Electronic Supplementary Materials

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Multi-directional wind energy harvesting based on the coupling effect between a piezoelectric beam and an elastic-supported sphere

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Section S1 Derivation of the coupled effect between the piezoelectric beam and sphere

The dynamics of vortex-induced vibration of an elastically mounted sphere are simplified as a linear oscillator, and its motion equation is

$$m_s\ddot{w}_s(t)+c\dot{w}_s(t)+(k_1+k_2)w_s(t)=F_{VIV} \tag{S1}$$

where $w_s(t)$, $\dot{w}_s(t)$, $\ddot{w}_s(t)$ are the displacement, velocity, and acceleration of the sphere, respectively. m_s, c, k_1 and k_2 are the total mass of the sphere and springs, the structural damping and the stiffness of spring 1 and spring 2, respectively. The term F_{VIV} stands for the vortex-induced force exerted by the wind flow on the sphere, and it is defined as

$$F_{VIV}=\frac{\pi C_L\rho_{air}D^2U^2}{16}q(t)-\frac{\pi C_D\rho_{air}D^2U}{8}\dot{w}_s(t) \tag{S2}$$

where C_L denotes the steady mean lift coefficient and C_D denotes the steady mean drag coefficient. U denotes the wind speeds. D represents the diameter of the sphere. ρ_{air} denotes the density of the air. $q(t)$ denotes a fluid–structure coupling term, given by

$$\ddot{q}(t)+\lambda\omega_s\left(q^2(t)-1\right)\dot{q}(t)+\omega_s^2q(t)=\frac{A}{D}\ddot{w}_s(t) \tag{S3}$$

Here, A represents a constant of the van der Pol wake oscillator model, A=23. λ is a constant describing vortex street fluctuation, λ=0.1. Both λ and A are determined by experiment according to Facchinetti[41]. ω_s denotes the vortex shedding frequency. The work of Facchinetti et al.[41] provided us with the theoretical foundation for vortex-induced forces and wake oscillator behavior. Equation (S2) is used to calculate the vortex-induced force, taking into account the periodicity of vortex shedding and the fluid pressure distribution on the surface of the sphere. Equation (S3) describes the dynamic behavior of the vortex-induced force wake oscillator. It considers the mass and damping characteristics of the wake oscillator, as well as the torque induced by the vortex force.

The vortex-induced force is transmitted to the piezoelectric beam through spring 2. According to Hooke's Law, the spring force is given by

$$F_{sp}=k_2(\Delta l_2+w_s(t)-w_b(L,t)) \tag{S4}$$

$$k_1\Delta l_1=k_2\Delta l_2 \tag{S5}$$

where $w_b(L,t)$ denotes the end displacement of the piezoelectric beam. Δl_1 and Δl_2 represent the initial elongation of spring 1 and spring 2, which are given by

$$L_0=l_1+\Delta l_1+D+l_2+\Delta l_2 \tag{S6}$$

where L_0, l_1 and l_2 represent the total height of the sphere-spring system, the length of spring 1, and the length of spring 2, respectively. These values can be obtained through measurements.

Assuming the vibrational response of the piezoelectric beam is denoted as $w_b(x,t)$, it can be expressed as the product of the beam mode function matrix and the generalized time coordinate matrix, given by

$$w_b(x,t)=\sum_{i=1}^{n}\Phi_i(x)Y_i(t) \tag{S7}$$

where $Y_i(t)$ is the mode vector and $\Phi_i(x)$ is the vibration mode amplitude of the piezoelectric beam. The calculation process of Galerkin method refers to the study of Dai et al[42].

The kinetic energy T of the system is given by

$$T=\frac{1}{2}\left[\int_{V_b}\rho_b\left[\frac{\partial w_b(x,t)}{\partial t}\right]^2 \mathrm{d}V_b+\int_{V_p}\rho_p\left[\frac{\partial w_b(x,t)}{\partial t}\right]^2 \mathrm{d}V_p\right] \tag{S8}$$

where V_b and ρ_b represent the volume and the density of the beam. V_p and ρ_p respectively represent the volume and the density of the piezoelectric layer. The potential energy U of the system is expressed as

$$U=\frac{1}{2}\left[\int_{V_b}\sigma_x^b\varepsilon_x^b\mathrm{d}V_b+\int_{V_p}\sigma_x^p\varepsilon_x^p\mathrm{d}V_p-\int_{V_p}E_3D_3\mathrm{d}V_p\right] \tag{S9}$$

where E_3 denotes the electric field induced by the piezoelectric effect. In unimorph piezoelectric layer, it is given by $E_3=-V_P/t_P$. ε_x^b and ε_x^p represent the strains of the beam and the piezoelectric layer in x direction, while σ_x^b and σ_x^p represent the stresses of the beam and the piezoelectric layer in x direction. The strains and stresses of the beam and piezoelectric layers are calculated by

