Electronic Supplementary Materials

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Robust self-triggered switching control of autonomous ground vehicles with varying linear parameters

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S1 Equations

$v_f = \delta - \frac{L_f\gamma}{v_x} - \beta, v_r = -\frac{L_r\gamma}{v_x} - \beta$	(S1)
$\begin{gathered} a_{11} = -\frac{K_f + K_r}{mv_x}, a_{12} = -\frac{L_fK_f - L_rK_r + mv_x^2}{mv_x^2} \\ a_{21} = \frac{L_rk_r - L_fk_f}{I_z}, a_{22} = \frac{L_f^2k_f + L_r^2k_r}{v_xI_z} \\ b_{11} = \frac{k_f}{mv_x}, b_{21} = \frac{L_fk_f}{I_z}, b_{22} = \frac{1}{I_z} \end{gathered}$	(S2)
$\rho \in \Phi_{C0} := C_0\{\chi_1, \dots, \chi_N\} = \begin{cases} \sum_{i=1}^{N} \wp_i(t)\chi_i \vert \wp_i(t) \vert \geq 0 \\ \sum_{i=1}^{N} \wp_i(t) = 1 \end{cases}$	(S3)
$[A(\rho), B(\rho)]^{\mathrm{T}} = \sum_{i=1}^{r} \wp_i(t)[A_i, B_i]^{\mathrm{T}}$	(S4)
$\dot{\mathcal{X}}(t) = \mathcal{A}_\sigma\mathcal{X}(t) + \mathcal{B}_\sigma u + \mathcal{C}_\sigma w(t) + \tilde{\Xi}(t)$	(S5)
$V_\sigma(t, \varsigma) = \mathcal{X}^{\mathrm{T}}(t)\Omega_k(t)\mathcal{X}(t)$	(S6)
$\kappa_{\tau+1} - \kappa_\tau \geq t_T$	(S7)
$[\kappa_\tau, \kappa_{\tau+1}) = [\kappa_{\tau,T_0}, \kappa_{\tau,t_T}) \cup [\kappa_{\tau,t_T}, \kappa_{\tau+1,T_0})$	(S8)
$\begin{gathered} \Upsilon_k = \{(\mathcal{X}, \wp) \mid V(Q_k) - V(Q_j) \leq 0\} \\ \Upsilon_j = \{(\mathcal{X}, \wp) \mid V(Q_k) - V(Q_j) = 0\} \end{gathered}$	(S9)
$\kappa_{\tau+1} = \min_{\kappa_\varphi}\{\kappa_\varphi \mid \kappa_\varphi \geq \kappa_\tau + t_T \& \mathcal{X}^{\mathrm{T}}(\kappa_\varphi)Q_k\mathcal{X}(\kappa_\varphi) - \varsigma^T(\kappa_\varphi)Q_j\varsigma(\kappa_\varphi) \geq 0\}$	(S10)

$$\Upsilon_j = \left\{(\mathcal{X}, \wp) \mid V(Q_k) - V(Q_j) \geq 0\right\}$$

(S11)

$$\mathcal{P}_{11} = \begin{bmatrix} \Pi_{11k,T_l} & 0 & \Pi_{13k,T_l} & \Pi_{14k,T_l} \\ * & \Pi_{22k,l} & 0 & 0 \\ * & * & -I & \Pi_{34k,T_l} \\ * & * & * & -I \end{bmatrix}$$

$$\mathcal{P}_{12} = \begin{bmatrix} \Pi_{15k,Tl} & \Pi_{16k,Tl} & \Pi_{17k,Tl} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{P}_{22} = \begin{bmatrix} -\varphi_1^2 I & 0 & 0 & \Pi_{58k,T_l} \\ 0 & -\varphi_1^2 I & 0 & 0 \\ 0 & 0 & -\varphi_2^2 I & 0 \\ 0 & 0 & 0 & -\varphi_2^2 I \end{bmatrix}$$

$$\mathcal{R}_{11} = \begin{bmatrix} \Gamma_{11k,t_T} & 0 & \Gamma_{13k,t_T} & \Gamma_{14k,t_T} \\ * & \Gamma_{22k,t_T} & 0 & 0 \\ * & * & -\frac{1}{\phi_1^2} I & 0 \\ * & * & * & -\frac{1}{\phi_2^2} I \end{bmatrix}$$

