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Supplementary materials for

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1 Gery wolf optimizer and multi-objective grey wolf optimizer

1.1 Gery wolf optimizer

In this subsection, a simple generalization operation and analysis of the single-objective algorithm gery wolf optimizer (GWO) (Mirjalili et al., 2014) is presented. GWO is a kind of evolutionary algorithm (EA) which mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. It is composed mainly of three parts: social hierarchy, encircling the prey, and hunting behaviors.

Grey wolf population has a strict pyramid-structured social hierarchy. Three grey wolves with the best fitness values in the grey wolf population are defined as alpha (α), beta (β), and delta (δ) wolves. They are the leaders of the population, and the omega (ω) wolves as the candidate solutions are equivalent to civilians in the population. Grey wolves encircle their prey during a hunt. To mathematically model the encircling behavior, the following equations are proposed:

$$\boldsymbol{D} = |\boldsymbol{C} \cdot \boldsymbol{X}_p - \boldsymbol{X}(t)|, \quad \text{(S1)}$$

$$\boldsymbol{X}(t+1) = \boldsymbol{X}_p(t) - \boldsymbol{A}_1 \cdot \boldsymbol{D}, \quad \text{(S2)}$$

where t is the current iteration, $\boldsymbol{X}_p$ is the position vector of the prey, $\boldsymbol{X}$ indicates the position vector of a grey wolf, and $\boldsymbol{A}$ and $\boldsymbol{C}$ are coefficient vectors, calculated as follows:

$$\boldsymbol{A} = 2a\boldsymbol{r}_1 - \boldsymbol{a}, \quad \text{(S3)}$$

$$\boldsymbol{C} = 2\boldsymbol{r}_2, \quad \text{(S4)}$$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are random vectors in [0, 1], the convergence factor $a=2-2t/T$, linearly decreasing from 2 to 0 throughout the iterations.

GWO uses the simulated social leadership and encircling mechanism to find the optimal solution for single objective optimization problems. This algorithm saves the three best solutions (α, β, and δ) obtained so far and obliges other individuals to update their positions with respect to them. The following formulae simulate the hunting process and find promising regions of the search space:

$$\boldsymbol{D}_\alpha = |\boldsymbol{C}_1 \cdot \boldsymbol{X}_\alpha - \boldsymbol{X}|, \boldsymbol{D}_\beta = |\boldsymbol{C}_2 \cdot \boldsymbol{X}_\beta - \boldsymbol{X}|, \boldsymbol{D}_\delta = |\boldsymbol{C}_3 \cdot \boldsymbol{X}_\delta - \boldsymbol{X}|, \quad \text{(S5)}$$

$$\boldsymbol{X}_1 = \boldsymbol{X}_\alpha - \boldsymbol{A}_1 \cdot \boldsymbol{D}_\alpha, \boldsymbol{X}_2 = \boldsymbol{X}_\alpha - \boldsymbol{A}_2 \cdot \boldsymbol{D}_\beta, \boldsymbol{X}_3 = \boldsymbol{X}_\alpha - \boldsymbol{A}_3 \cdot \boldsymbol{D}_\delta, \quad \text{(S6)}$$

$$\boldsymbol{X}(t+1) = (\boldsymbol{X}_1(t) + \boldsymbol{X}_2(t) + \boldsymbol{X}_3(t))/3, \quad \text{(S7)}$$

where $\boldsymbol{A}$ is an important parameter that affects the exploration and exploitation of algorithms. A_i (i=1, 2, ..., N) is a random value in the interval [−2a, 2a], where a decreases from 2 to 0 over the course of iterations. When random values of A_i (i=1, 2, ..., N) are in [−1, 1], the next position of an individual can be any one between its current position and the position of the prey. Therefore, when $|\boldsymbol{A_i}| < 1$ $(i = 1, 2, \ldots, N)$, the wolves are forced to pursue an attack towards the prey (exploitation). GWO uses $|\boldsymbol{A_i}|$ $(i = 1, 2, \ldots, N)$ with random values greater than 1 or less than −1 to oblige the individuals to diverge from the prey to obtain mathematical model divergence. Therefore, when $|\boldsymbol{A_i}| > 1$ (i=1, 2, ..., N), the grey wolves are forced to diverge from the prey to hopefully find a fitter prey (exploration).

To sum up, the search process starts by creating a random population of grey wolves (candidate solutions) in GWO. Over the course of iterations, α, β, and δ wolves estimate the probable position of the prey. Each candidate solution updates its distance from the prey. The parameter a decreases from 2 to 0, emphasizing exploration and exploitation, respectively. The candidate solutions tend to diverge from the prey when $|\boldsymbol{A_i}| > 1$ $(i = 1, 2, \ldots, N)$ and converge towards the prey when $|\boldsymbol{A_i}| < 1$ $(i = 1, 2, \ldots, N)$. Finally, the GWO algorithm is terminated with the satisfaction of an end criterion.

1.2 Multi-objective grey wolf optimizer

The multi-objective grey wolf optimizer (MOGWO) adopts two important components, similar to the multi-objective particle swarm optimization algorithm (MOPSO) (Coello and Lechuga, 2002). The first component is an archive, which is responsible for storing previously obtained non-dominated Pareto optimal solutions. Once it is full, the grid mechanism is called to improve the diversity of the final approximated Pareto optimal front. The grid mechanism functions as follows: first, it rearranges the segmentation of the objective space and finds the most crowded segment to omit one of its solutions. Then, the new solution should be inserted to the least crowded segment. The second component is a leader selection strategy that assists in choosing α, β, and δ solutions from the archive as the leaders in the hunting process. Three leaders are chosen sequentially from the least, second least, and third least crowded segments of the search space from the archive by the roulette-wheel method in MOGWO. Roulette is a method of random selection, where the selection probability is the possibility that the fitness value of each hypercube is proportional to the sum of the fitness values of all hypercubes. The hypercubes are obtained by dividing the currently obtained Pareto solution set. The fitness value of the i^{th} hypercube that may be selected is as follows:

$$P_i = c/N_i, \tag{S8}$$

where c is a constant number greater than one, and N_i is the number of Pareto optimal solutions obtained in the i^{th} segment. Consequently, the search is always directed toward the unexplored areas of the search space because of the movement of the leader.