$$\begin{gathered}\varepsilon_x^b=\varepsilon_x^p=\varepsilon_x=-z\frac{\partial w_b(x,t)}{\partial x}\\ \sigma_x^b=E^b\varepsilon_x\\ \sigma_x^p=E^p\left(\varepsilon_x-d_{31}E_3\right)=E^p\varepsilon_x-e_{31}E_3\end{gathered} \tag{S10}$$

where d_{31} denotes the strain coefficient of the piezoelectric layer. e_{31} denotes the piezoelectric stress coefficient and is given by $e_{31}=E^pd_{31}$. D_3 denotes the electric displacement obtained by

$$D_3=d_{31}E^p\varepsilon_x+\varepsilon_{33}E_3=e_{31}\varepsilon_x+\varepsilon_{33}E_3 \tag{S11}$$

where ε_{33} is the permittivity component under constant strain. According to Ohm's law, the voltage $V(t)$ across a resistor is directly proportional to the current passing through it. Considering the definition of current, the following equation is obtained

$$V(t)=R\dot{Q}(t) \tag{S12}$$

Non-conservative work includes the electric work of resistance load R, the work of vortex-induced force F_{VIV} and the work of damping force.

$$\delta W=-V(t)\delta Q(t)+F_{sp}\delta w(t)-\int_0^{L_0}c\frac{\partial w_b(x,t)}{\partial t}\delta w_b(x,t)dx \tag{S13}$$

Equation (S7) is introduced into Equations (S8), (S9), and (S13) to facilitate the discretization process. Based on the Euler-Lagrange equation, where $L=T-U$, the following equations are obtained

$$\begin{gathered}\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{Y}_i}-\frac{\partial L}{\partial Y_i}=\frac{\delta W}{\delta Y_i}\\ \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{Q}}-\frac{\partial L}{\partial Q}=\frac{\delta W}{\delta Q}\end{gathered} \tag{S14}$$

Substituting equations (S1), (S3), (S4) concerning vortex-induced force into the Euler-Lagrange equation (S14), the analysis model of the coupled effect between the piezoelectric beam and sphere is obtained

$$m_s\ddot{w}_s(t)+c\dot{w}_s(t)+(k_1+k_2)w_s(t)=\frac{\pi C_L\rho_{air}D^2U^2}{16}q(t)-\frac{\pi C_D\rho_{air}D^2U}{8}\dot{w}_s(t) \tag{S15}$$

$$\ddot{q}(t)+\lambda\omega_s\left(q^2(t)-1\right)\dot{q}(t)+\omega_s^2 q(t)=\frac{A}{D}\ddot{w}_s(t) \tag{S16}$$

$$F_{sp}=k_2(\Delta l_2+w_s(t)-w_b(L,t)) \tag{S17}$$

$$\ddot{Y}_i+2\varsigma_i\omega_i\dot{Y}_i+\omega_i^2 Y_i-\theta_i V=F_{sp} \tag{S18}$$

$$C_P\dot{V}+\frac{V}{R}+\sum_1^n\theta_i\dot{Y}_i=0 \tag{S19}$$

Section S2 The setups of wind tunnel experiment and the fabrication of the proposed device

In this study, we validated the proposed theoretical model through wind tunnel experiments and investigated the potential for energy harvesting by coupling VIV of a sphere with the oscillations of a piezoelectric cantilever beam. The experiments were conducted in a wind tunnel with dimensions of 260 mm × 260 mm × 2000 mm. Wind speed was controlled by adjusting the fan speed with a velocity controller and was measured precisely using an anemometer. The wind speed during experiments ranged from 2.39 m/s to 9.48 m/s.

The polystyrene foam sphere was transversely supported in the airflow. The sphere's structural stiffness was controlled by springs at both ends, with each spring connected to an upper and lower steel frame. Free decay tests were performed to determine the system's natural frequency and structural damping in air. The displacement of the sphere was recorded at 240 fps and a resolution of 1280×720 pixels using a high-speed camera. Experimental data were analyzed using a custom MATLAB-based machine vision processing program. For each dataset, displacement signals were sampled at 240 Hz over at least 50 vibration cycles to ensure high accuracy, enabling reliable measurements of velocity, acceleration, and frequency.

For energy harvesting experiments involving VIV of the sphere, the experimental setup was adjusted accordingly. Each end of the sphere was attached to a spring; the upper spring was connected to the upper steel frame, and the lower spring was connected to the free end of a cantilevered piezoelectric beam. The beam's fixed end was attached to the steel frame. This setup enabled the VIV of the sphere to generate electrical energy through the periodic bending of the cantilevered piezoelectric beam. The experimental setup consisted of a polystyrene foam sphere, two springs, a flexible aluminum beam substrate, a piezoelectric patch (MFC-M2807P2), and a fixed frame. The generated voltage was monitored using a resistance box and an oscilloscope by adjusting the load resistance on the piezoelectric patch.

