



Supplementary materials for

Huanpei LYU, Libin ZHANG, Dapeng TAN, Fang XU, 2022. A collaborative assembly for low-voltage electrical apparatuses. *Front Inform Technol Electron Eng*

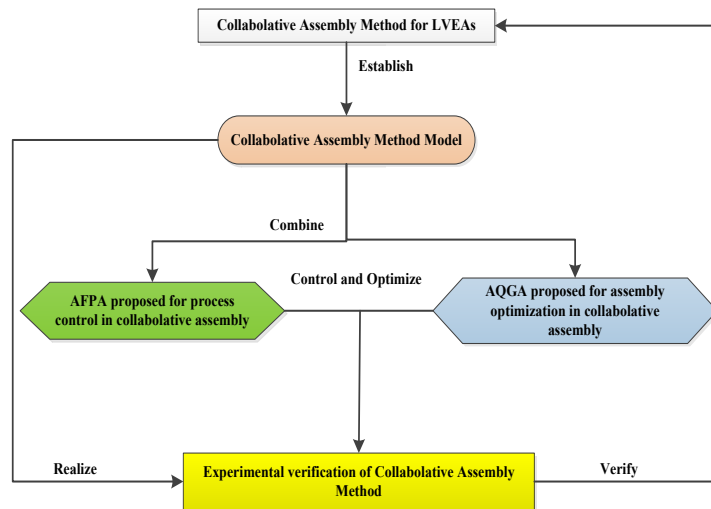


Fig. S1 The main content of this paper

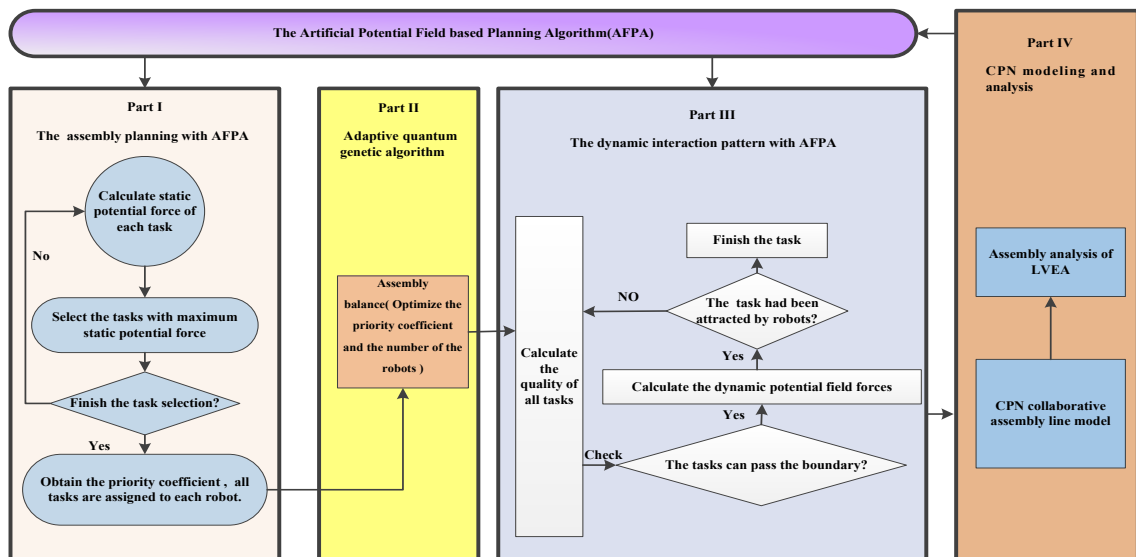


Fig. S2 The architecture of collaborative assembly methodology

If $\mathbf{X}(x, y)$ is the position of the robots, the attracted potential between a robot and task U_{att} is calculated as follows:

$$U_{att} = \frac{1}{2} k_{att} (\mathbf{X} - \mathbf{X}_g)^2, \quad (1)$$

where k_{att} is an attracted potential scaling factor and \mathbf{X}_g is the position of the goal. Thus, the attracted force \mathbf{F}_{att} can be defined as follows:

$$\mathbf{F}_{att} = -\nabla U_{att}(\mathbf{X}) = -k_{att} |\mathbf{X} - \mathbf{X}_g|. \quad (2)$$

The definition of repelled potential between a robot and an obstacle is calculated as follows:

$$U_{rep} = \begin{cases} \frac{1}{2} k_{rep} \left(\frac{1}{\mathbf{X} - \mathbf{X}_o} - \frac{1}{\rho_0} \right)^2, & \mathbf{X} \cdot \mathbf{X}_o \leq \rho_0, \\ 0, & \mathbf{X} \cdot \mathbf{X}_o \geq \rho_0 \end{cases}, \quad (3)$$

where k_{rep} is the repelled potential scaling factor, \mathbf{X}_o is the position of the obstacle, and ρ_0 is the interference distance of the obstacle. Hence, the repelled force \mathbf{F}_{rep} can be defined as follows:

$$\mathbf{F}_{rep} = -\nabla U_{rep}(\mathbf{X}) = \begin{cases} \frac{1}{2} k_{rep} \left(\frac{1}{\mathbf{X} \cdot \mathbf{X}_o} - \frac{1}{\rho_0} \right) \frac{1}{(\mathbf{X} \cdot \mathbf{X}_o)^2} \frac{\partial(\mathbf{X} \cdot \mathbf{X}_o)}{\partial \mathbf{X}}, & \mathbf{X} \cdot \mathbf{X}_o \leq \rho_0, \\ 0, & \mathbf{X} \cdot \mathbf{X}_o \geq \rho_0 \end{cases}. \quad (4)$$

Therefore, the global potential of robot U_{gro} is calculated as follows:

$$U_{gro} = U_{att} + U_{rep}, \quad (5)$$

where U_{rep} is the repelled potential between the robot and the obstacle.

