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Frequency-domain analysis of fluid-structure interaction in aircraft hydraulic pipeline systems: numerical and experimental studies

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S1 Fourteen equations model of FSI for straight pipe

Axial force, pressure and velocities:

$$\frac{1}{\rho_f}\frac{\partial p}{\partial z}+\frac{\partial v_z}{\partial t}+\frac{2\tau_0(t)}{r_i\rho_f}=0, \tag{S1}$$

$$\frac{1}{K^*}\frac{\partial p}{\partial t}-\frac{2\mu}{EA_p}\frac{\partial f_z}{\partial t}+\frac{\partial v_f}{\partial z}=0, \tag{S2}$$

$$\frac{\partial f_z}{\partial z}-\rho_p A_p\frac{\partial \dot{u}_z}{\partial t}+2\pi r_i\tau_0(t)=0, \tag{S3}$$

$$\frac{\partial f_z}{\partial t}-A_p\mu\frac{r}{e}\frac{\partial p}{\partial t}-EA_p\frac{\partial \dot{u}_z}{\partial z}=0. \tag{S4}$$

Shear and bending in the *y*-*z* plane:

$$\frac{\partial f_y}{\partial z}-(\rho_f A_f+\rho_p A_p)\frac{\partial \dot{u}_y}{\partial t}=0, \tag{S5}$$

$$\frac{\partial f_y}{\partial t}-\kappa_G GA_p(\frac{\partial \dot{u}_y}{\partial z}+\dot{\varphi}_x)=0, \tag{S6}$$

$$\frac{\partial m_x}{\partial z}-(\rho_f I_f+\rho_p I_p)\frac{\partial \dot{\varphi}_x}{\partial t}-f_y=0, \tag{S7}$$

$$\frac{\partial m_x}{\partial t}-EI_p\frac{\partial \dot{\varphi}_x}{\partial z}=0. \tag{S8}$$

Shear and bending in the *x*-*z* plane:

$$\frac{\partial f_x}{\partial z} - (\rho_f A_f + \rho_p A_p)\frac{\partial \dot{u}_x}{\partial t} = 0\,, \tag{S9}$$

$$\frac{\partial f_x}{\partial t} - \kappa_G G A_p (\frac{\partial \dot{u}_x}{\partial z} - \dot{\varphi}_y) = 0\,, \tag{S10}$$

$$\frac{\partial m_y}{\partial z} - (\rho_f I_f + \rho_p I_p)\frac{\partial \dot{\varphi}_y}{\partial t} + f_x = 0\,, \tag{S11}$$

$$\frac{\partial m_y}{\partial t} - EI_p \frac{\partial \dot{\varphi}_y}{\partial z} = 0\,. \tag{S12}$$

Torsion in the pipe wall:

$$\frac{\partial m_z}{\partial z} - \rho_p J_p \frac{\partial \dot{\varphi}_z}{\partial t} = 0\,, \tag{S13}$$

$$\frac{\partial m_z}{\partial t} - GJ_p \frac{\partial \dot{\varphi}_z}{\partial z} = 0\,, \tag{S14}$$

where $\frac{1}{K^*} = \frac{1}{K} + \frac{2r_i}{Ee}$, $\kappa_G = \frac{2(1+\mu)}{4+3\mu}$, $I_f = \frac{\pi r_i^4}{4}$, $I_p = \frac{\pi(r_o^4 - r_i^4)}{4}$, $J_p = \frac{A_p(r_o^2 + r_i^2)}{2}$.

S2 Fourteen equations model of FSI for curved pipe

Axial stress, pressure and velocities:

$$\frac{1}{K^*}\frac{\partial p}{\partial t} + \frac{\partial v_f}{\partial l} - \frac{2\mu}{EA_p}\frac{\partial f_z}{\partial t} + \frac{\dot{u}_y}{R} = 0\,, \tag{S15}$$

$$\frac{1}{\rho_f}\frac{\partial p}{\partial l} + \frac{\partial v_f}{\partial t} + \frac{2\tau_0(t)}{r_i \rho_f} = 0\,, \tag{S16}$$

$$\frac{\partial f_z}{\partial l} - \rho_p A_p \frac{\partial \dot{u}_z}{\partial t} + \frac{f_y}{R} + 2\pi r_i \tau_0(t) = 0\,, \tag{S17}$$

$$\frac{\partial f_z}{\partial t} - A_p \mu \frac{r}{e}\frac{\partial p}{\partial t} - EA_p \frac{\partial \dot{u}_z}{\partial l} - EA_p \frac{\dot{u}_y}{R} = 0\,, \tag{S18}$$

where $\frac{1}{K^*} = \frac{1}{K} + \frac{2r}{Ee}$.

