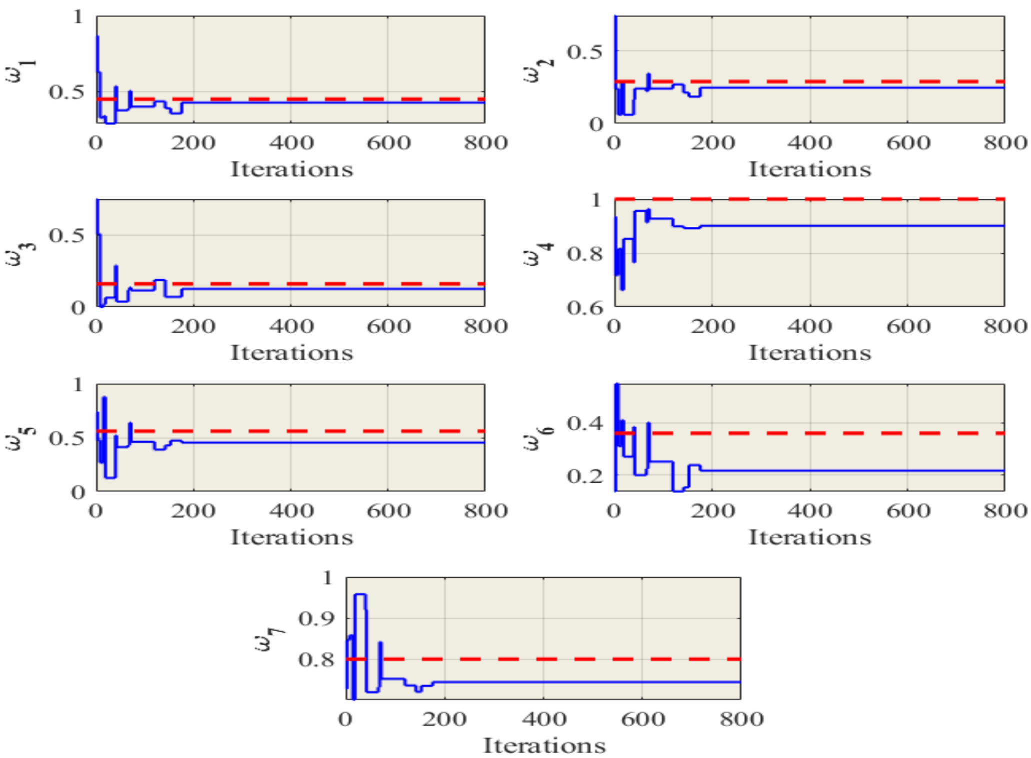


Fig. S7 Case study 2: parameter estimation curves for the best independent run for *u*=0.15

To further analyze the effectiveness of EEFOA, the average accuracy for all the noise scenarios is presented in bar charts in Fig. S8. Additionally, the performance statistics for all five noise scenarios are presented in Table

S2. The bar chart representation and the statistics table indicate EEFOA's stability and consistency. The trend shows that the bar chart follows the average accuracy and that the mean fitness value decreases with increasing noise interference. The mean fitness values for the no-noise scenario are 9.62590E-06, while for noise values u=0.00015, u=0.0015, u=0.015, and u=0.15, the mean fitness values achieved are 1.51770E-05, 5.62263E-05, 1.17199E-04, and 6.33318E-03, respectively, as mentioned in Table S2.

Table S2 Comparison: ranks of fitness, from the best to the worst, along with standard deviations for case study 2

Noise levels	Best fitness	Mean fitness	Worst fitness	Standard deviation
u=0	4.11393E-12	9.62590E-06	7.72156E-05	1.56884E-05
u=0.00015	4.19625E-08	1.51770E-05	8.02964E-05	1.80374E-05
u=0.0015	7.09322E-07	5.62263E-05	1.56890E-03	2.06801E-04
u=0.015	3.27075E-05	1.17199E-04	7.29374E-04	1.01215E-04
u=0.15	2.78103E-03	6.33318E-03	9.18258E-03	1.15436E-03