The total force \mathbf{F}_{total} acting on a robot is calculated as follows:

$$\mathbf{F}_{total} = \mathbf{F}_{att} + \mathbf{F}_{rep}. \quad (6)$$

Fig. S3 The definition of the APF

The CPN is a multivariate group, where $CPN = (\Sigma, P, T, A, N, C, G, E, IN)$ and which satisfies the following conditions:

- 1) Σ is a type of non-empty finite set called a color set
- 2) P is a finite set of places
- 3) T is a finite set of transitions
- 4) A is a finite set of arcs, where $P \cap T = P \cap A = T \cap A = \Phi$, and Φ is an empty set
- 5) N is a node function, where $N: A \rightarrow (P \times TUT \times P)$
- 6) C is a color function, where $C: P \rightarrow \Sigma$
- 7) G is a guard function; it is defined from T into expressions such that: $\forall t \in T: [Type(G(t)) = Bool \wedge Type(Var(G(t))) \subseteq \Sigma]$, and has its own threshold, which is used to judge whether the input value is satisfied
- 8) E is an arc expression function; it is defined from E into expressions such that: $\forall a \in A: [Type(E(a)) = C(p(a))_{MS} \wedge Type(Var(E(a))) \subseteq \Sigma]$, where $p(a)$ is the place of $N(a)$
- 9) IN is an initial function; it is defined from P into expressions such that: $\forall p \in P: [Type(IN(p)) = C(p)_{MS} \wedge Var(IN(p)) = \Phi, \Phi$ is an empty set.

Fig. S4 The formal definition of CPN

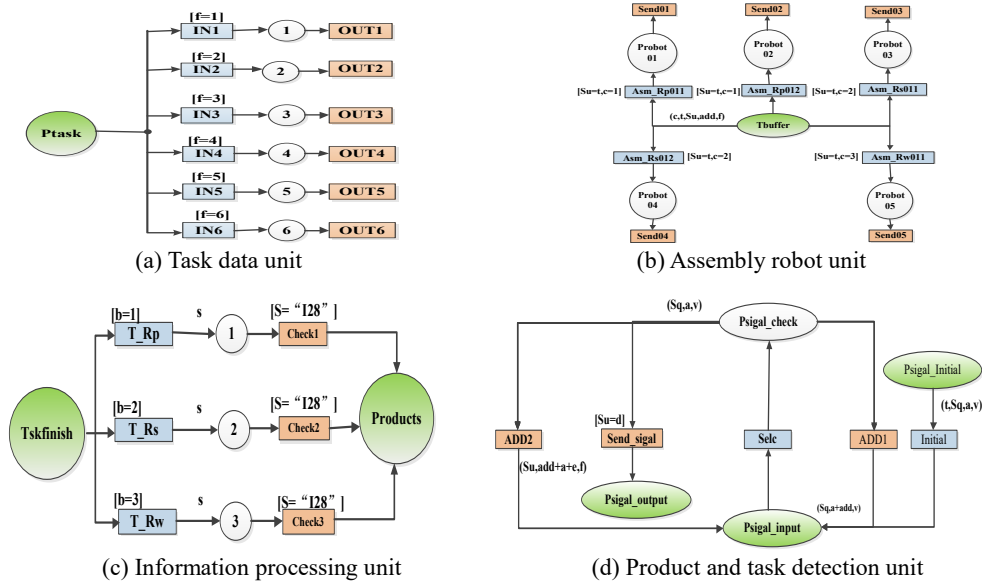


Fig. S5 Components of the CPN collaborative assembly model

Table S3 shows the setting of places in the model.

Thus, according to the theoretical model of the assembly, $Ptask: \{task|taskn = (s_n, t_n, b_n, sq_n, add_n, f_n), s_n \in Pname, t_n \in Ptnum, sq_n \in Ptchng, add_n \in Ptchng, f_n \in Ptslect, n \in N\}$. $Probot: \{robot|robotn = (t_n, b_n, sq_n, add_n, f_n), t_n \in Ptkind, b_n \in Ptnum, sq_n \in Ptchng, add_n \in Ptchng, f_n \in Ptslect, n \in N\}$. $Psigal: \{tsignal|tsignaln = (su_n \text{ and } f_n), add_n \in Ptchng, f_n \in Ptslect, su_n \in Ptchng, n \in N\}$. So in software CPN Tools, some places can be defined as follows.

Assembly task place colset:

$Ptask = \text{product } Pname * Ptkind * Ptnum * Ptsequ * Ptadd * Ptslect;$

Task caching place colset:

$Tbuffer = \text{product } Pname * Ptkind * Ptnum * Ptsequ * Ptadd;$

Completed task place colset:

$Tskfinish = \text{product } Pname * Ptkind * Ptslect;$

Assembly robot place colset:

$Probot = \text{product } Ptkind * Ptnum * Ptsequ * Ptadd * Ptslect;$

Task information place colset:

$Psigal = \text{product } Ptkind * Ptnum * Ptsequ * Ptadd * Ptslect.$

Fig. S6 Setting of places in the CPN collaborative assembly model

1) Transitions between task place and task buffer place

The variable $ts = \langle s, b, t, su, add, f \rangle$, and there is a guard function $G(ts)$:

$$G(ts) = \begin{cases} 1 & f = Prio \\ 0 & \end{cases}, \quad 1)$$

where $Prio$ is the assembly priority coefficient, $Prio \in [1, 2, 3, 4, 5, 6]$. That is, when the f is equal to the corresponding $Prio$, the task proceeds, and $Prio$ is applied as a threshold value in function $G(ts)$. In Fig. S5(a), “IN1”, “IN2”, “IN3”, “IN4”, “IN5”, and “IN6” belong to the transitions between the task place and the task buffer place.

2) Transitions between task buffer place and assembly robot place

The variable $tr_n = \langle b_n, t_n, sq_n, add_n, f_n \rangle$, and there is a guard function $G(tr_n)$:

$$G(tr_n) = \begin{cases} 1 & b_n = rk, t_n = sq_{next} \\ 0 & \end{cases}, \quad 2)$$

where rk is the robot-type coefficient, $rk \in [1, 2, 3]$; n is the serial number of the tasks, $n \in \{1, 2, 3, \dots, n, n+1, \dots\}$. When the value of b_n is equal to rk and when the signal of $t_n = sq_{next}$ is obtained, the task is accepted by the robot. In the function $G(tr_n)$, rk and sq_{next} are used as a threshold. In Fig. S5(b), “Asm_Rp012”, “Asm_Rs011”, “Asm_Rs012”, “Asm_Rp011”, and “Asm_Rw011” belong to the transitions between the task buffer place and the assembly robot place.