Section S3 The expression of the aeroelectromechanical efficiency ψ_{aem} and the electromechanical efficiency ψ_{em}

The aeroelectromechanical efficiency is defined as

$$\Psi_{aem}=\frac{P_{avg}}{\bar{P}_f} \tag{S20}$$

where P_{avg} is the average output power and $\bar{P}_f$ is the input mechanical power extracted from fluid flow. This input mechanical power is calculated by

$$\bar{P}_f=\frac{A_f}{2}\rho_a U^3 \tag{S21}$$

Here, A_f is the fontal area of the harvester in operation and given by

$$A_f=2(w_s+\frac{D}{2})D \tag{S22}$$

The electromechanical efficiency refers to conversion of mechanical vibration power, and is given by

$$\Psi_{em} = \frac{\bar{P}_{avg}}{\bar{P}_m} \tag{S23}$$

where $\bar{P}_m$ is the average mechanical power and is calculated by the instantaneous mechanical power

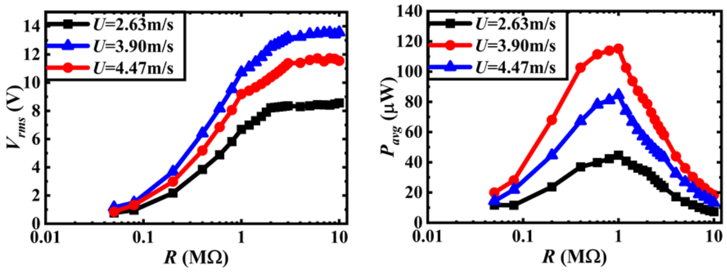
$$P_{\mathrm{m}} = F_{sp}\dot{w}_{\mathrm{b}} \tag{S24}$$

The mechanical energy of the piezoelectric beam is then given by

$$E_{\mathrm{m}} = \int_{t_0}^{t_1} P_{\mathrm{m}} \mathrm{d}t \tag{S25}$$

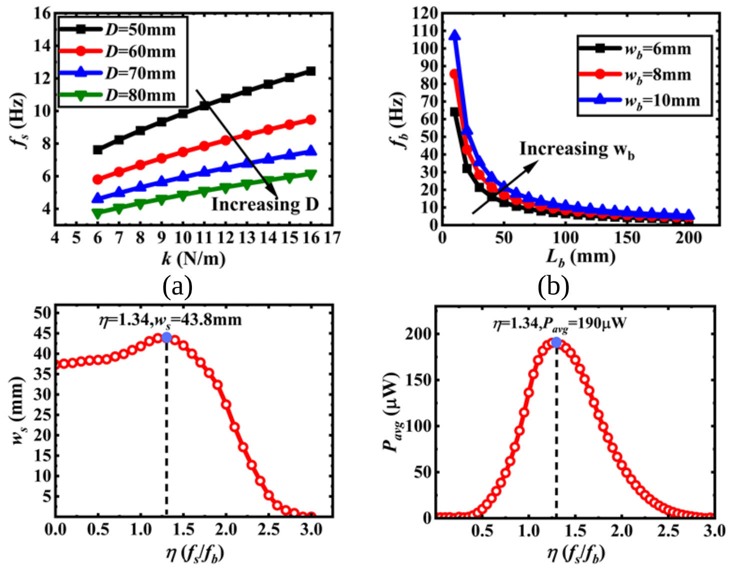
Therefore, the average mechanical power is

$$\bar{P}_{\mathrm{m}} = \frac{E_{\mathrm{m}}}{t_1 - t_0} \tag{S26}$$



(a) (b)

Fig. S1 (a) The RMS voltage and (b) the average power varies with load resistance under three representative wind speeds.



(c) (d)

Fig. S2 (a) The natural frequency of the sphere-spring (f_s) system varies with the stiffness of springs of various diameters. (b) The natural frequency of the piezoelectric beam system (f_b) varies with beam length for various widths. (c) The vibration amplitude of the harvester's sphere varies with dimensionless frequency (η). (d) The output power (P) of the harvester varies with dimensionless frequency (η).

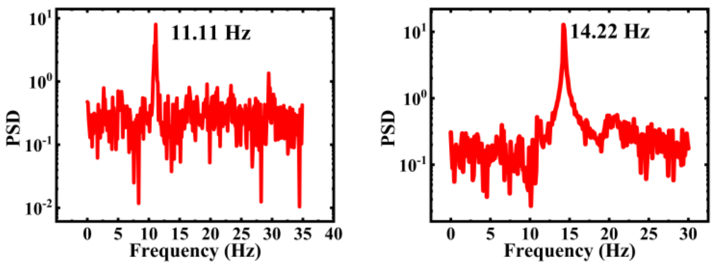


Fig. S3 FFT of the output power signal at a wind speed of U=3.90m/s for various dimensionless frequencies of (a) η=0.569 and (b) η=1.341.