$$\mathcal{P}_{12} = \mathcal{R}_{12}, \mathcal{P}_{21} = \mathcal{R}_{21}, \mathcal{P}_{22} = \mathcal{R}_{22}, (T_l = t_T)$$

$$\Pi_{11k,T_l} = A_k^T Q_{k,T_l} A_k + K_k^T B_k^T Q_{k,T_l} + Q_{k,T_l} B_k K_k + \dot{\wp} \frac{\partial Q_{k,T_l}}{\partial \wp} + l\left(Q_{k,T_{l+1}} - Q_{k,T_l}\right)$$

$$\Pi_{22k,T_l} = A_k^T Q_{k,T_l} + Q_{k,T_l} A_r + K_k^T B_k^T Q_{k,T_l} + Q_{k,T_l} B_k K_k + \sum_{i=1}^{N} \left\{\underline{\wp}_i, \overline{\wp}_i\right\} \frac{\partial Q_{k,T_l}}{\partial \wp}$$
$$+ l\left(Q_{k,T_{l+1}} - Q_{k,T_l}\right)$$

$$\Pi_{22k,t_T} = A_r^T Q_{k,t_T} + Q_{k,t_T} A_r + K_k^T B_k^T Q_{k,t_T} + Q_{k,t_T} B_k K_k + \dot{\wp} \frac{\partial Q_{k,t_T}}{\partial \wp}$$
$$+ \sum_{j=1,k\neq j}^{N} \lambda_{kj} \left(Q_{j,T_0} - Q_{k,T_l}\right)$$

$$\Gamma_{11k,T_l} \quad = A_k^T P_{k,T_l} + P_{k,T_l} A_k + \Psi + \frac{1}{\phi_1^2} + K_k^T$$
$$B_k^T P_{k,T_l} + P_{k,T_l} B_k K_k + \sum_{i=1}^{N} \left\{\underline{\wp}_i, \overline{\wp}_i\right\}$$
$$\frac{\partial Q_{k,t_T}}{\partial \wp} + l\left(P_{k,T_{l+1}} - P_{k,T_l}\right)$$

$$\Gamma_{22k,T_l} \quad = A_r^T Q_{k,t_T} + \sum_{i=1}^{N} \left\{\underline{\wp}_i, \overline{\wp}_i\right\} \frac{\partial Q_{k,t_T}}{\partial \wp} +$$
$$K_k^T B_k^T Q_{k,t_T} + Q_{k,t_T} B_k K_k Q_{k,t_T} A_r +$$
$$\sum_{j=1,k\neq j}^{N} \lambda_{kj} \left(Q_{j,T_0} - Q_{k,T_l}\right)$$

(S12)

$$\mathit{\Gamma}_{13k,Tl} = \mathit{\Gamma}_{34k,Tl} = P_{k,Tl}(\varphi), \mathit{\Gamma}_{14k,Tl} = C_k(\varphi)P_{k,Tl}(\varphi)$$

$$\begin{aligned}\mathit{\Gamma}_{11k,t_T} \quad &= A_r^T P_{k,t_T} + P_{k,t_T} A_r + \mathit{\Psi} + \frac{1}{\phi_1^2} + K_k^T B_k^T P_{k,t_T} \\ &+Q_{k,t_T} B_k K_k + \sum_{i=1}^{N} \left\{\underline{\wp}_i, \overline{\wp}_i\right\} \frac{\partial P_{k,t_T}}{\partial \wp} + \\ &\sum_{j=1,k\neq j}^{N} \lambda_{kj} \left(P_{j,T_0} - P_{k,T_l}\right)\end{aligned}$$

$$\mathit{\Pi}_{13k,t_T} = \mathit{\Pi}_{34k,t_T} = \mathit{\Pi}_{13k,T_l}$$

$$\mathit{\Pi}_{14k,t_T} = \mathit{\Pi}_{14k,T_l}, (T_l = t_T)$$

$$\mathit{\Pi}_{15k,T_l} = \frac{1}{\varphi_1^2} T_k, \mathit{\Pi}_{17k,T_l} = \frac{1}{\varphi_2^2} T_k$$

$$\mathit{\Pi}_{16k,T_l} = \frac{1}{\varphi_1^2} B_k K_k R_k, \mathit{\Pi}_{58k,T_l} = \frac{1}{\varphi_2^2} A_r R_k$$