3) Transitions between the completed task place colset and the task or product name place

The variable $tf_n = \langle s_n, b_n, f_n \rangle$, and there is a guard function $G(tf_n)$:

$$G(tf_n) = \begin{cases} 1 & tf_n = rk, t_n = sq_{next} \\ 0 & \end{cases}, \quad 3)$$

where rk and sq_{next} are the thresholds of $G(tf_n)$.

In Fig. S5(c), “T_Rp”, “T_Rs”, “T_Rw”, “Check1”, “Check2”, and “Check3” belong to the transitions between the completed task place colset and the task or product name place.

4) Transitions between input information place and output information place

The variable $Inform_n = \langle sq_n, add_n, f_n \rangle$, and there is a guard function:

$$G(Inform_n) = \begin{cases} 1 & sq_n = add_n \\ 0 & \end{cases}. \quad 4)$$

In the function $G(Inform_n)$, the threshold is the input value $Inform_n (\langle sq_n, add_n, f_n \rangle)$, which is used for comparison with the next input value $Inform_{n+1}$, when $sq_n = sq_{n+1}, add_n \neq add_{n+1}, sq_n \neq add_n$ and $sq_n \neq add_n$, the threshold is equal to $\langle sq_n, add_n + add_n, f_n \rangle$.

In Fig. S5(d), “ADD1”, “ADD2”, “Send_signal”, “Selc”, and “Initial” belong to the transitions between the input information place and the output information place.

Accordingly, the corresponding assembly unit is designed in CPN Tools by combining the places and transitions mentioned above, as shown in Fig. S5.

Fig. S7 Setting of the model transitions in the CPN collaborative assembly model

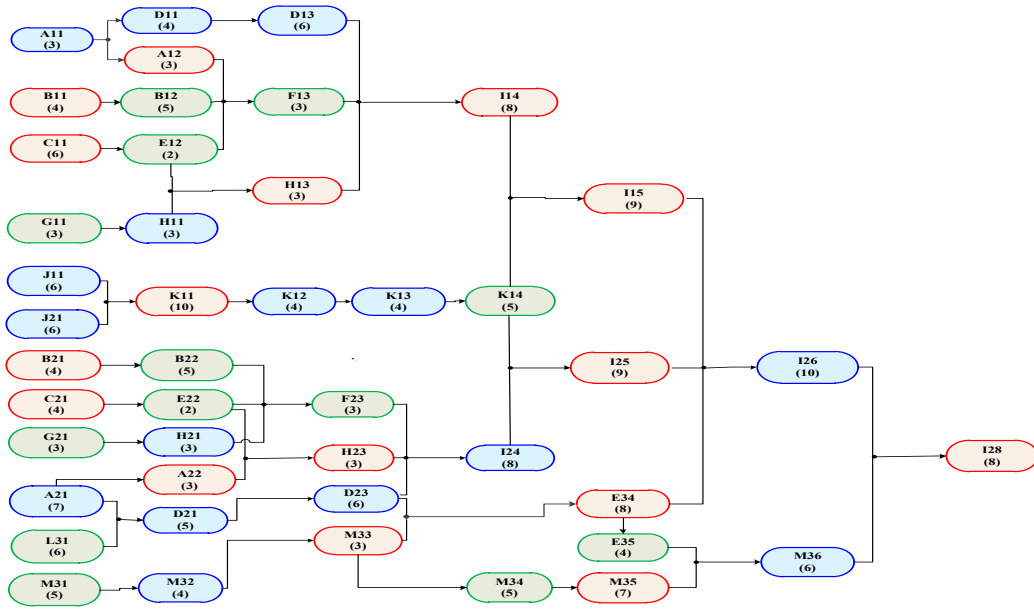


Fig. S8 The relationship between the tasks required for a TPCLP

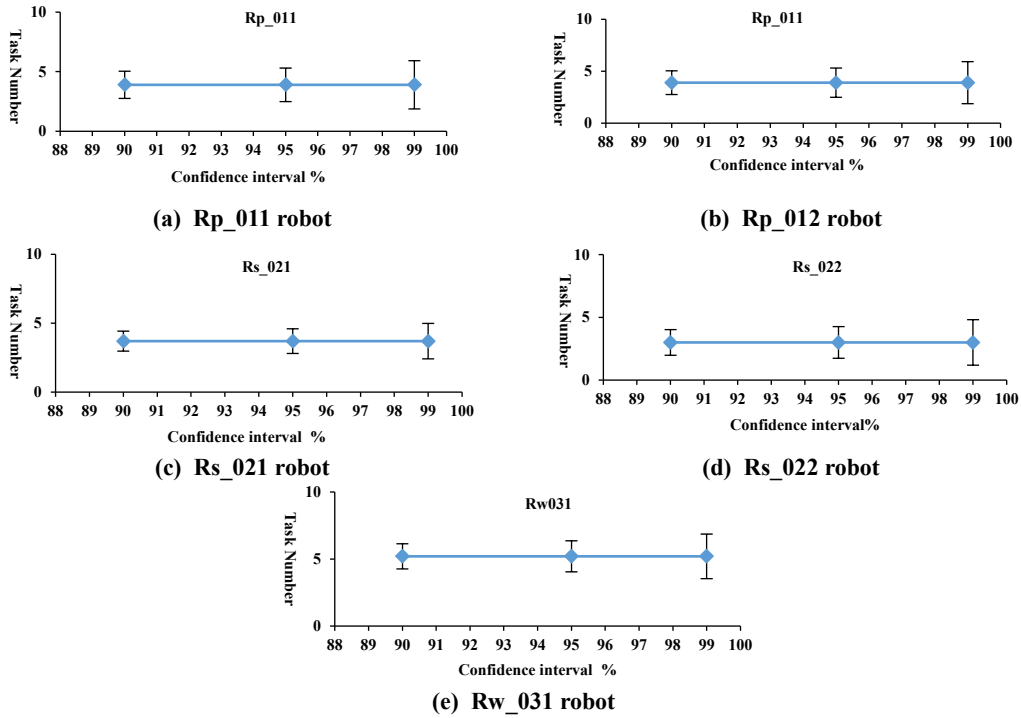


Fig. S9 The average number of assembly tasks of the assembly robots for different confidence intervals