2 Exploration and exploitation analysis

According to the analysis of MOGWO in the literature (Mirjalili et al., 2016), A_i (i=1, 2, ..., N) is an important value that affects exploration and exploitation and varies randomly within (−2a, 2a), where a decreases linearly from 2 to 0 with the iterations. When $t < 2/T$ where t is the current number of iterations and T is the total number of iterations, or $1 < a < 2$, the probability of $|\boldsymbol{A}_i|$>1 (i=1, 2, ..., N) ranges from 1 to $1/2$. This means that search agents tend to diverge from the prey when a_i > 1 (i=1, 2, ..., N). When $t \geq 2/T$, that is to say, $0 < a < 1$, it is inevitable that $|\boldsymbol{A}_i|$<1 (i=1, 2, ..., N),. All search agents move towards the prey when a_i<1 (i=1, 2, ..., N). Through the above analysis, it can be inferred that the search is too extensive at the beginning of the iteration, whereas late in the iteration, $|\boldsymbol{A}_i|$<1 (i=1, 2, ..., N) causes the search to be too concentrated.

Therefore, we can conclude that there is no satisfactory balance between exploration and exploitation throughout the iteration.

3 Procedure 1

Procedure 1 CBMOGWO
// Step 1: initialization
Initialize the population $\boldsymbol{X}$ ($\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_N$) and archive
Select leaders X_α, X_β, and X_δ
For t=1:MaxIter
// Step 2: population division
Calculate convergence factor a and a_{new}
For num=1:$N/2$
Randomly select $\boldsymbol{x}_i$, $\boldsymbol{x}_j$ $(i \neq j)$
Divide the population: $\boldsymbol{X}_{\mathrm{w}}^1$, $\boldsymbol{X}_{\mathrm{w}}^2$, $\boldsymbol{X}_{\mathrm{l}}^1$, and $\boldsymbol{X}_{\mathrm{l}}^2$
End For
// Step 3: population position update
Determine the threshold Θ ($p = \Theta \times N$)
Update the positions of individuals ${\boldsymbol{x}_{\mathrm{w}i}}^t$ ($i = 1, 2, ..., p$)
Put ${\boldsymbol{x}_{\mathrm{w}j}}^t$ ($j = p + 1, p + 2, \ldots, m + n$)into $\boldsymbol{X}^{t+1}$
Update the positions of individuals ${\boldsymbol{x}_{li}}^t$ ($i = 1, 2, \ldots, m$)
Update the positions of individuals ${\boldsymbol{x}_{lj}}^t$ ($j = 1, 2, \ldots, n$)
Maintain archive and reselect leaders X_α, X_β, and X_δ
End For
// Step 4: return archive
Return archive

4 Multi-objective test problems and multi-objective knapsack problems

4.1 Multi-objective test problems (MOTPs)

4.1.1 Mathematical formulae for MOTPs

See Tables S1–S3.

4.1.2 Statistics of MOTPs

Inverted generational distance (IGD): see Table S4, Figs. S1 and S2.
Hypervolume (HV): see Table S5, Figs. S3 and S4.

4.2 Multi-objective knapsack problems (MOKPs)

Statistics of MOKPs: see Table S6, Figs. S5 and S6.

Table S1 Bi-objective MOTPs

Name	Mathematical formulation
UF1	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}[x_j - \sin(6\pi x_1 + \frac{jn}{\pi})]^2$, $f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}[x_j - \sin(6\pi x_1 + \frac{jn}{\pi})]^2$, $J_1 = \{j \mid j$ is odd and $2 \le j \le n\}$, $J_2 = \{j \mid j$ is even and $2 \le j \le n\}$.
UF2	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} y_i^2$, $f_2 = 1 - \sqrt{x} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} y_i^2$, $J_1 = \{j \mid j$ is odd and $2 \le j \le n\}$, $J_2 = \{j \mid j$ is even and $2 \le j \le n\}$, $y_j = \begin{cases} x_j - [0.3x_1{}^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\cos(6\pi x_1 + \frac{4j\pi}{n}), & j \in J_1, \\ x_j - [0.3x_1{}^2\cos(24\pi x_1 + \frac{4j\pi}{n}) + 0.6x_1]\cos(6\pi x_1 + \frac{4j\pi}{n}), & j \in J_2. \end{cases}$
UF3	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_1} y_i^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$, $f_2 = 1 - \sqrt{x_1} + \frac{2}{\lvert J_2\rvert}(4\sum_{j\in J_1} y_j^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 2)$, J_1 and J_2 are the same as those of UF1, $y_j = x_j - x_1{}^{0.5(1.0+\frac{3(j-2)}{n-2})}$, $j = 2, 3, \ldots, n$.
UF4	$f_1 = x_1 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j)$, $f_2 = 1 - x_1{}^2 + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j)$, J_1 and J_2 are the same as those of UF1, $y_j = x_j - \sin(6\pi x_1 + \frac{j\pi}{n})$, $j = 2, 3, \ldots, n$, $h(t) = \frac{\lvert t\rvert}{1 + e^{2\lvert t\rvert}}$.
UF5	$f_1 = x_1 + \left(\frac{1}{2N} + \epsilon\right)\lvert\sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} h(y_j)$, $f_2 = 1 - x_1 + \left(\frac{1}{2N} + \epsilon\right)\lvert\sin(2N\pi x_1)\rvert + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} h(y_j)$, J_1 and J_2 are the same as those of UF1, $\epsilon > 0$, $y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right)$, $j = 2, 3, \ldots, n$, $h(t) = 2t^2 - \cos(4\pi t) + 1$.
UF6	$f_1 = x_1 + \max\{0, 2(\frac{1}{2N} + \epsilon)\sin(2N\pi x_1)\} + \frac{2}{\lvert J_1\rvert}(4\sum_{j\in J_1} y_i^2 - 2\prod_{j\in J_1}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 1)$, $f_2 = 1 - x_1 + \max\{0, 2(\frac{1}{2N} + \epsilon)\sin(2N\pi x_1)\} + \frac{2}{\lvert J_2\rvert}(4\sum_{j\in J_2} y_i^2 - 2\prod_{j\in J_2}\cos(\frac{20y_j\pi}{\sqrt{j}}) + 1)$, J_1 and J_2 are the same as those of UF1, $\epsilon > 0$, $y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right)$, $j = 2, 3, \ldots, n$.
UF7	$f_1 = \sqrt[5]{x_1} + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1} y_i^2$, $f_2 = 1 - \sqrt[5]{x_1} + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2} y_i^2$, J_1 and J_2 are the same as those of UF1, $\epsilon > 0$, $y_j = x_j - \sin\left(6\pi x_1 + \frac{j\pi}{n}\right)$, $j = 2, 3, \ldots, n$.

