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Supplementary materials for

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Proof of Theorem 1

On the basis of the selected V, shown as (15), then, take derivative of it with respect to time, one will lead to

$$\begin{aligned}\dot{V} &= s_\chi \dot{s}_\chi \\ &= -s_\chi \varepsilon_{\chi F} sat(s_\chi) - k_{\chi F} s_\chi^2 + s_\chi \bar{d}_a.\end{aligned} \tag{S1}$$

Considering that the exponential approach law (11) affects the performance of the controlled system directly, corresponding analysis for stability within different range of s_χ is the key problem. To tackle the above situation, three cases are put forward.

Case 1: $|s_\chi| \le \varDelta_\chi$, $sat(s_\chi) = s_\chi/\varDelta_\chi$.

Based on (16) and the condition of $\varepsilon_{\chi_F} > 0$, we can obtain

$$\frac{\bar{d}_a}{s_\chi} < \frac{\varepsilon_{\chi_F} + \bar{d}_a}{s_\chi} \le k_{\chi_F}. \tag{S2}$$

Substitute (11) into (S1), it will lead

$$\dot{V} = {s_\chi}^2 (-\frac{\varepsilon_{\chi_F}}{\varDelta_\chi} - k_{\chi_F} + \frac{\bar{d}_a}{s_\chi}), \tag{S3}$$

where, $\varDelta_\chi > 0$, $k_{\chi_F} > 0$, afterwards, it is clear that $-\frac{\varepsilon_{\chi_F}}{\varDelta_\chi}$ is non-positive, accordingly. Hence, the following inequality:

$$\dot{V} \le {s_\chi}^2 (-\frac{\varepsilon_{\chi_F}}{\varDelta_\chi} - k_{\chi_F} + \frac{\bar{d}_a + \varepsilon_{\chi_F}}{s_\chi}) \le 0 \tag{S4}$$

is hold.

Case 2: $s_\chi > \varDelta_\chi$, $sat(s_\chi) = 1$.

Given that $\varepsilon_{\chi_F} > 0$, then, after appropriate reduction, one can obtain

$$\frac{\bar{d}_a - \varepsilon_{\chi_F}}{s_\chi} < \frac{\bar{d}_a + \varepsilon_{\chi_F}}{s_\chi} \le k_{\chi_F}, \tag{S5}$$

along with the calculation result based on (S1)

$$\dot{V} = {s_\chi}^2 (\frac{\bar{d}_a - \varepsilon_{\chi_F}}{s_\chi} - k_{\chi_F}), \tag{S6}$$

then, we can acquire

$$\dot{V} \le {s_\chi}^2 (\frac{\bar{d}_a + \varepsilon_{\chi_F}}{s_\chi} - k_{\chi_F}) \le 0 \tag{S7}$$

using (S5) and (S6).

Case 3: $s_\chi < -\Delta_\chi$, $sat(s_\chi) = -1$.

Substitute (16) into (S1), we also can draw the conclusion directly:

$$\dot{V} = {s_\chi}^2 \left(\frac{\varepsilon_{\chi_F} + \bar{d}_a}{s_\chi} - k_{\chi_F} \right) \leq 0 \tag{S8}$$

Based on the above three analyses, regardless of the type of sliding surface selected, the system state can ultimately reach a stable state.