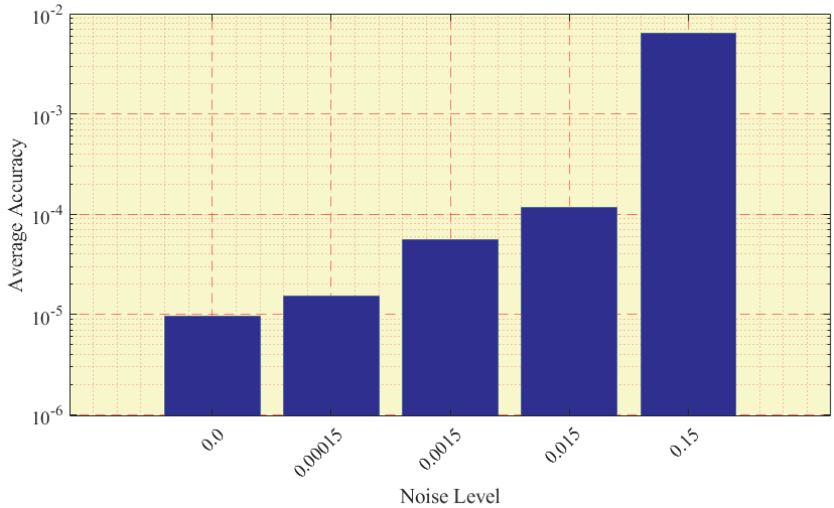
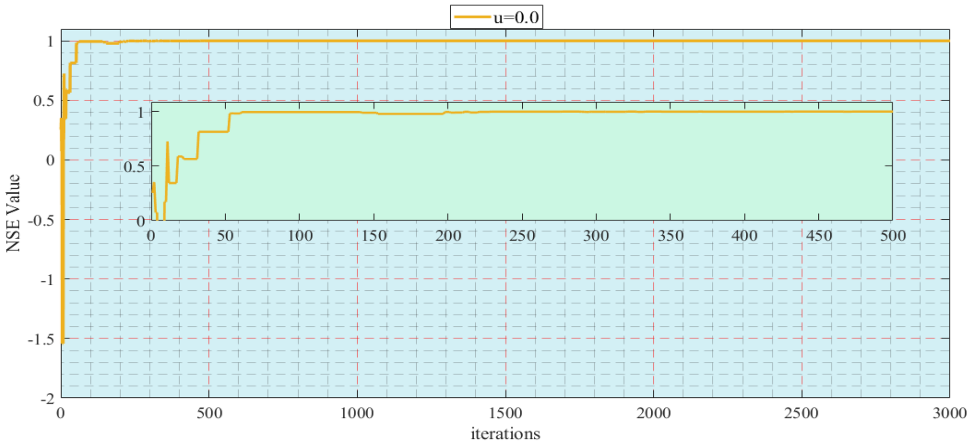
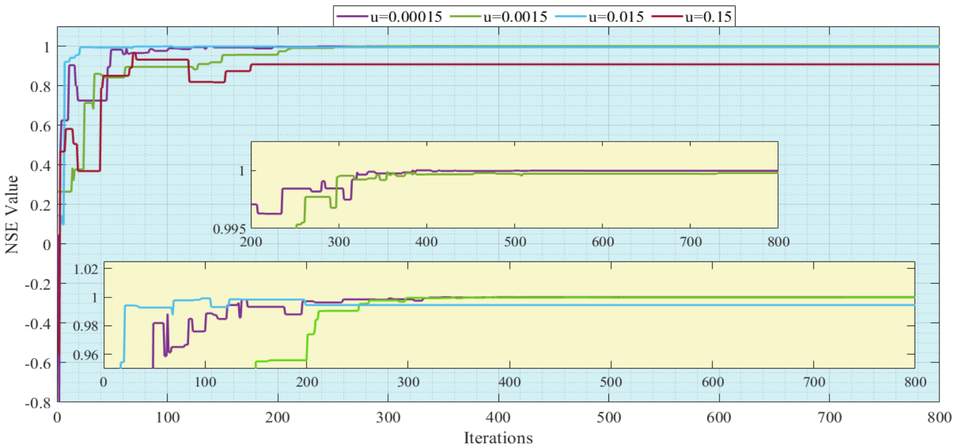


Fig. S8 Average accuracy bar chart for all disturbance variations of case study 2

The Nash–Sutcliffe efficiency (NSE) analysis was carried out to evaluate EEFOA's efficiency by comparing the estimated system response of the model with the actual response. Fig. S9 represents the NSE convergence plots of EEFOA, giving NSE values equal to 1 for the no-noise scenario, illustrating a perfect match in terms of NSE. However, for higher order values of noise other than the maximum noise level of u=0.15, the NSE value remains between 0.99 and 1, demonstrating the accuracy of EEFOA for the fractional-HARX model.