Shear and bending in the *y*-*z* plane:

$$\frac{\partial f_y}{\partial l} - (\rho_f A_f + \rho_p A_p)\frac{\partial \dot{u}_y}{\partial t} - \frac{f_z}{R} + \frac{A_f}{R} p = 0\,, \tag{S19}$$

$$\frac{\partial f_y}{\partial t} - \kappa_G G A_p (\frac{\partial \dot{u}_y}{\partial l} + \dot{\varphi}_x + \frac{\dot{u}_z}{R}) = 0\,, \tag{S20}$$

$$\frac{\partial m_x}{\partial l} - (\rho_p I_p + \rho_f I_f)\frac{\partial \dot{\varphi}_x}{\partial t} - f_y = 0\,, \tag{S21}$$

$$\frac{\partial m_x}{\partial t} - \kappa_E EI_p \frac{\partial \dot{\varphi}_x}{\partial l} = 0\,, \tag{S22}$$

where $\kappa_G = \dfrac{2(1+\mu)}{4+3\mu}$, $I_f = \dfrac{\pi r_i^4}{4}$, $I_p = \dfrac{\pi(r_o^4 - r_i^4)}{4}$.

Shear and bending in the *x-z* plane:

$$\frac{\partial f_x}{\partial l} - (\rho_f A_f + \rho_p A_p)\frac{\partial \dot{u}_x}{\partial t} = 0 , \tag{S23}$$

$$\frac{\partial f_x}{\partial t} - \kappa_G G A_p (\frac{\partial \dot{u}_x}{\partial l} - \dot{\varphi}_y)=0 , \tag{S24}$$

$$\frac{\partial m_y}{\partial l} - (\rho_p I_p + \rho_f I_f)\frac{\partial \dot{\varphi}_y}{\partial t} + f_x - \frac{m_z}{R} = 0 , \tag{S25}$$

$$\frac{\partial m_y}{\partial t} - \kappa_E E I_p (\frac{\partial \dot{\varphi}_y}{\partial l} - \frac{\dot{\varphi}_z}{R}) = 0 . \tag{S26}$$

Torsion in the pipe wall:

$$\frac{\partial m_z}{\partial l} - \rho_p J_p \frac{\partial \dot{\varphi}_z}{\partial t} + \frac{m_y}{R} = 0 , \tag{S27}$$

$$\frac{\partial m_z}{\partial t} - G J_p \frac{\partial \dot{\varphi}_z}{\partial l} - \frac{\dot{\varphi}_y}{R} = 0 , \tag{S28}$$

where $J_p = \dfrac{A_p(r_o^2 + r_i^2)}{2}$, $\kappa_E = \dfrac{\lambda}{1.65}$ and $\lambda = \dfrac{e_p R}{r^2}$.

S3 Derivation of valve's impedance

The equation for the flow pressure at the throttle can be expressed as

$$Q_v = C_q A_c \sqrt{\frac{2P_v}{\rho}} , \tag{S29}$$

where P_v is the pressure difference between the front and back of the valve, and Q_v is the flow rate through the valve, C_q is flow coefficient and A_c is flow area.

Then this expression can be linearized at the mean pressure difference P_v^0 as

$$\frac{\Delta Q_v}{\Delta P_v} = \frac{1}{2} C_q A_c \sqrt{\frac{2}{\rho}} \frac{1}{\sqrt{P_v}}\bigg|_{P_v = P_v^0} , \tag{S30}$$

and its Laplace transform can be derived as

$$\frac{Q_v(s)}{P_v(s)} = \frac{1}{2} C_q A_c \sqrt{\frac{2}{\rho}} \frac{1}{\sqrt{P_v^0}} . \tag{S31}$$

Considering the stable status, the relation between the mean pressure difference and mean flow rate still follows Eq. (S29) here, yielding

$$Q_v^0 = C_q A_c \sqrt{\frac{2P_v^0}{\rho}} \, . \tag{S32}$$

By submitting Eq. (S32) into Eq. (S31), one may have

$$\frac{P_v(s)}{Q_v(s)} = 2\frac{P_v^0}{Q_v^0} \, , \tag{S33}$$

which is the hydraulic impedance.