Table S2 Tri-objective MOTPs

Name	Mathematical formulation
UF8	$f_1 = \cos(0.5x_1\pi)\cos(0.5x_2\pi) + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $f_2 = \cos(0.5x_1\pi)\sin(0.5x_2\pi) + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $f_3 = \sin(0.5x_1\pi) + \frac{2}{j_3}\sum_{j\in J_3}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $J_1 = \{j \mid 3 \le j \le n,$ and $j - 1$ is a multiplication of 3$\}$, $J_2 = \{j \mid 3 \le j \le n,$ and $j - 2$ is a multiplication of 3$\}$, $J_3 = \{j \mid 3 \le j \le n,$ and j is a multiplication of 3$\}$, $\epsilon = 0.1$.
UF9	$f_1 = 0.5[\max\{0, (1+\epsilon)(1-4(2x_1-1)^2)\} + 2x_1]x_2 + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $f_2 = 0.5[\max\{0, (1+\epsilon)(1-4(2x_1-1)^2)\} + 2x_1]x_2 + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $f_3 = 1 - x_2 + \frac{2}{\lvert j_3\rvert}\sum_{j\in J_3}(x_j - 2x_2\sin\left(2\pi x_1 + \frac{j\pi}{n}\right)^2)$, $J_1 = \{j \mid 3 \le j \le n,$ and $j - 1$ is a multiplication of 3$\}$, $J_2 = \{j \mid 3 \le j \le n,$ and $j - 2$ is a multiplication of 3$\}$, $J_3 = \{j \mid 3 \le j \le n,$ and j is a multiplication of 3$\}$, $\epsilon = 0.1$.
UF10	$f_1 = \cos(0.5x_1\pi)\cos(0.5x_2\pi) + \frac{2}{\lvert J_1\rvert}\sum_{j\in J_1}(4{y_j}^2 - \cos(8\pi y_j) + 1)$, $f_2 = \cos(0.5x_1\pi)\sin(0.5x_2\pi) + \frac{2}{\lvert J_2\rvert}\sum_{j\in J_2}(4{y_j}^2 - \cos(8\pi y_j) + 1)$, $f_3 = \sin(0.5x_1\pi) + \frac{2}{\lvert j_3\rvert}\sum_{j\in J_3}(4{y_j}^2 - \cos(8\pi y_j) + 1)$, $J_1 = \{j \mid 3 \le j \le n,$ and $j - 1$ is a multiplication of 3$\}$, $J_2 = \{j \mid 3 \le j \le n,$ and $j - 2$ is a multiplication of 3$\}$, $J_3 = \{j \mid 3 \le j \le n,$ and j is a multiplication of 3$\}$, $\epsilon = 0.1$.

Table S3 Five-objective MOTPs

Name	Mathematical formulation
UF11	$f_1(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\cos(z'_{M-1}\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_1)((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\cos(z'_{M-1}\pi/2)+1), & \text{otherwise},\end{cases}$ $f_2(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\sin(z'_{M-1}\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_2)((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\sin(z'_{M-1}\pi/2)+1), & \text{otherwise},\end{cases}$ $\vdots$ $f_{M-1}(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\sin(z'_2\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_{M-1})((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\sin(z'_2\pi/2)+1), & \text{otherwise},\end{cases}$ $f_M(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\sin(z'_2\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_M)((1+g(\boldsymbol{x}_M))\sin(z'_2\pi/2)+1), & \text{otherwise},\end{cases}$ $g(x_M) = \sum_{x_i\in \boldsymbol{x}_M}(z'_i - 0.5)^2$.
UF12	$f_1(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\cos(z'_{M-1}\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_1)((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\cos(z'_{M-1}\pi/2)+1), & \text{otherwise},\end{cases}$ $f_2(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\sin(z'_{M-1}\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_2)((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\cos(z'_2\pi/2)\ldots\cos(z'_{M-2}\pi/2)\sin(z'_{M-1}\pi/2)+1), & \text{otherwise},\end{cases}$ $\vdots$ $f_{M-1}(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\sin(z'_2\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_{M-1})((1+g(\boldsymbol{x}_M))\cos(z'_1\pi/2)\sin(z'_2\pi/2)+1), & \text{otherwise},\end{cases}$ $f_M(\boldsymbol{x}) = \begin{cases}(1+g(\boldsymbol{x}_M))\sin(z'_2\pi/2)+1, & z_i \ge 0,\\ S(\text{psum}_M)((1+g(\boldsymbol{x}_M))\sin(z'_2\pi/2)+1), & \text{otherwise},\end{cases}$ $g(x_M) = 100(\lvert\boldsymbol{x}_M\rvert + \sum_{x_i\in \boldsymbol{x}_M}(z'_i - 0.5)^2 - \cos(20\pi(z'_i - 0.5)))$.

