Electronic Supplementary Materials

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Deformation and stability of the seawall, considering the strength uncertainty of cement mixing piles

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Section S1

Refer to El-Kadi and Williams (2000), the strength of the cement soil within an individual CM pile is assumed to follow a normal distribution as Eq. (S1).

$$f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{S1}$$

where, the random variable x, in this study, represents the undrained shear strength; μ and σ represent the mean and standard deviation, respectively.

The autocorrelation function is given by Eq. (S2) in 2-D conditions. In the one-dimensional case, it degenerates to Eq. (S3).

$$\boldsymbol{\rho}_{m\times m},\rho_{i,j}=\exp\left[-\sqrt{\left(\frac{d_{i,j}^{x}}{L_x}\right)^2+\left(\frac{d_{i,j}^{z}}{L_z}\right)^2}\right] \tag{S2}$$

$$\rho_{i,j}=\exp\left(-\frac{d_{i,j}^{z}}{L_z}\right) \tag{S3}$$

where, L_z is the correlation distance in the z direction, which is set to be 1m; $d_{i,j}^{z}$ is the distance in the z direction between point i and point j. Once the autocorrelation matrix $\boldsymbol{\rho}_{m\times m}$ is obtained, the corresponding covariance matrix $\boldsymbol{C}_{m\times m}$ can be calculated using the following formula:

$$\boldsymbol{C}_{m\times m},C_{i,j}=\sigma^2\rho_{i,j} \tag{S4}$$

The upper triangular matrix $\boldsymbol{s}$ is obtained through standard Cholesky decomposition, such that $\boldsymbol{C}=\boldsymbol{s}\boldsymbol{s}^{\mathrm{T}}$.

The recursive expression for Cholesky decomposition is shown in Eq. (S5), where c_{ij} represents the element in the row *i* and column *j* of the original matrix $\boldsymbol{C}$.

$$\boldsymbol{s}, s_{ij} = \begin{cases} \left(c_{ij} - \sum_{k=1}^{j-1} s_{jk}^2\right)^{1/2} & i = j \\ \frac{1}{s_{jj}}\left(c_{ij} - \sum_{k=1}^{j-1} s_{ik}s_{jk}\right) & i > j \\ 0 & i < j \end{cases} \tag{S5}$$

The expression for the non-drained shear strength random field at each point in space can be obtained by superimposing the mean and fluctuation components, as shown below:

$$\boldsymbol{s}_u = \boldsymbol{s}\epsilon + \boldsymbol{\mu} \tag{S6}$$

where, ϵ is a vector that follows a standard normal distribution.

Based on this, an improvement method was proposed by Zhu et al. (2017), where the average strength changes linearly with depth, as shown in Eq. (S7). Although the mean strength at each depths *z* is different, within the range of the strength random field, the standard deviation of the undrained shear strength remains constant, and that is to say $\sigma_{\ln c_{zi}} = \sigma_{\ln c_{0i}} = \text{const}$.

$$\mu_{c_{uz}} = \mu_{c_{u0}} + \rho z \tag{S7}$$

where, $\mu_{c_{uz}}$ represents the strength mean at depth *z*, $\mu_{c_{u0}}$ represents the surface soil strength mean, and ρ represents the gradient of the undrained shear strength with increasing depth.

Finally, we can obtain Eq. (S8).

$$\begin{aligned} c_{zi} &= \exp\left(\frac{\ln c_{0i} - \mu_{\ln c_{0i}}}{\sigma_{\ln c_{0i}}} \times \sigma_{\ln c_{zi}} + \mu_{\ln c_{zi}}\right) = \exp\left(\frac{\ln c_{0i} - \mu_{\ln c_{0i}}}{\sigma_{\ln c_{0i}}} \times \sigma_{\ln c_{0i}} + \mu_{\ln c_{zi}}\right) \\ &= \exp\left(\ln c_{0i} - \mu_{\ln c_{0i}} + \mu_{\ln c_{zi}}\right) = \exp\left(\ln c_{0i} - \mu_{\ln c_{0i}} + \ln \mu_{c_{zi}} - \frac{1}{2}\sigma_{\ln c_{zi}}^2\right) \\ &= \exp\left(\ln c_{0i} - \mu_{\ln c_{0i}} + \ln \mu_{c_{zi}} - \frac{1}{2}\sigma_{\ln c_{0i}}^2\right) = \exp\left(\ln c_{0i} - \ln \mu_{c_{0i}} + \ln \mu_{c_{zi}}\right) \\ &= c_{0i}\frac{\mu_{c_{zi}}}{\mu_{c0i}} = c_{0i}\frac{\mu_{c_{u0}} + \rho z}{\mu_{c_{u0}}} \end{aligned} \tag{S8}$$