(a)



(b)

Fig. S9 NSE-based convergence curves for case study 2: (a) no noise scenario; (b) noisy scenario

NSE-based statistical plots for 60 independent executions are presented in Fig. S10. The statistical analysis demonstrates EEFOA's stable and unfluctuating behavior. The stability is endorsed by the average NSE value achieved over 60 independent executions, i.e., [0.9606], [0.9513], [0.9294], [0.9170], and [0.8628] for u=0.0, u=0.00015, u=0.0015, u=0.015, and u=0.15 correspondingly. However, increasing noise affects the average NSE value but a value close to 1 evidences EEFOA's promising performance for fractional-HARX. Fig. S11 presents a comparison of the designed methodology with its state-of-the-art counterparts, including the whale optimization algorithm (WOA), the African vulture optimization algorithm (AVOA), Harris hawk's optimizer (HHO), and the reptile search algorithm (RSA). It can be seen that the proposed EEFOA outperforms its counterparts for all noise scenarios of a fractional-HARX system with 3rd order nonlinearity. The proposed EEFOA is also analyzed in terms of its computational complexity by calculating an average execution time and the corresponding standard deviation for both case studies and all scenarios of the fractional-HARX system. The results are presented in Table S3 and indicate that the computational time increases by increasing the iterations in the EEFOA. In addition, the EEFOA requires a greater computational budget for greater orders of nonlinearity or degrees of freedom in the fractional-HARX system.

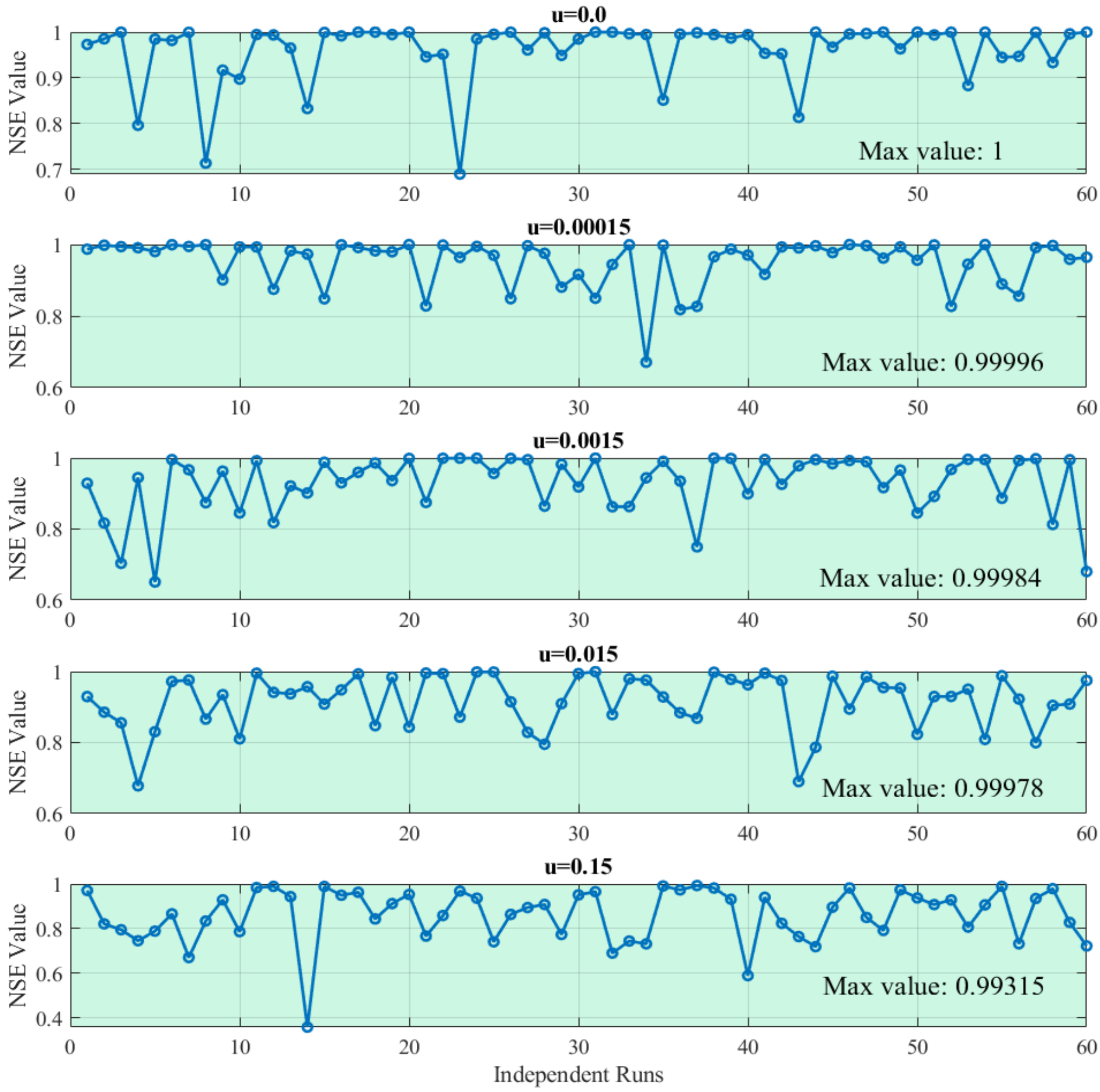


Fig. S10 NSE-based statistical analysis over 60 independent runs of case study 2

Table S3 EEFOA's computational complexity analysis

Time complexity				
Noise level	**Case study**	**Iterations**	**Average time (sec)**	**Standard deviation**
u=0.00015	01	500	6.519	0.138
u=0.0015			6.621	0.138
u=0.015			6.511	0.030
u=0.15			6.651	0.121
u=0.00015	02	800	11.898	0.088
u=0.0015			11.923	0.045
u=0.015			11.857	0.044
u=0.15			11.908	0.058