$$A_{l1} = \begin{bmatrix} -\mathit{\Psi}_{11k,T_l} & Q_{k,T_l} \\ * & -\mathit{\Psi}_{22k,T_l} \end{bmatrix},$$

$$A_{l2} = \begin{bmatrix} 0 & 0 \\ \frac{1}{\varpi_1^2} B_k R & Q_{k,T_l} T_k^T \end{bmatrix},$$

$$A_{T1} = \begin{bmatrix} -\mathit{\Psi}_{11k,T_T} & Q_{k,T_T} \\ * & -\mathit{\Psi}_{22k,T_T} \end{bmatrix},$$

$$\mathfrak{I} = \begin{bmatrix} -\varpi_2^2 I & 0 \\ * & -\varpi_2^2 I \end{bmatrix},$$

$$A_{T2} = \begin{bmatrix} 0 & 0 \\ \frac{1}{\varpi_1^2} B_k R & Q_{k,T_T} T_k^T \end{bmatrix},$$

$$\mathit{\Psi}_{11k,T_l} = \dot{\wp} \frac{\partial Q_{k,T_l}}{\partial \wp} + l\left(Q_{k,T_{l+1}} - Q_{k,T_l}\right)$$

$$\mathit{\Psi}_{11k,t_T} = \sum_{i=1}^{N} \left\{\underline{\wp}_i, \overline{\wp}_i\right\} \frac{\partial Q_{k,t_T}}{\partial \wp} + l\left(Q_{j,T_0} - Q_{k,t_T}\right)$$

$$\mathit{\Psi}_{22k,t_T} = \left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right)^T \Phi_{k,t_T}(t) + Q_{k,t_T} \left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right) + \varpi_1^2 \mathcal{C}_k^T Q_{k,t_T}$$

$$\mathit{\Psi}_{22k,T_l} = \left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right)^T \Phi_{k,T_l}(t) + \Phi_{k,T_l}(t) \left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right) + \varpi_1^2 \mathcal{C}_k^T \Phi_{k,T_l}(t)$$

(S13)

S2 Proof of Theorem 1

Proof: For the interval $[\kappa_\tau, \kappa_{\tau+1})$, the time derivative of $V_k(t, \varsigma)$ determined by (S6) is calculated as

$$\begin{aligned}\dot{V}_k(t, \varsigma) = \mathcal{X}^T(t)(\tilde{\mathcal{A}}_k^T \Omega_k(t) + \Omega_k(t) \tilde{\mathcal{A}}_k + \frac{\partial \Omega_k(t)}{\partial t} + \dot{\wp} \frac{\partial \Omega_k(t)}{\partial \wp}\Big) \mathcal{X}(t) \\ + \mathcal{X}^T(t)(\Omega_k(t) \mathcal{C}_k) w(t)\end{aligned}$$

(S14)

Combining with Lemma 2, we have

$$\begin{aligned} V_k(t,\varsigma) = \mathcal{X}^T(\tilde{\mathcal{A}}_k^T\Omega_k(t) + \Omega_k(t)\tilde{\mathcal{A}}_k + \frac{\partial\Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial\Omega_k(t)}{\partial\wp} + \varphi_1^2 + \varphi_2^2)\mathcal{X} \\ + \frac{1}{\varphi_1^2}w^T(t)\mathcal{C}_k^T\mathcal{C}_k\Omega_k(t)w(t) + \frac{1}{\varphi_2^2}\tilde{\Xi}^T(t)\Omega_k^2(t,\wp)\tilde{\Xi}(t) \end{aligned} \tag{S15}$$

Then, we design $\Omega_{k,L}(t)$ as

$$\Omega_{k,L}(t) = \begin{cases} l(t-\kappa_\tau,T_l)\big(Q_{k,l+1} - Q_{k,l}\big), & Q_{k,l}, t \in [\kappa_\tau, T_l], \\ Q_{k,l}, & t \in [\kappa_\tau, T_l] \cup [\kappa_\tau, T_1], \\ Q_{k,tT}, & t \in [\kappa_\tau, tT], \end{cases} \tag{S16}$$

