Electronic supplementary materials

For https://doi.org/10.1631/jzus.A2600099

Dynamics of a two-disk dynamo loaded with a dual neuron circuit and its application

Fuqiang WU[1,2], Ying XU[3], Huimin QI[1], Jun MA[4]

[1]*School of Mathematics and Statistics, Ningxia University, Yinchuan 750021, China*
[2]*Ningxia Basic Science Research Center of Mathematics, Yinchuan 750021, China*
[3]*School of Mathematics and Statistics, Shandong Normal University, Jinan 250014, China*
[4]*Department of Physics, Lanzhou University of Technology, Lanzhou 730050, China*

Section S1

The equilibrium point of Eq. (10) is $E^*=(x_1^*, y_1^*, x_2^*, y_2^*, z^*)$. For the derivation of the equilibrium point E^*, we impose the condition that the right-hand side of Eq. (10) is equal to zero as

$$\begin{cases} i_{ext} - a_1 x_1^* + y_1^* = 0; \\ x_1^* - b_1 y_1^* + c y_2^* + d_1 y_2^* z^* = 0; \\ -a_2 x_2^* + y_2^* = 0; \\ x_2^* - b_2 y_2^* - c y_1^* + d_2 y_1^* z^* = 0; \\ m_0 - e y_1^* y_2^* = 0; \end{cases} \tag{S1}$$

By calculating Eq. (S1), we can obtain:

$$\begin{cases} x_1^* = \dfrac{i_{ext}}{a_1} + \dfrac{m_0}{a_1 e y_2^*}; y_1^* = \dfrac{m_0}{e y_2^*}; \\ x_2^* = \dfrac{y_2^*}{a_2}; z^* = \dfrac{m_0 (a_1 b_1 - 1)}{e a_1 d_1 y_2^{*2}} \\ \quad -\dfrac{i_{ext}}{a_1 d_1 y_2^*} - \dfrac{c}{d_1}; \end{cases} \tag{S2}$$

and y_2^* in Eq. (S2) can be derived from the following equation

$$\begin{aligned} &(1 - \frac{1}{a_2 b_2}) y_2^{*4} + \frac{c m_0 (d_1 + d_2)}{e b_2 d_1} y_2^{*2} \\ &+ \frac{i_{ext} d_2 m_0}{a_1 d_1 e b_2} y_2^* - \frac{d_2 m_0^2 (a_1 b_1 - 1)}{e^2 a_1 b_2 d_1} = 0, \end{aligned} \tag{S3}$$

To analyze the stability property of E^*, we compute the Jacobian matrix of Eq. (10) at E^* as

$$J = \begin{pmatrix} J_{11} & J_{12} & 0 & 0 & 0 \\ J_{21} & J_{22} & 0 & J_{24} & J_{25} \\ 0 & 0 & J_{33} & J_{34} & 0 \\ 0 & J_{42} & J_{43} & J_{44} & J_{45} \\ 0 & J_{52} & 0 & J_{54} & 0 \end{pmatrix}, \tag{S4}$$

where $J_{11}=-a_1/g_1$, $J_{12}=1/g_1$, $J_{21}=1/g_2$, $J_{22}=-b_1/g_2$, $J_{24}=(c+d_1z^*)/g_2$, $J_{25}= d_1y_2^*/g_2$, $J_{33}=-a_2/g_3$, $J_{34}=1/g_3$, $J_{42}=(-c+d_2z^*)/g_4$, $J_{43}=1/g_4$, $J_{44}=-b_2/g_4$, $J_{45}=d_2y_1^*/g_4$, $J_{52}=-ey_2^*$, $J_{54}=-ey_1^*$.

Then, the characteristic equation can be expressed as

$$f(\lambda) = |\lambda - J| = 0. \tag{S5}$$

By algebraic simplification, the characteristic equation in Eq. (S5) can be derived as

$$\begin{aligned} f(\lambda) = {} & A_0 \lambda^5 + A_1 \lambda^4 \\ & + A_2 \lambda^3 + A_3 \lambda^2 \\ & + A_4 \lambda + A_5 = 0, \end{aligned} \tag{S6}$$

where A_0, A_1, A_2, A_3, A_4 and A_5 are shown in Eq. (S13).