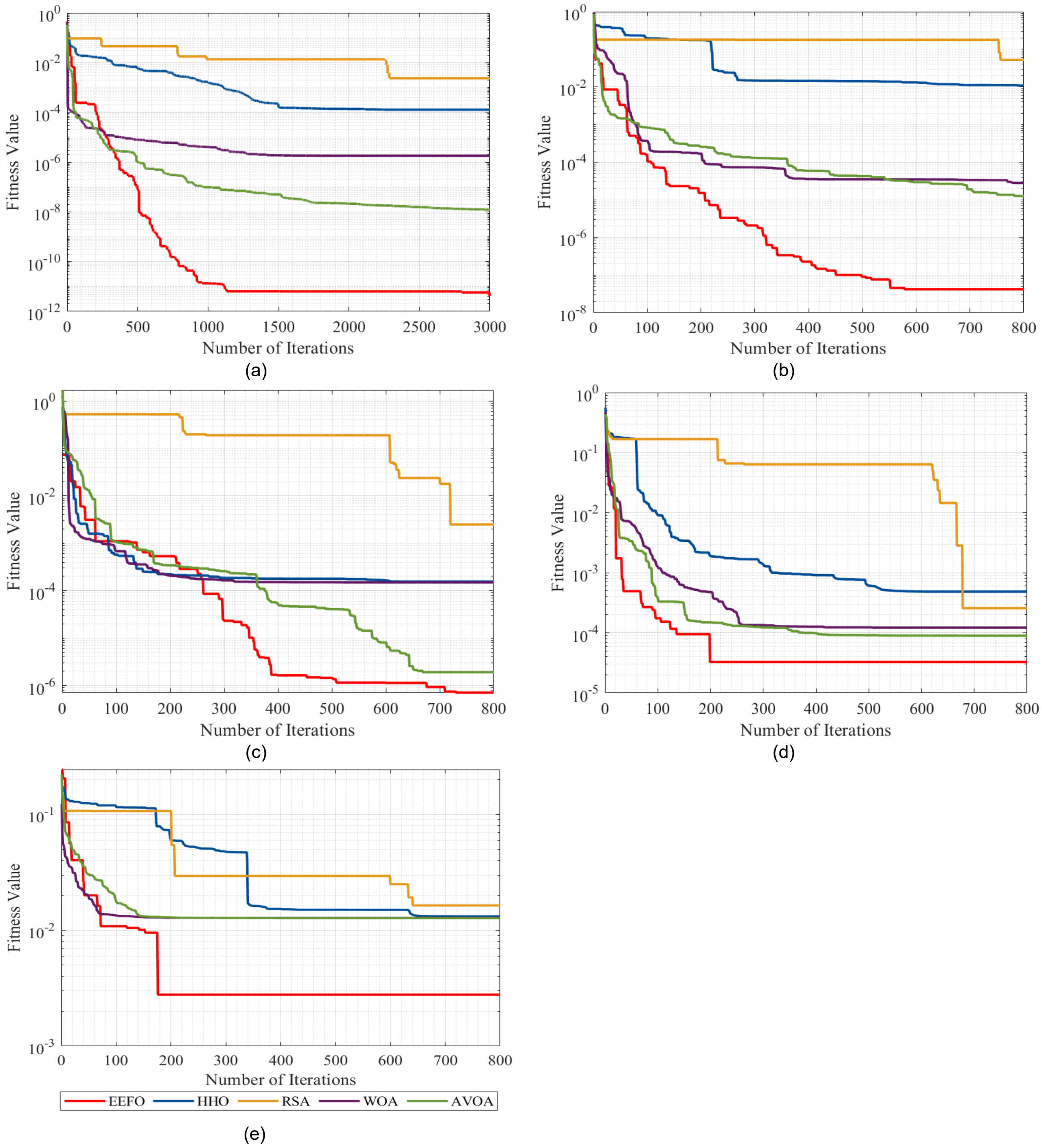


Fig. S11 Comparison between EEFOA and recent equivalent algorithms for case study 2: (a) *u*=0.0; (b) *u*=0.00015; (c) *u*=0.0015; (d) *u*=0.015; (e) *u*=0.15