Table S4 Statistical results for IGD on UF1 to UF12 on MOTPs

IGD	UF1 (bi-objective)			UF2 (bi-objective)			UF3 (bi-objective)			UF4 (bi-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.2626	0.1187	0.9995	0.1740	0.0643	0.0584	1.0315	0.4157	0.3681	0.1080	0.0808	0.0721
Median	0.2588	0.1081	0.0929	0.1752	0.0642	0.0571	1.0473	0.3796	0.3332	0.1077	0.7775	0.0716
STD	0.0350	0.0360	0.0243	0.0154	0.0106	0.0080	0.0689	0.1275	0.0727	0.0027	0.0085	0.0050
Worst	0.3249	0.2010	0.1883	0.2047	0.0976	0.0811	0.1350	0.7733	0.5663	0.1137	0.1070	0.0836
Best	0.2030	0.0704	0.0689	0.1468	0.0427	0.0447	0.8841	0.2797	0.2643	0.1035	0.0697	0.0634
IGD	UF5 (bi-objective)			UF6 (bi-objective)			UF7 (bi-objective)			UF8 (tri-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	2.2744	0.6072	0.4774	2.2269	0.6910	0.6040	0.3513	0.2401	0.1916	0.6710	0.3019	0.2725
Median	2.334	0.5711	0.4586	2.2160	0.6787	0.5638	0.3548	0.2543	0.2296	0.6728	0.2834	0.2580
STD	0.3105	0.2608	0.1109	0.1674	0.1951	0.1765	0.0387	0.1003	0.0969	0.0912	0.0525	0.0590
Worst	2.7984	1.2352	0.8085	2.5836	1.1048	1.2630	0.4115	0.3817	0.3523	0.8635	0.4407	0.4950
Best	1.5274	0.2867	0.3136	1.9492	0.3728	0.3398	0.2300	0.0658	0.0515	0.4758	0.2144	0.2032
IGD	UF9 (tri-objective)			UF10 (tri-objective)			UF11 (five-objective)			UF12 (five-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.6991	0.4046	0.3577	5.0888	1.4454	1.0922	4.6901	1.1321	1.0804	3.1113e+03	2.2892e+03	2.0433e+03
Median	0.6947	0.4079	0.3650	5.3457	1.2131	1.0070	4.7163	1.1149	1.0839	3.1109e+03	2.1682e+03	2.0180e+03
STD	0.1014	0.1068	0.0620	0.9887	0.8435	0.6798	0.4749	0.1676	0.1520	2.5991e+02	3.4281e+02	2.4282e+02
Worst	0.9425	0.6274	0.4852	6.5355	3.9874	3.9888	5.7214	1.5838	1.4035	3.7143e+03	2.9965e+03	2.5081e+03
Best	0.4807	0.2642	0.2412	2.4069	0.3082	0.2547	3.7176	0.8140	0.8325	2.5684e+03	1.8379e+03	1.6317e+03