Based on the Routh-Hurwitz stability criterion, the equilibrium point E^* is locally asymptotically stable if and only if all coefficients of the characteristic equation are positive. In addition, all the elements in the first column of the Routh array are positive. That is,

$$\begin{cases} B_1 = 1 > 0, B_2 = A_1 > 0, \\ B_3 = A_2 - \dfrac{A_3}{A_1} > 0, \\ B_4 = A_3 - \dfrac{M_1 A_1}{M_2} > 0, \\ B_6 = A_5 > 0, \\ B_5 = \dfrac{(M_1 - M_2 A_5) M_2}{A_1 A_3 M_2 - A_1^2 M_1} > 0, \end{cases} \tag{S7}$$

where $M_1 = A_1 A_4 - A_5$, $M_2 = A_1 A_2 - A_3$.

It is clear that if the inequality given in Eq. (S7) is satisfied, the equilibrium point E^* is asymptotically stable. In contrast, if the condition in Eq. (S7) is violated, E^* becomes an unstable equilibrium point.

The vector field of the system in Eq. (10) is described as

$$F = \begin{pmatrix} \dfrac{1}{g_1}(i_{ext} - a_1 x_1 + y_1) \\ \dfrac{1}{g_2}\left(x_1 - b_1 y_1 + c y_2 + d_1 y_2 z\right) \\ \dfrac{1}{g_3}\left(-a_2 x_2 + y_2\right) \\ \dfrac{1}{g_4}(x_2 - b_2 y_2 + c y_1 + d_2 y_1 z) \\ m_0 - e y_1 y_2 \end{pmatrix}. \tag{S8}$$

Then, we compute the divergence of the system in Eq. (10) as follows:

$$\begin{aligned} \nabla V &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial y_1} + \frac{\partial f_3}{\partial x_2} + \frac{\partial f_4}{\partial y_2} + \frac{\partial f_5}{\partial z} \\ &= -\left(\frac{a_1}{g_1} + \frac{b_1}{g_2} + \frac{a_2}{g_3} + \frac{b_2}{g_4}\right). \end{aligned} \tag{S9}$$

Since the values of a_1/g_1, b_1/g_2, a_2/g_3 and b_2/g_4 in Eq. (S3) are all positive, it follows that $\nabla V < 0$. The dynamical model given by Eq. (10) represents a dissipative system with exponential convergence as

$$\frac{dV}{d\tau} = e^{-\left(\frac{a_1}{g_1} + \frac{b_1}{g_2} + \frac{a_2}{g_3} + \frac{b_2}{g_4}\right)}. \tag{S10}$$

When the time tends to infinity, the trajectory of the electromechanical system in Eq. (10) shrinks to an attractor.

Next, we calculate the Lyapunov exponents when the initial values of the dimensionless electromechanical system model in Eq. (10) are selected to be ($x_1(0)$, $y_1(0)$, $x_2(0)$, $y_2(0)$, $z(0)$)=(0.2, 0.1, 0.1, 0.2, 0.1), and the parameters m_0 and i_{ext} are assigned values of 1 and 0.05, respectively. The Lyapunov exponents are obtained as $LE_1 \approx 0.2031$, $LE_2 \approx 0.0007286$, $LE_3 \approx -0.00048762$, $LE_4 \approx -0.0015$, and $LE_5 \approx -2.2028$. The Lyapunov dimension in Eq. (10) is defined as

$$\begin{aligned} \nabla V &= \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial y_1} + \frac{\partial f_3}{\partial x_2} + \frac{\partial f_4}{\partial y_2} + \frac{\partial f_5}{\partial z} \\ &= -\left(\frac{a_1}{g_1} + \frac{b_1}{g_2} + \frac{a_2}{g_3} + \frac{b_2}{g_4}\right), \end{aligned} \tag{S11}$$

where LE_i represents the Lyapunov exponent of Eq. (10). Thus, the model in Eq. (10) has

$$\begin{aligned} D_L &\approx 4 + \frac{LE_1 + LE_2 + LE_3 + LE_4}{\left|LE_5\right|} \\ &\approx 4.0916. \end{aligned} \tag{S12}$$