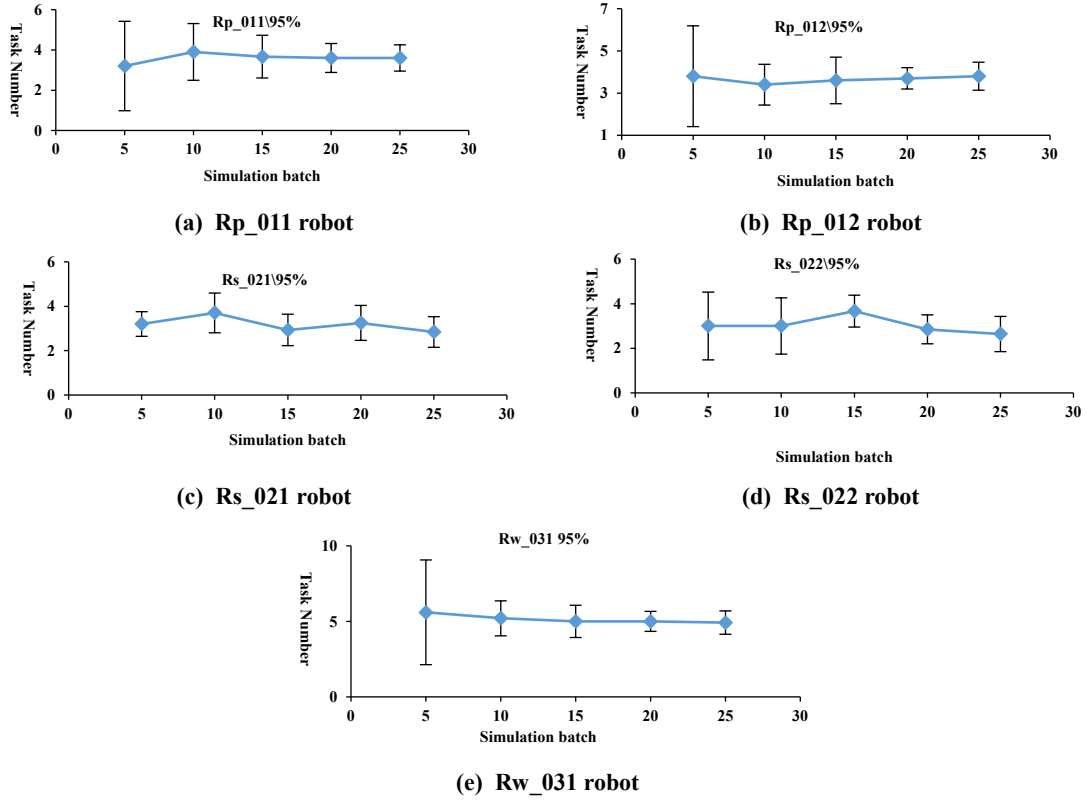
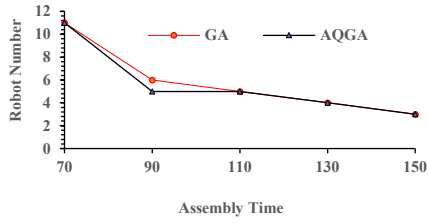


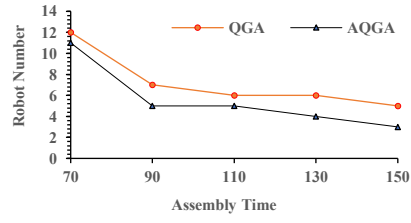
Fig. S10 The number of assembly tasks for each assembly robot for different simulation batches

- 1) GA: The population size is 40, the maximum genetic generation is 20, the individual variable dimension is 45, the crossover probability is 0.75, and the mutation rate is 0.04.
- 2) GASA: The population size is 40, the maximum genetic generation is 20, the individual variable dimension is 45, the crossover probability is 0.75, and the mutation rate is 0.04.
- 3) QGA: The population size is 40, the maximum genetic generation is 20, the individual variable dimension is 45, and the quantum rotation angle is 0.05.
- 4) RG-QGA: The population size is 40, the maximum genetic generation is 20, the individual variable dimension is 45, and the quantum rotation angle is 0.05.
- 5) AQGA: The population size is 40, the maximum genetic generation is 20, the individual variable dimension is 45, and the quantum rotation angle is 0.05.

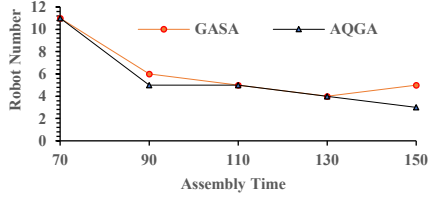
Fig. S11 The setting of relevant parameters of the compared algorithms



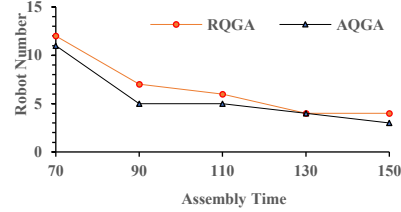
(a) Comparison between GA and AQGA



(b) Comparison between QGA and AQGA

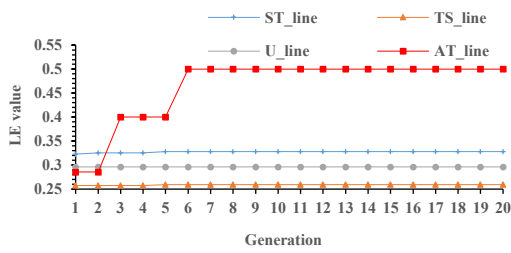


(c) Comparison between GASA and AQGA

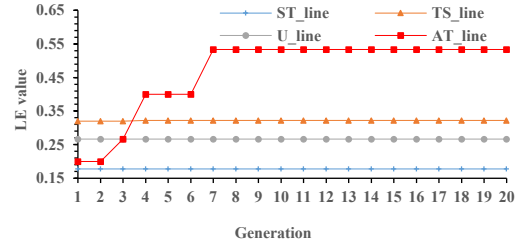


(d) Comparison between RQGA and AQGA

Fig. S12 A comparison of the number of robots required for the different algorithms.