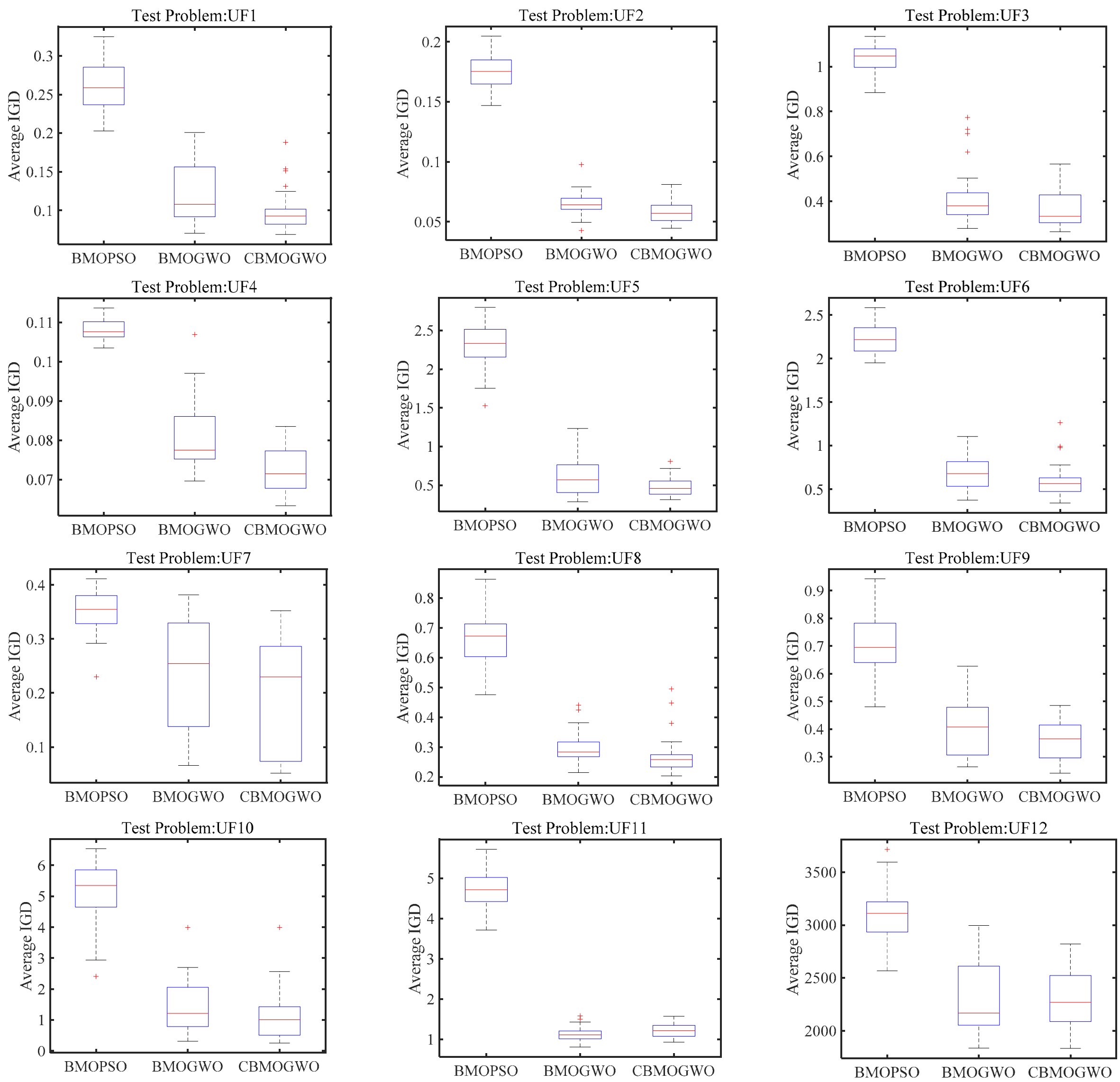

Fig. S1 Boxplot of the statistical results for IGD on UF1 to UF12 on MOTPs

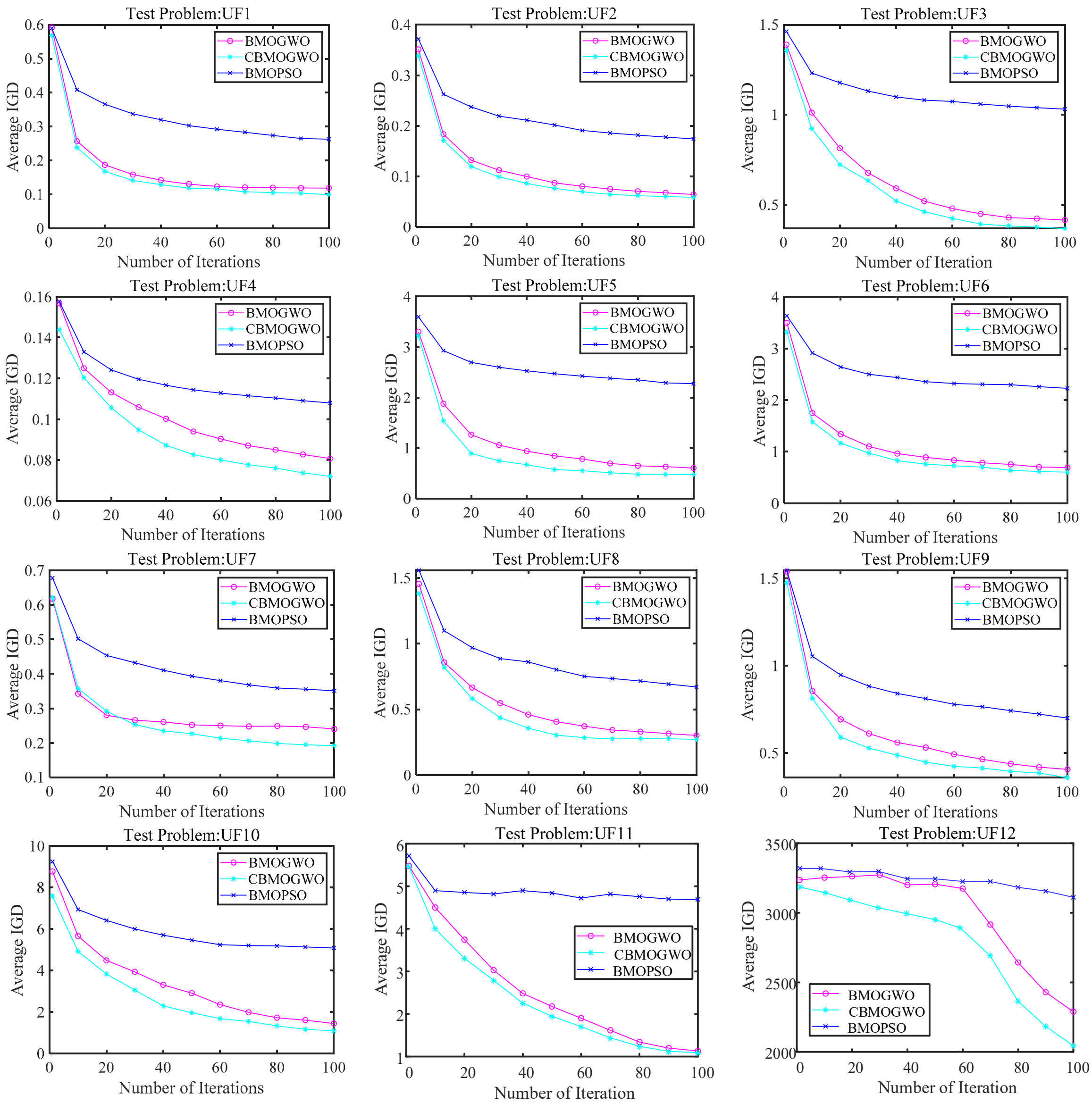

Fig. S2 Evolution of the average IGD value on UF1 to UF12 on MOTPs

Table S5 Statistical results for HV on UF1 to UF12 on MOTPs

HV	UF1 (bi-objective)			UF2 (bi-objective)			UF3 (bi-objective)			UF4 (bi-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOPSO	BMOPSO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.3570	0.5598	0.6170	0.4727	0.6406	0.7025	7.19e-05	0.2358	0.2529	0.2938	0.3308	0.3625
Median	0.3533	0.5669	0.6302	0.4728	0.6420	0.7023	0	0.2561	0.2764	0.2938	0.3354	0.3636
STD	0.0400	0.0370	0.0411	0.0170	0.0137	0.0117	2.38e-04	0.0968	0.0938	0.0033	0.0120	0.0086
Worst	0.2945	0.4866	0.4810	0.4401	0.6053	0.6659	0	0.0052	0	0.2872	0.3014	0.3452
Best	0.4446	0.6134	0.6650	0.5131	0.6659	0.7256	0.0010	0.3635	0.3993	0.3021	0.3487	0.3803