For $\forall t \in \big[\kappa_{\tau,T_l}, \kappa_{\tau,T_{l+1}}\big)$ and $\forall k \in H$, the integration of (26) and (27) leads to

$$\dot{V}_k(t,\mathcal{X}) < 0, \mathcal{X} \neq 0 \tag{S17}$$

Therefore, $V_k(t,\mathcal{X}) = \mathcal{X}^T\Omega_{k,L}(t)\mathcal{X}$ can be obtained for $\forall t \in \big[\kappa_{\tau,t_T}, \kappa_{\tau+1}\big)$. By using the designed switching law, we have $\mathcal{X}^T(t)Q_k\mathcal{X}(t) - \mathcal{X}^T(t)Q_j\mathcal{X}(t) \leq 0$, $j \in H, j \neq k$. Since we have weighting function $\lambda_{kj} \geq 0$, it is known that $\forall t \in [\kappa_\tau, \kappa_{\tau+1})$, $\dot{V}_k(t,\mathcal{X}) < 0, \mathcal{X} \neq 0$.

Along this line then, we have

$$\begin{aligned} \int_0^t e^T(\tau)\Lambda e(\tau)d\tau &= \int_0^t (x(\tau) - x_r(\tau))^T\,\Lambda(x(\tau) - x_r(\tau))d\tau \\ &= \int_0^t \Big(\mathcal{X}^T(\tau)\Psi\mathcal{X}(\tau) + \dot{V}_k(\tau,\mathcal{X})\Big)d\tau \\ &\quad -V_k(t,\mathcal{X}) + V(0,\mathcal{X}) \end{aligned} \tag{S18}$$

with Ψ is the intermediate matrix. From (S17) and (S18) yields

$$\begin{aligned} \int_0^t e^T(\tau)\Lambda e(\tau)d\tau &\leq \int_0^t \Big(\mathcal{X}^T(\tau)\Psi\mathcal{X}(\tau) + \dot{V}_k(\tau,\mathcal{X})\Big)d\tau \\ &\leq \int_0^t \bigg(\mathcal{X}^T(\tau)\bigg(\tilde{\mathcal{A}}_k^T\Omega_k(t) + \Omega_k(t)\tilde{\mathcal{A}}_k + \frac{\partial\Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial\Omega_k(t)}{\partial\wp} \\ &+ \Psi\bigg)\mathcal{X}(t) + \mathcal{X}^T(t)(\Omega_k(t)\mathcal{C}_k)\omega(t) + \tilde{\Xi}^T(t)\Omega_k(t)\mathcal{X}(t) \\ &+ \mathcal{X}^T(t)\Omega_k(t)\tilde{\Xi}(t) + \omega(t)(\mathcal{C}_k^T\Omega_k(t,\wp))\mathcal{X}(t)\bigg)d\tau \end{aligned} \tag{S19}$$

It follows from (S19) that

$$\begin{aligned} \int_0^t e^T(\tau)\Lambda e(\tau)d\tau \leq \int_0^t \bigg(X^T(\tau)\Big(\tilde{A}_k^T\Omega_k(\tau) + \Omega_k(\tau)A_k\Big) + \frac{\partial\Omega_k(\tau)}{\partial t} + \phi\frac{\partial\Omega_k(\tau)}{\partial\varphi} + \Psi + \frac{1}{\phi_1^2} \\ + \frac{1}{\phi_2^2}\Omega_k^2(\tau)\bigg)d\tau \\ + \int_0^t \big(X^T(\tau) + \phi_2^2\Xi^T(\tau)\Xi(\tau) + \omega(\tau)(\phi_1^2\Omega_k(\tau)) + C_k^TC_k\Omega_k(\tau)\omega^T(\tau)\big)d\tau \end{aligned} \tag{S20}$$

When the external disturbance $w(t) = 0$, from (S19), one can achieve $\dot{V}_k(t,\mathcal{X}) < 0$

for $t \in [\kappa_\tau, \kappa_{\tau+1})$. With $\lambda = \max_{k=1}^{N}(\phi_1^2 \Omega_k^T(\tau) \mathcal{C}_k^T \mathcal{C}_k^T \Omega_k^T(\tau))$, by combining (S14), (S15) and (S20), it can be guaranteed that $\int_0^t e^T(\tau)\,\Lambda e(\tau) d\tau \leq \lambda^2 \int_0^t \omega^T(\tau)\,\omega(\tau) d\tau$ for $\forall t \in [\kappa_\tau, \kappa_{\tau+1})$.