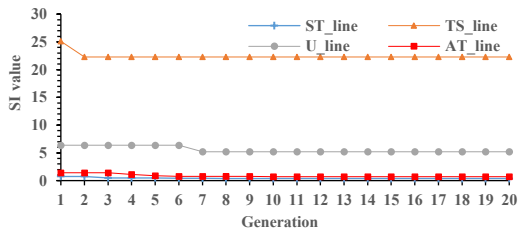


(a) The LE value for an assembly cycle of 90

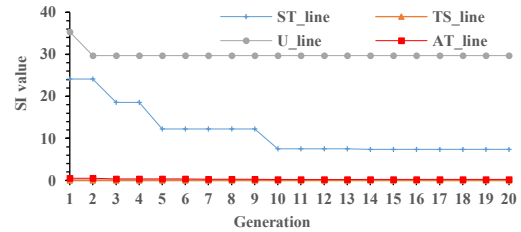


(b) The LE value for an assembly cycle of 150

Fig. S13 The LE values of each assembly mode for different assembly cycles.

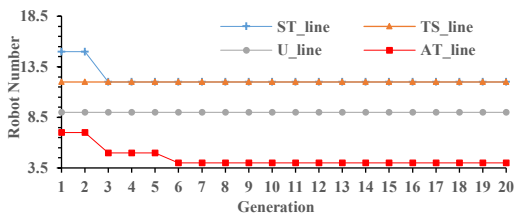


(a) SI value for an assembly cycle of 90

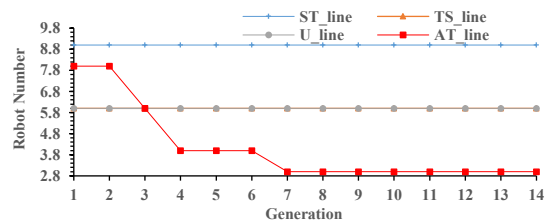


(b) The SI value for an assembly cycle of 150

Fig. S14 The SI value of each assembly mode for different assembly cycles

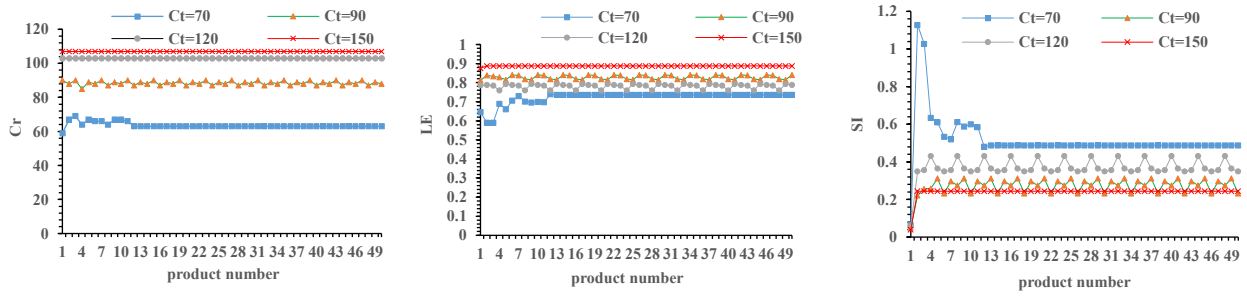


a) The number of robots required when the assembly cycle is 90



b) The number of robots required when the assembly cycle is 150

Fig. S15 The number of robots required for each assembly mode for different assembly cycles.



(a) Actual assembly cycle (b) The LE value (c) The SI value
 Fig. S16 Analysis of the robot assembly performance with the dynamic balance of AFPA.