The expressions for A_0, A_1, A_2, A_3, A_4 and A_5 in Eq. (S6) are as follows:

$$
\begin{aligned}
A_0 &= 0;\\
A_1 &= \frac{a_1}{g_1}+\frac{b_1}{g_2}+\frac{a_2}{g_3}+\frac{b_2}{g_4};\\
A_2 &= \frac{a_1b_1-1}{g_1g_2}+\frac{a_1a_2}{g_1g_3}+\frac{a_2b_1}{g_2g_3}+\frac{a_1b_2}{g_1g_4}+\frac{a_2b_2-1}{g_3g_4}+\frac{d_2ey_1^{*2}}{g_4}\\
&\quad+\frac{b_1b_2+c^2+cd_1z^*-cd_2z^*-d_1d_2z^{*2}}{g_2g_4}+\frac{d_1ey_2^{*2}}{g_2};\\
A_3 &= \frac{a_1a_2b_1-a_2}{g_1g_2g_3}+\frac{a_1b_1b_2-b_2+a_1c^2+a_1cd_1z^*-a_1cd_2z^*-a_1d_1d_2z^{*2}}{g_1g_2g_4}\\
&\quad+\frac{b_1d_2ey_1^{*2}-cd_1ey_1^*y_2^*+cd_2ey_1^*y_2^*+b_2d_1ey_2^{*2}+2d_1d_2ey_1^*y_2^*z^*}{g_2g_4}\\
&\quad+\frac{a_1d_2ey_1^{*2}}{g_1g_4}+\frac{a_2d_2ey_1^{*2}}{g_3g_4}+\frac{a_1d_1ey_2^{*2}}{g_1g_2}+\frac{a_2d_1ey_2^{*2}}{g_2g_3}+\frac{a_1a_2b_2-a_1}{g_1g_3g_4}\\
&\quad+\frac{a_2b_1b_2-b_1+a_2c^2-a_2cd_2z^*-a_2d_1d_2z^{*2}+a_2cd_1z^*}{g_2g_3g_4};\\
A_4 &= \frac{1-a_1b_1-a_2b_2+a_1a_2b_1b_2+a_1a_2c^2+a_1a_2cd_1z^*-a_1a_2cd_2z^*-a_1a_2d_1d_2z^{*2}}{g_1g_2g_3g_4}\\
&\quad+\frac{a_1b_1d_2ey_1^{*2}-d_2ey_1^{*2}-a_1cd_1ey_1^*y_2^*+a_1cd_2ey_1^*y_2^*+a_1b_2d_1ey_2^{*2}+2a_1d_1d_2ey_1^*y_2^*z^*}{g_1g_2g_4}\\
&\quad+\frac{a_2b_1d_2ey_1^{*2}-a_2cd_1ey_1^*y_2^*+a_2cd_2ey_1^*y_2^*-d_1ey_2^{*2}+a_2b_2d_1ey_2^{*2}+2a_2d_1d_2ey_1^*y_2^*z^*}{g_2g_3g_4}\\
&\quad+\frac{a_1a_2d_2ey_1^{*2}}{g_1g_3g_4}+\frac{a_1a_2d_1ey_2^{*2}}{g_1g_2g_3};\\
A_5 &= \frac{1}{g_1g_2g_3g_4}(-a_2d_2ey_1^{*2}+a_1a_2b_1d_2ey_1^{*2}-a_1a_2cd_1ey_1^*y_2^*\\
&\quad+a_1a_2cd_2ey_1^*y_2^*-a_1d_1ey_2^{*2}+a_1a_2b_2d_1ey_2^{*2}+2a_1a_2d_1d_2ey_1^*y_2^*z^*).
\end{aligned}
\tag{S13}
$$