HV	UF5 (bi-objective)			UF6 (bi-objective)			UF7 (bi-objective)			UF8 (tri-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0	0.0892	0.1424	0	0.0696	0.1032	0.1834	0.3479	0.4120	0.0133	0.2342	0.2730
Median	0	0.0575	0.1617	0	0.0591	0.1188	0.1831	0.3461	0.3727	0.0079	0.2782	0.3141
STD	0	0.0926	0.0886	0	0.0657	0.0649	0.0319	0.0654	0.0786	0.0745	0.0820	0.0745
Worst	0	0	0	0	0	0	0.1204	0.2193	0.2705	0	0.0652	0.0465
Best	0	0.2771	0.3224	0	0.1858	0.2282	0.2723	0.4872	0.5475	0.0705	0.3225	0.3413

HV	UF9 (tri-objective)			UF10 (tri-objective)			UF11 (five-objective)			UF12 (five-objective)		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.0583	0.2788	0.2873	0	0.0222	0.0453	0	8.4070e-05	5.2127e-04	0	0	0
Median	0.0477	0.2589	0.2554	0	0	0	0	0	3.3919e-04	0	0	0
STD	0.0434	0.1253	0.0912	0	0.0607	0.0778	0	1.5602e-04	6.6569e-04	0	0	0
Worst	6.97e-04	0.0427	0.1285	0	0	0	0	0	0	0	0	0
Best	0.1429	0.4587	0.4595	0	0.3017	0.2977	0	6.2683e-04	3.4000e-03	0	0	0