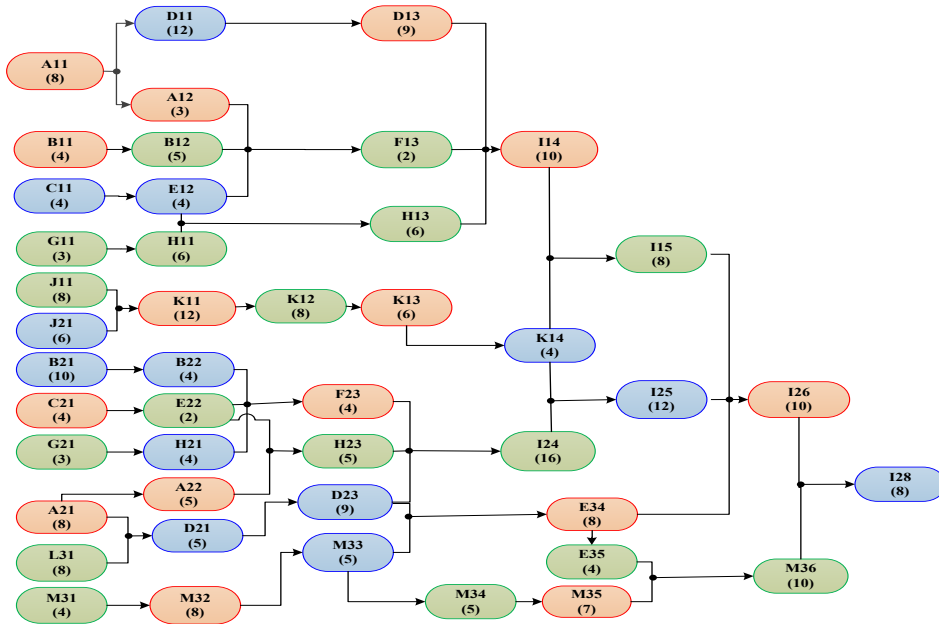


Fig. S17 The relationship between the assembly tasks required for Type 2

Table S1 Studies of assembly line construction and balance optimization

Author	Line type	Solution Approach	Assembly Problem
Yu MY, et al., 2018	Flexible assembly	Dynamic management of flexible assembly process based on actual information	The agility requirements of multi-variety and small-batch production
Cil ZA, et al., 2017	Robotic parallel assembly line	Iterative beam search (IBS), best search method based on IBS (BIBS) and cutting BIBS (CBIBS) algorithms	Min. joint cycle time
Tavakoli A, 2020	Multi-product assembly line	Tabu search and simulated annealing algorithms	Multi-criteria optimization of multi-product assembly line
Yan P, et al., 2011	Parallel assembly line	An improved PSO-SS(particle swarm optimization-Scatter Search) co-evolution algorithm	Multi-robot parallel assembly optimization
Li XL, et al., 2021	Flexible assembly	Hybrid POS algorithm	The scheduling problem of flexible assembly systems (FASs) without intermediate buffers
Samouei P, et al., 2016	Fuzzy mixed-model assembly line	Hierarchical-cyclic heuristic approach	Fuzzy mixed-model assembly line balancing problem
Özcan U, et al., 2009	Simple straight and U-type assembly line	Adaptive learning approach and simulated annealing	Simple straight and U-type assembly line balancing problems
Rizwan M, et al., 2020	Human-robot collaborative assembly	Hybrid conditional planning, answer set programming	Human-robot collaborative assembly planning
Johannsmeier L, et al., 2017	Human-robot collaborative assembly	A hierarchical human-robot interaction-planning framework	Assembly task allocation
He F, et al., 2021	Robot assembly	Improved deep deterministic policy gradient reinforcement learning algorithm	Axle-hole assembly

Table S2 Assembly tasks corresponding to each assembly robot

Robot type	Corresponding assembly tasks
R_p	A11 J11 J21 A21 D11 H11 H21 D21 M32 D13 K12 D23 I14 K13 I24 I15 M36 I26
R_s	A12 B11 C21 C11 B21 K11 H13 H23 M33 E34 M35 I25 I28 A22
R_w	G11 A11 G21 L31 M31 B12 E12 E22 F13 F23 M34 K14 E35

Table S3 The setting of places in the model

Place Name	Color Set	Variable
Task or product name and its variables	<i>Ptname</i> = string;	var <i>s</i>
Task type and its variables	<i>Ptkind</i> = int;	var <i>t</i>
Task feature and its variables	<i>Ptnum</i> = int;	var <i>a, b, c, e</i>
Follow-up task feature and its variables	<i>Ptsequ</i> = int;	var <i>Su, Sq, Sv</i>
Task feature change and its variables	<i>Ptadd</i> = int;	var <i>add, p, v, u</i>
Task priority coefficient and its variables	<i>Ptslect</i> = int;	var <i>f</i>

**Table S4 The relationship between
the assembly cycle and the robot number**

Assembly Cycle(time units)	Robot Number
70	11
90	5
110	5
130	4
150	3

Table S5 The number of assembly tasks for each type of robot

System status	Monitoring points	Status	Number of tasks completed	Number of products completed
All robots are functioning normally	ASM_Rp011	Normal	166	10
	ASM_Rp012	Normal	164	
	ASM_Rs011	Normal	98	
	ASM_Rs012	Normal	82	
	ASM_Rw011	Normal	210	
	T_Rp	Normal	330	
	T_Rs	Normal	180	
One robot failed	T_Rw	Normal	210	10
	ASM_Rp011	Normal	330	
	ASM_Rp012	Trouble	0	
	ASM_Rs011	Normal	90	
	ASM_Rs012	Normal	90	
	ASM_Rw011	Normal	210	
	T_Rp	Normal	330	
Two robots failed	T_Rs	Normal	180	10
	T_Rw	Normal	210	
	ASM_Rp011	Normal	330	
	ASM_Rp012	Trouble	0	
	ASM_Rs011	Normal	180	
	ASM_Rs012	Trouble	0	
	ASM_nh011	Normal	210	
Three robots failed	T_Rp	Normal	330	0
	T_Rs	Normal	180	
	T_Rw	Normal	210	
	ASM_Rp011	Normal	160	
	ASM_Rp012	Trouble	0	
	ASM_Rs011	Normal	100	
	ASM_Rs012	Trouble	0	
ASM_Rw011	Trouble	210		
T_Rp	Normal	160		
T_Rs	Normal	100		
T_Rw	Trouble	0		

Table S6 The value range of the critical parameters

Parameter	Population size	Genetic generation	Crossover probability	Mutation rate
Value range	20–100	20–500	0.4–0.99	0.0001–0.1

Table S7 Comparison of optimal target values for each algorithm.

Ct	Algorithms	Min Fitness	Max LE	Min SI	Min S
70	QGA	1.133	0.079	0.6627	12
	GA	1.132	0.08571	0.676	11
	GASA	1.123	0.08571	0.683	11
	RQGA	1.132	0.2857	0.778	12
	AQGA	1.130	0.3117	0.683	11
90	QGA	1.115	0.129	0.621	7
	GA	1.113	0.167	0.549	6
	GASA	1.113	0.162	0.609	6
	RQGA	1.112	0.381	0.602	7
	AQGA	1.108	0.533	0.493	5
110	QGA	1.113	0.1379	0.5064	6
	GA	1.111	0.1745	0.4311	5
	GASA	1.112	0.1745	0.4595	5
	RQGA	1.11	0.3636	0.6005	6
	AQGA	1.109	0.4928	0.4364	5
130	QGA	1.113	0.1167	0.4152	6
	GA	1.11	0.2212	0.3784	4
	GASA	1.11	0.2308	0.3832	4
	RQGA	1.107	0.4615	0.3678	4
	AQGA	1.107	0.4615	0.3253	4
150	QGA	1.11	0.1773	0.3898	5
	GA	1.108	0.32	0.2798	3
	GASA	1.111	0.1533	0.4253	5
	RQGA	1.106	0.2	0.4	4
	AQGA	1.106	0.5333	0.2411	3

Table S8 Statistical data of different algorithms.

