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Multi-scale analysis of the self-vibration of a liquid crystal elastomer fiber-spring system exposed to constant-gradient light

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Section S1: Derivations of Eq. (8)

Combining Eqs. (4)-(7), we have

$$\bar{\tau}\frac{\mathrm{d}\bar{T}(\bar{X},\bar{t})}{\mathrm{d}\bar{t}}=\bar{b}\left\{\bar{X}\left[\bar{w}(t)-\bar{C}\int_0^1\bar{T}(\bar{X},\bar{t})d\bar{X}\right]+C\int_0^{\bar{X}}\bar{T}(\bar{X},\bar{t})d\bar{X}+\bar{X}\right\}+\bar{Q}-\bar{T}(\bar{X},\bar{t}) \tag{S1}$$

and the tension of the LCE fiber is:

$$\bar{F}_L=\bar{K}_{\mathrm{L}}\left[\bar{w}(\bar{t})-\bar{C}\int_0^1\bar{T}(\bar{X},\bar{t})d\bar{X}\right] \tag{S2}$$

The temperature field can be written as:

$$\bar{T}(\bar{X},\bar{t})=\bar{T}^{(0)}(\bar{X},\bar{t})+\bar{\tau}\bar{T}^{(1)}(\bar{X},\bar{t})+O(\bar{\tau}^2) \tag{S3}$$

Combining Eqs. (S1)-(S3), we obtain the temperature field in a different form:

$$\bar{T}(\bar{X},\bar{t})=\frac{\bar{b}\left[\bar{w}(\bar{t})+1\right]}{e^{\bar{C}\bar{b}}-1}\left(e^{\bar{C}\bar{b}}-1\right)+\bar{Q}+\bar{\tau}\frac{\bar{b}\dot{w}(\bar{t})}{e^{\bar{C}\bar{b}}-1}\left[\frac{\left(e^{\bar{C}\bar{b}}-1\right)\left(\bar{C}\bar{b}e^{\bar{C}\bar{b}}-e^{\bar{C}\bar{b}}+1\right)}{e^{\bar{C}\bar{b}}-1}-\bar{C}\bar{b}\bar{X}e^{\bar{C}\bar{b}\bar{X}}\right] \tag{S4}$$

Correspondingly, Eq. (S5) can be written as:

$$\bar{F}_{\mathrm{L}}(\bar{t})=\frac{\bar{K}_{\mathrm{L}}\bar{C}\bar{b}}{e^{\bar{C}\bar{b}}-1}\bar{w}(\bar{t})+\bar{K}_{\mathrm{L}}\bar{C}\bar{b}\,\bar{\tau}\frac{1-e^{\bar{C}\bar{b}}+\bar{C}\bar{b}e^{\bar{C}\bar{b}}}{\left(e^{\bar{C}\bar{b}}-1\right)^2}\dot{\bar{w}}(t)+\bar{K}_{\mathrm{L}}\left(\frac{\bar{C}\bar{b}}{e^{\bar{C}\bar{b}}-1}-1-\bar{C}\bar{Q}\right) \tag{S5}$$

Section S2: Derivations of Eqs. (17) and (18)

And by defining $x=\bar{w}+\frac{a_3}{a_2},\omega_0=\sqrt{a_2}$, Eq. (16) can be re-expressed as:

$$\ddot{x}-\varepsilon_0(\dot{x}-a_1|\dot{x}|\dot{x})+{\omega_0}^2x=0 \tag{S6}$$

Utilizing the linear perturbation method, we can derive the linearized equation as follows:

$$\ddot{x} - \varepsilon_0 \dot{x} + {\omega_0}^2 x = 0 \tag{S7}$$

Eq. (S7) can alternatively be written as:

$$\lambda^2 - \varepsilon_0 \lambda + {\omega_0}^2 = 0 \tag{S8}$$

Therefore, the Hurwitz criterion ε_0 is:

$$\tau \frac{\overline{K}_{\mathrm{L}} \overline{C} \overline{b} (e^{\overline{C}\overline{b}} - 1 - \overline{C}\overline{b} e^{\overline{C}\overline{b}})}{\left(e^{\overline{C}\overline{b}} - 1\right)^2} - \overline{\beta}_1 > 0 \tag{S9}$$

By solving the governing equation, the analytical solution can be obtained as follows:

$$\overline{w} = (\frac{3\pi}{8\sqrt{a_2} a_1} + \frac{1}{e^{\frac{1}{2}(\varepsilon t + \omega_0)}}) \cos(\sqrt{a_2} t + \theta_0) - \frac{a_3}{a_2} + o(\varepsilon) \tag{S10}$$

The amplitude is:

$$A = \frac{3\pi}{8\sqrt{\frac{\overline{K}_{\mathrm{L}} \overline{C} \overline{b}}{e^{\overline{C}\overline{b}} - 1} + \overline{K}_{\mathrm{S}}} \frac{\overline{\beta}_2 \left(1 - e^{\overline{C}\overline{b}}\right)^2}{\overline{\tau} \overline{C} \overline{b} \overline{K}_{\mathrm{L}} (e^{\overline{C}\overline{b}} - \overline{C}\overline{b} e^{\overline{C}\overline{b}} - 1) - \overline{\beta}_1 \left(1 - e^{\overline{C}\overline{b}}\right)^2}} \tag{S11}$$

and the frequency is:

$$f = \sqrt{\frac{\overline{K}_{\mathrm{L}} \overline{C} \overline{b}}{e^{\overline{C}\overline{b}} - 1} + \overline{K}_{\mathrm{S}}} \tag{S12}$$