Then, it is noted that the disturbance rejection level is satisfied for $t \to \infty$. Thus, the augmented system is guaranteed to be asymptotically stable. Therefore, this completes the proof of **Theorem 1**.

S3 Proof of Theorem 2

Proof: It is known that the condition can be rewritten as

$$\begin{bmatrix} \Lambda_k(t) & \Omega_k(t)\mathcal{C}_k \\ * & \frac{1}{\varpi_1^2} I \end{bmatrix} < 0 \tag{S21}$$

With

$$\Lambda_k(t) = \tilde{\mathcal{A}}_k^T \Omega_k(t) + \Omega_k(t)\tilde{\mathcal{A}}_k + \frac{\partial \Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial \Omega_k(t)}{\partial \wp} \tag{S22}$$

According to Lemma 1, we have $\Lambda_k(t) < 0$, define $\Phi_k(t) = \Omega_k^{-1}(t)$, pre- and post-multiplying $\Lambda_{1k}(t)$ by $\Phi_k(t)$, we obtain

$$\begin{gathered} \Phi_k(t)\tilde{\mathcal{A}}_k^T + \tilde{\mathcal{A}}_k \Phi_k(t) + \varpi_1^2 \Phi_k(t) \mathcal{C}_k^T \\ +\Phi_k(t)\left(\frac{\partial \Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial \Omega_k(t)}{\partial \wp}\right)\Phi_k(t) < 0 \end{gathered} \tag{S23}$$

By using the Lemma 1, we have

$$\begin{bmatrix} -\left(\frac{\partial \Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial \Omega_k(t)}{\partial \wp}\right) & \Phi_k(t) \\ * & \Theta_1 \end{bmatrix} < 0 \tag{S24}$$

with $\Theta_1 = \tilde{\mathcal{A}}_k^T \Phi_k(t) + \Phi_k(t)\tilde{\mathcal{A}}_k + \varpi_1^2 \mathcal{C}_k^T \Phi_k(t)$.

Thus, one can obtain

$$\begin{bmatrix} -\left(\frac{\partial \Omega_k(t)}{\partial t} + \dot{\wp}\frac{\partial \Omega_k(t)}{\partial \wp}\right) & \Phi_k(t) \\ * & \Theta_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \Theta_3 \end{bmatrix} < 0 \tag{S25}$$

with

$$\Theta_2 = \left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right)^T \Phi_k(t) + \Phi_k(t)\left(\mathcal{A}_\sigma + \mathcal{B}_\sigma \tilde{K}_\sigma\right) + \varpi_1^2 \mathcal{C}_k^T \Phi_k(t)$$

$$\Theta_3 = \frac{1}{\varpi_2^2}\mathcal{B}_k R_k \mathcal{B}_k^T R_k^T + \varpi_2^2 \Phi_k(t) T_k^T(\kappa) T_k^T(\kappa) \Phi_k(t)$$

Thus, we have

$$\begin{bmatrix} \overline{A}_{l1} & \overline{A}_{l2} \\ * & \mathfrak{J} \end{bmatrix} < 0 \tag{S26}$$

with

$$\bar{A}_{l2} = \begin{bmatrix} -\frac{\partial \Omega_k(t)}{\partial t} - \dot{\wp}\frac{\partial \Omega_k(t)}{\partial \wp} & \Phi_k(t) \\ * & \Theta_2 \end{bmatrix},$$

$$\bar{A}_{l1} = \begin{bmatrix} 0 & 0 \\ \frac{1}{\varpi_1^2} B_k R & \Phi_k(t) T_k^T(k) \end{bmatrix}$$

Then, the system control gains are determined by (36). For any time t within the interval $\forall t \in \left[\kappa_{\tau,T_l}, \kappa_{\tau,T_{l+1}}\right)$, when $\dot{\wp}_i$ is determined by $\underline{\wp}_i$ or $\overline{\wp}_i$, $\sum_{i=1}^{g}\left\{\underline{\wp}_i, \overline{\wp}_i\right\}\frac{\partial}{\partial \wp_i}$ can be defined as $\dot{\wp}_i \frac{\partial}{\partial \wp_i}$, according to Theorem 1.

In this way, when $\forall t \in [\kappa_{\tau,t_T}, \kappa_{\tau+1})$, the dwell time is large than t_T with a constant parameter matrix $P_2(t)$, implying that the system stability condition can be derived. Therefore, the proof of **Theorem 2** is completed.