Items No.	RBN_A QGA	RBN_ GA	RBN_ GASA	RBN_Q GA	RBN_R QGA	SI_AQ GA	SI_GA SA	SI_GA A	SI_QG A	SI_RQ GA
1	5.000	6.000	7.000	6.000	6.000	0.608	0.608	0.764	0.594	0.609
2	6.000	7.000	6.000	6.000	6.000	0.539	0.735	0.609	0.508	0.518
3	5.000	5.000	6.000	7.000	6.000	0.463	0.505	0.609	0.583	0.530
4	5.000	6.000	6.000	6.000	6.000	0.505	0.602	0.609	0.563	0.529
5	5.000	5.000	6.000	6.000	7.000	0.492	0.431	0.609	0.609	0.431
6	5.000	7.000	6.000	7.000	7.000	0.478	0.583	0.609	0.596	0.583
7	5.000	5.000	7.000	6.000	6.000	0.500	0.431	0.735	0.492	0.508
8	5.000	6.000	5.000	6.000	6.000	0.414	0.598	0.434	0.492	0.609
9	5.000	6.000	7.000	7.000	6.000	0.508	0.608	0.763	0.554	0.508
10	5.000	6.000	7.000	6.000	6.000	0.459	0.603	0.735	0.589	0.463
Items No.	LE_AQ GA	LE_G A	LE_G ASA	LE_QG A	LE_RQ GA	FIT_A QGA	FIT_G A	FIT_G ASA	FIT_Q GA	FIT_R QGA
1	0.444	0.161	0.138	0.161	0.444	1.110	1.113	1.116	1.113	1.110
2	0.444	0.124	0.161	0.161	0.444	1.110	1.115	1.111	1.113	1.110
3	0.533	0.198	0.161	0.138	0.444	1.108	1.112	1.113	1.115	1.110
4	0.533	0.167	0.161	0.161	0.444	1.108	1.113	1.113	1.113	1.110
5	0.533	0.200	0.161	0.161	0.381	1.108	1.111	1.113	1.113	1.113
6	0.533	0.124	0.161	0.129	0.381	1.108	1.115	1.113	1.115	1.113
7	0.533	0.200	0.124	0.161	0.444	1.108	1.111	1.115	1.113	1.110
8	0.533	0.161	0.200	0.161	0.444	1.108	1.113	1.111	1.113	1.110
9	0.533	0.161	0.138	0.138	0.444	1.108	1.113	1.116	1.115	1.111
10	0.533	0.167	0.124	0.161	0.444	1.108	1.113	1.115	1.113	1.110

Table S9 Descriptive statistics of different algorithms.

Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
RBN_AQGA	10	5.10000	0.316228	5.000	6.000	5.00000	5.00000	5.00000
RBN_GA	10	5.90000	0.737865	5.000	7.000	5.00000	6.00000	6.25000
RBN_GASA	10	6.30000	0.674949	5.000	7.000	6.00000	6.00000	7.00000
RBN_QGA	10	6.30000	0.483046	6.000	7.000	6.00000	6.00000	7.00000
RBN_RQGA	10	6.20000	0.421637	6.000	7.000	6.00000	6.00000	6.25000
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
SI_AQGA	10	0.49660	.051803	0.414	0.608	0.46200	0.49600	0.51575
SI_GA	10	0.57040	.091920	0.431	0.735	0.48650	0.60000	0.60800
SI_GASA	10	0.64760	.102858	0.434	0.764	0.60900	0.60900	0.74200
SI_QGA	10	0.55800	.044944	0.492	0.609	0.50400	0.57300	0.59450
SI_RQGA	10	0.52880	.058385	0.431	0.609	0.49675	0.52350	0.58950
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
LE_AQGA	10	0.51520	0.037526	0.444	0.533	0.51075	0.53300	0.53300
LE_GA	10	0.16630	0.027769	0.124	0.200	0.15175	0.16400	0.19850
LE_GASA	10	0.15290	0.022757	0.124	0.200	0.13450	0.16100	0.16100
LE_GQA	10	0.15320	0.012796	0.129	0.161	0.13800	0.16100	0.16100
LE_RQGA	10	0.43140	0.026563	0.381	0.444	0.42825	0.44400	0.44400
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
FIT_AQGA	10	1.10840	0.000843	1.108	1.110	1.10800	1.10800	1.10850
FIT_GA	10	1.11290	0.001370	1.111	1.115	1.11175	1.11300	1.11350
FIT_GASA	10	1.11360	0.001838	1.111	1.116	1.11250	1.11300	1.11525
FIT_QGA	10	1.11360	0.000966	1.113	1.115	1.11300	1.11300	1.11500
FIT_RQGA	10	1.11070	0.001252	1.110	1.113	1.11000	1.11000	1.11150

Table S10 Ranks for different algorithms.

Variable	MeanRank	Variable	MeanRank	Variable	MeanRank	Variable	MeanRank
RBN_AQGA	1.35	SI_AQGA	1.80	LE_AQGA	4.90	FIT_AQGA	1.10
RBN_GA	2.95	SI_GA	3.05	LE_GA	2.40	FIT_GA	3.60
RBN_GASA	3.60	SI_GASA	4.55	LE_GASA	1.80	FIT_GASA	4.05
RBN_QGA	3.65	SI_QGA	2.85	LE_QGA	1.80	FIT_QGA	4.10
RBN_RQGA	3.45	SI_RQGA	2.75	LE_RQGA	4.10	FIT_RQGA	2.15

Table S11 Friedman test statistics of different algorithms.