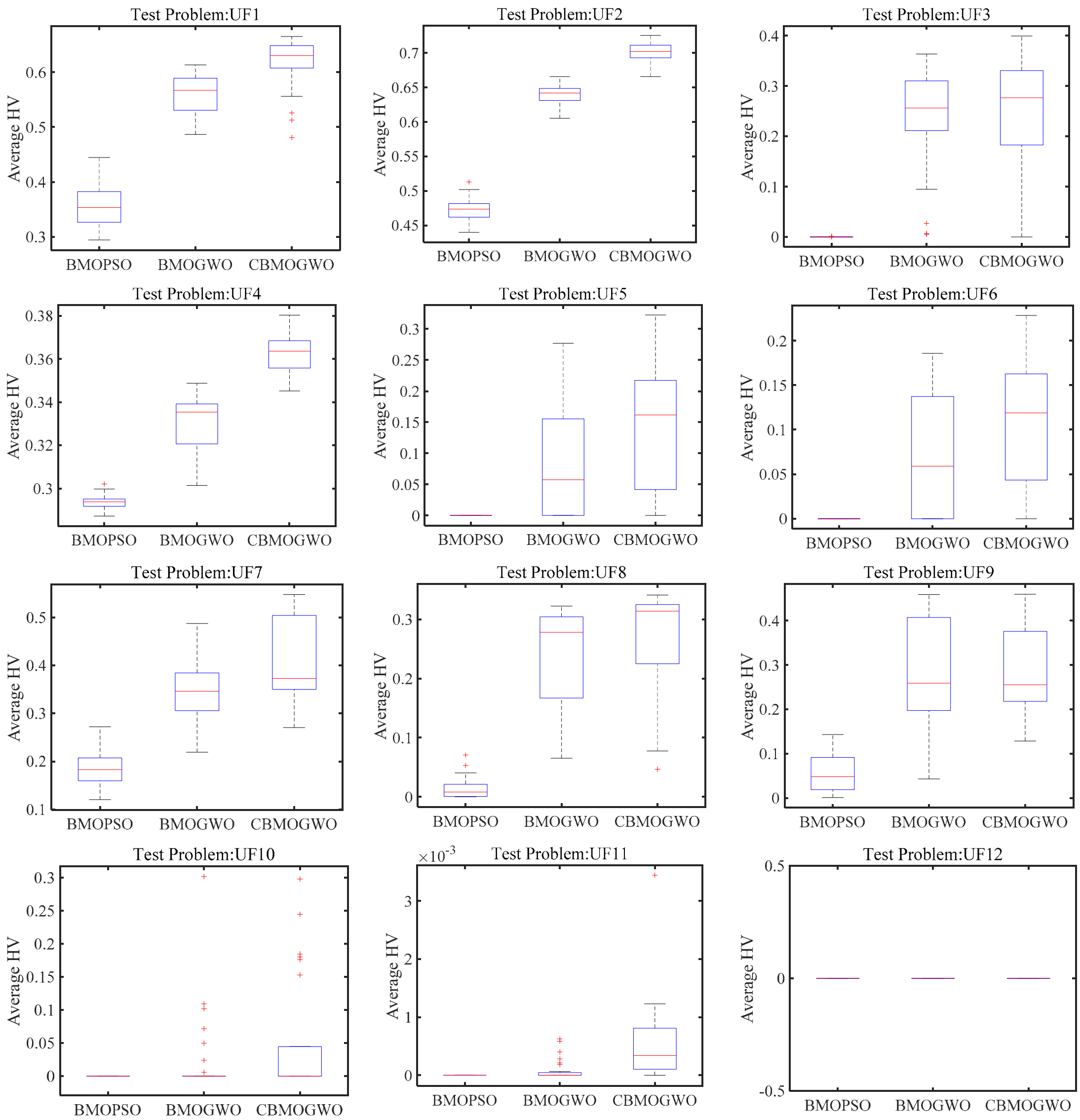

Fig. S3 Boxplot of the statistical results for HV on UF1 to UF12 on MOTPs

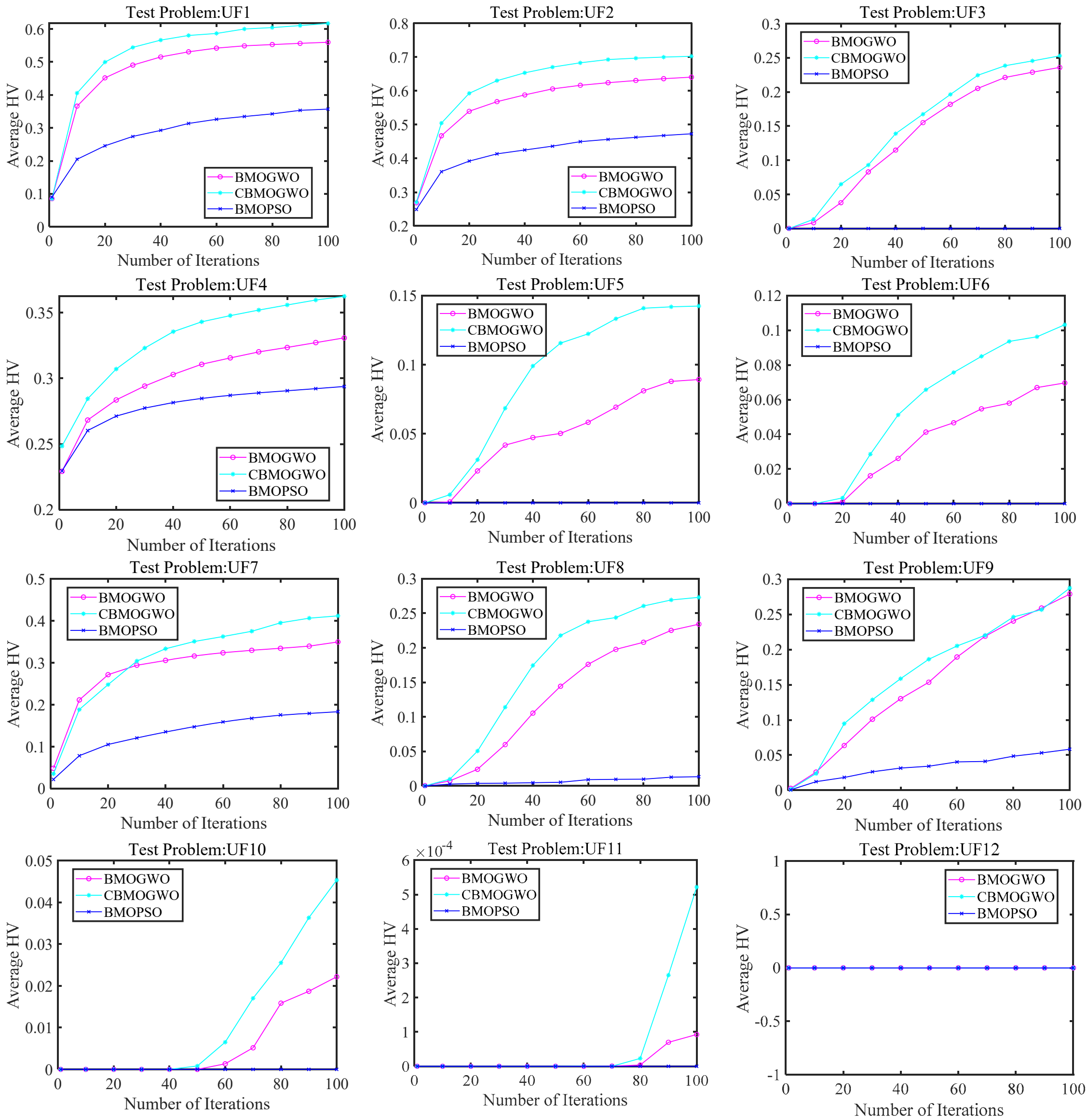

Fig. S4 Evolution of the average HV value on UF1 to UF12 on MOTPs

Table S6 Statistical results for HV on MOKPs

HV	*M*=2			*M*=3		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.3921	0.5195	0.5652	0.2270	0.3081	0.3340
Median	0.3915	0.5194	0.5645	0.2269	0.3120	0.3339
STD	0.0062	0.0034	0.0049	0.0032	0.0106	0.0038
Worst	0.3789	0.5138	0.5541	0.2220	0.2871	0.3246
Best	0.4059	0.5251	0.5780	0.2553	0.3218	0.3428
HV	*M*=4			*M*=5		
	BMOPSO	BMOGWO	CBMOGWO	BMOPSO	BMOGWO	CBMOGWO
Average	0.1318	0.1929	0.2015	0.0745	0.1173	0.1197
Median	0.1340	0.1927	0.2016	0.0744	0.1174	0.1199
STD	0.0018	0.0038	0.0035	0.0014	0.0012	0.0019
Worst	0.1285	0.1864	0.1941	0.0722	0.1149	0.1150
Best	0.1348	0.2014	0.2078	0.0783	0.1194	0.1241