RBN		SI		LE		FIT	
Variable	Value	Variable	Value	Variable	Value	Variable	Value
N	10	N	10	N	10	N	10
Chi-square	19.148	Chi-square	16.123	Chi-square	33.583	Chi-square	30.292
df	4	df	4	df	4	df	4
Asymp. Sig.	0.001	Asymp. Sig.	0.003	Asymp. Sig.	0.000	Asymp. Sig.	0.000

Table S12 Statistical data of different assembly lines

Items No.	ST_RBN	ST_LE	ST_SI	ST_FIT	TS_RBN	TS_LE	TS_SI	TS_FIT
1	12.000	0.332	0.633	1.333	12.000	0.319	3.808	1.150
2	12.000	0.332	0.894	1.400	12.000	0.319	4.123	1.163
3	12.000	0.332	0.633	1.400	12.000	0.311	8.515	1.185
4	12.000	0.331	1.000	1.400	12.000	0.324	3.606	1.155
5	12.000	0.332	0.633	1.400	12.000	0.324	2.916	1.176
6	12.000	0.332	0.633	1.333	12.000	0.320	4.301	1.155
7	12.000	0.331	1.155	1.400	12.000	0.317	2.916	1.184
8	12.000	0.332	0.633	1.333	12.000	0.311	7.106	1.184
9	12.000	0.331	1.000	1.400	12.000	0.324	2.916	1.145
10	12.000	0.332	0.633	1.333	12.000	0.317	4.301	1.174
Items No.	U_RBN	U_LE	U_SI	U_FIT	AT_RBN	AT_LE	AT_SI	AT_FIT
1	9.000	0.296	1.291	1.121	5.000	0.533	0.412	1.107
2	9.000	0.296	2.582	1.108	6.000	0.444	0.608	1.110
3	9.000	0.296	8.042	1.108	6.000	0.444	0.608	1.110
4	9.000	0.296	1.291	1.121	5.000	0.533	0.508	1.108
5	9.000	0.296	4.203	1.150	5.000	0.533	0.533	1.108
6	9.000	0.296	7.594	1.108	5.000	0.412	0.533	1.107
7	9.000	0.296	1.291	1.108	5.000	0.431	0.533	1.108
8	9.000	0.296	1.291	1.121	5.000	0.533	0.505	1.108
9	9.000	0.296	1.291	1.121	5.000	0.533	0.434	1.108
10	9.000	0.296	1.291	1.121	6.000	0.444	0.608	1.110

Table S13 Descriptive statistics of different assembly lines

Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
ST_RBN	10	12.00000	0.000000	12.000	12.000	12.00000	12.00000	12.00000
TS_RBN	10	12.00000	0.000000	12.000	12.000	12.00000	12.00000	12.00000
U_RBN	10	9.00000	0.000000	9.000	9.000	9.00000	9.00000	9.00000
AT_RBN	10	5.30000	0.483046	5.000	6.000	5.00000	5.00000	6.00000
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
ST_LE	10	0.33170	0.000483	0.331	0.332	0.33100	0.33200	0.33200
TS_LE	10	0.31861	0.004818	0.311	0.324	0.31553	0.31900	0.32400
U_LE	10	0.29600	0.000000	0.296	0.296	0.29600	0.29600	0.29600
AT_LE	10	0.48403	0.052522	0.412	0.533	0.44075	0.48850	0.53300
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
ST_SI	10	0.78470	0.205441	0.633	1.155	0.63300	0.63300	1.00000
TS_SI	10	4.45080	1.883078	2.916	8.515	2.91600	3.96550	5.00225
U_SI	10	3.01670	2.701231	1.291	8.042	1.29100	1.29100	5.05075
AT_SI	10	0.52820	0.068715	0.412	0.608	0.48725	0.53300	0.60800
Variable	N.	Mean	Sd.	Min.	Max.	P25	P50	P75
ST_FIT	10	1.37323	0.034560	1.333	1.400	1.33300	1.40000	1.40000
TS_FIT	10	1.16710	0.015308	1.145	1.185	1.15375	1.16850	1.18400
U_FIT	10	1.11870	0.012755	1.108	1.150	1.10800	1.12100	1.12100
AT_FIT	10	1.10840	0.001174	1.107	1.110	1.10775	1.10800	1.11000

Table S14 Ranks of different assembly lines

Variable	MeanRank	Variable	MeanRank	Variable	MeanRank	Variable	MeanRank
ST_RBN	3.50	ST_LE	3.00	ST_SI	2.00	ST_FIT	4.00
TS_RBN	3.50	TS_LE	2.00	TS_SI	3.80	TS_FIT	3.00
U_RBN	2.00	U_LE	1.00	U_SI	3.20	U_FIT	1.75
AT_RBN	1.00	AT_LE	4.00	AT_SI	1.00	AT_FIT	1.25

Table S15 Friedman test statistics of different assembly lines

RBN		SI		LE		FIT	
Variable	Value	Variable	Value	Variable	Value	Variable	Value
N	10	N	10	N	10	N	10
Chi-square	30.000	Chi-square	28.080	Chi-square	30.000	Chi-square	28.030
df	3	df	3	df	3	df	3
Asymp. Sig.	0.000	Asymp. Sig.	0.000	Asymp. Sig.	0.000	Asymp. Sig.	0.000

Table S16 A comparison between the collaborative assembly and basic assembly lines

Theoretical cycle	Com- pared parame- ters	One-sided linear assembly /simulated Annealing	Double-sided linear assembly /tabu search	U-shaped assembly /genetic algorithm	Collaborative assembly /AFPA-AQGA
$C_r=70$	LE	0.33	0.35	0.29	0.74
	SI	1	2.2361	3.74	0.49
	S	12	12	9	5
$C_r=90$	LE	0.33	0.2574	0.3	0.82–0.84
	SI	1.41	26.7067	2.45	0.23–0.31
	S	9	12	9	4
$C_r=120$	LE	0.33	0.1986	0.22	0.76–0.79
	SI	1.18	64.3623	1.29	0.35–0.43
	S	9	12	6	3
$C_r=150$	LE	0.32	0.3067	0.27	0.89
	SI	1.2	0	1.41	0.24
	S	5	6	6	3
	LE	0.32	0.3067	0.27	0.89