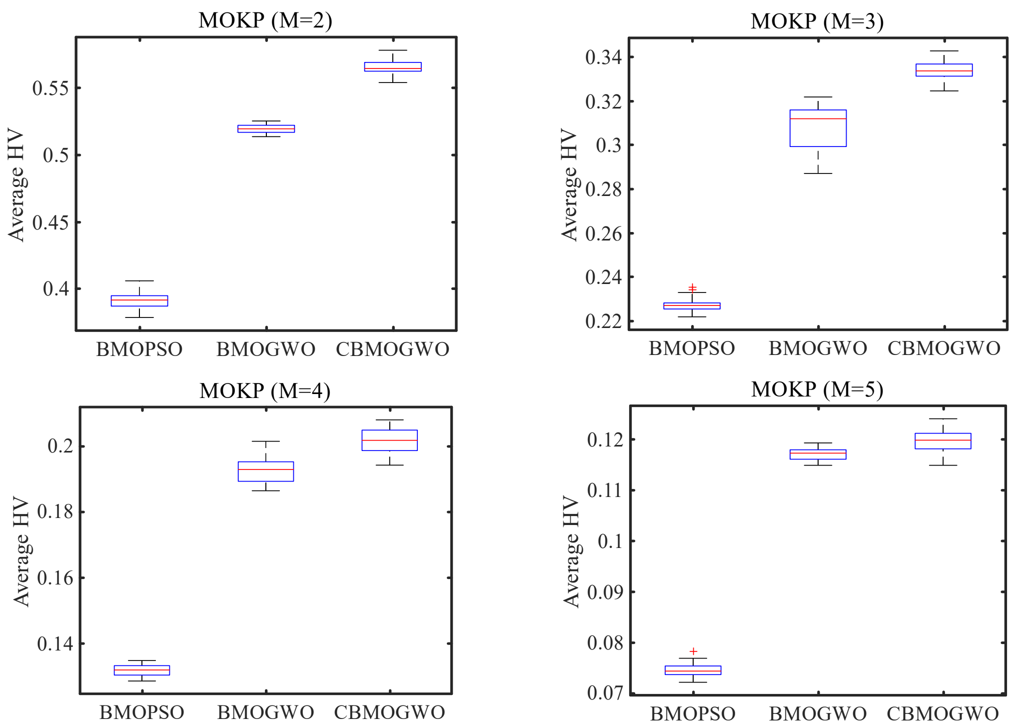

Fig. S5 Boxplot of the statistical results for HV on MOKPs

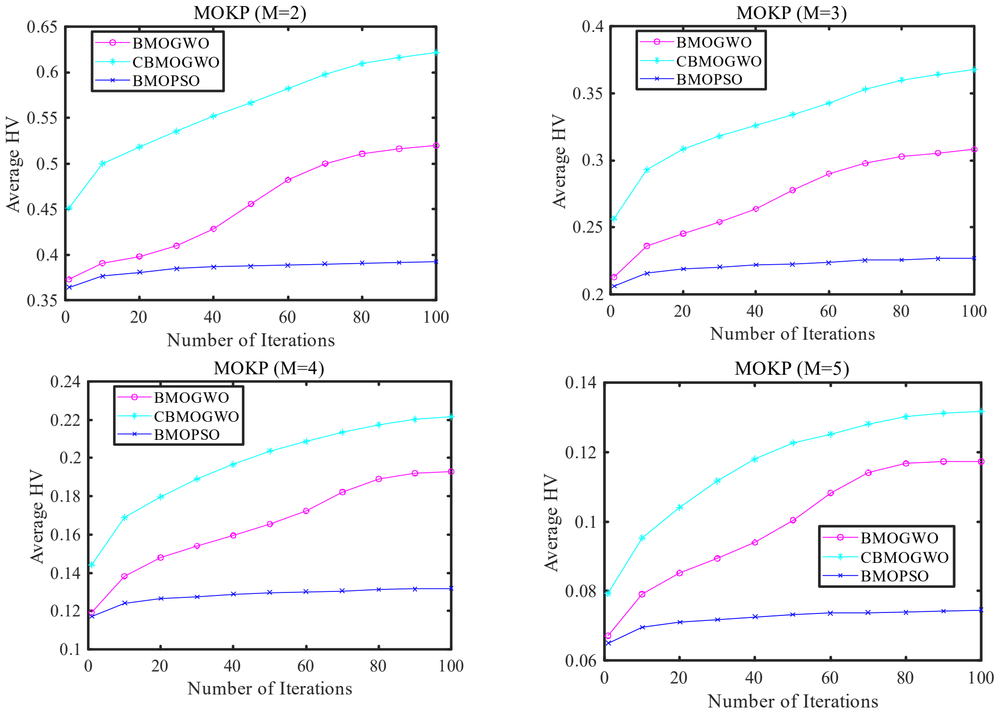

Fig. S6 Evolution of the average HV value on MOKPs

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