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E-mail: jzus@zju.edu.cn

Supplementary materials for

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1 Further discussion of the proposed HS-CS algorithm

The entire flowchart based on the steps in our manuscript is illustrated in Fig. S1.

The highlights of the proposed HS-CS algorithm can be summarized in two points.

On one hand, the population updating using Levy flight enables the CS operator to have the characteristic of the intermittent and irregular search. This characteristic ensures that the modified HS-CS algorithm effectively avoids falling into the local optima.

On the other hand, "pitch adjusting and selecting the best" can avoid looping search operations by improving the efficiency, the convergence accuracy, and the execution speed of the HS-CS algorithm.

Compared to the standard HS, the modified strategies can effectively improve the global search ability of the HS-CS algorithm.

2 Function optimization using HS-CS

2.1 Experiment results and analysis

Figs. S2–S13 depict the corresponding logarithmic convergence curves of these 12 classical functions using the selected four HS variants and three CS variants under different dimensions.

In these convergence graphs, there are seven polylines to show the downward trend, approaching the global optimum solution. Finally, a higher accuracy of each HS variant and CS variant is obtained.

After analyzing Figs. S2–S13, it is easy to find that the proposed HS-CS algorithm is superior to the six other SI algorithms in terms of convergence speed and accuracy.

In the case of 10-dimensional problems (Figs. S2a–S13a), the HS-CS and AGOHS algorithms indicate that they can quickly converge to the theoretical global optimum, but the convergence efficiency of AGOHS is slightly worse than that of HS-CS when optimizing functions such as F1–F12. HS-CS shows good exploration and exploitation capabilities both in multimodal functions (F2–F6, F12) and unimodal functions (F1, F7–F11), which can converge to the global optimum within 500 iterations. However, despite the fact that IDHS can nearly reach the best solution on 9 out of all 12 test instances on functions F2–F6 and F9–F12, ECS has a better performance than IDHS in convergence efficiency on functions F1, F5–F7, F9, and F11, and it also exhibits a higher convergence speed than AGOHS on functions F9 and F11. It can be seen from Figs. S3a–S5a and S13a that when optimizing functions (e.g., F2–F4, F12) with many local optima, due to the fact that ECS lacks the self-adaptive learning ability, its local exploitation capability declines in the later searching stage, falls into a local optimum, and cannot reach the global optimum. Moreover, Figs. S2a–S13a show that the search accuracy obtained by HSDM is slightly better than that by MCS and CS, but the optimal solution obtained by those algorithms is significantly worse than HS-CS.

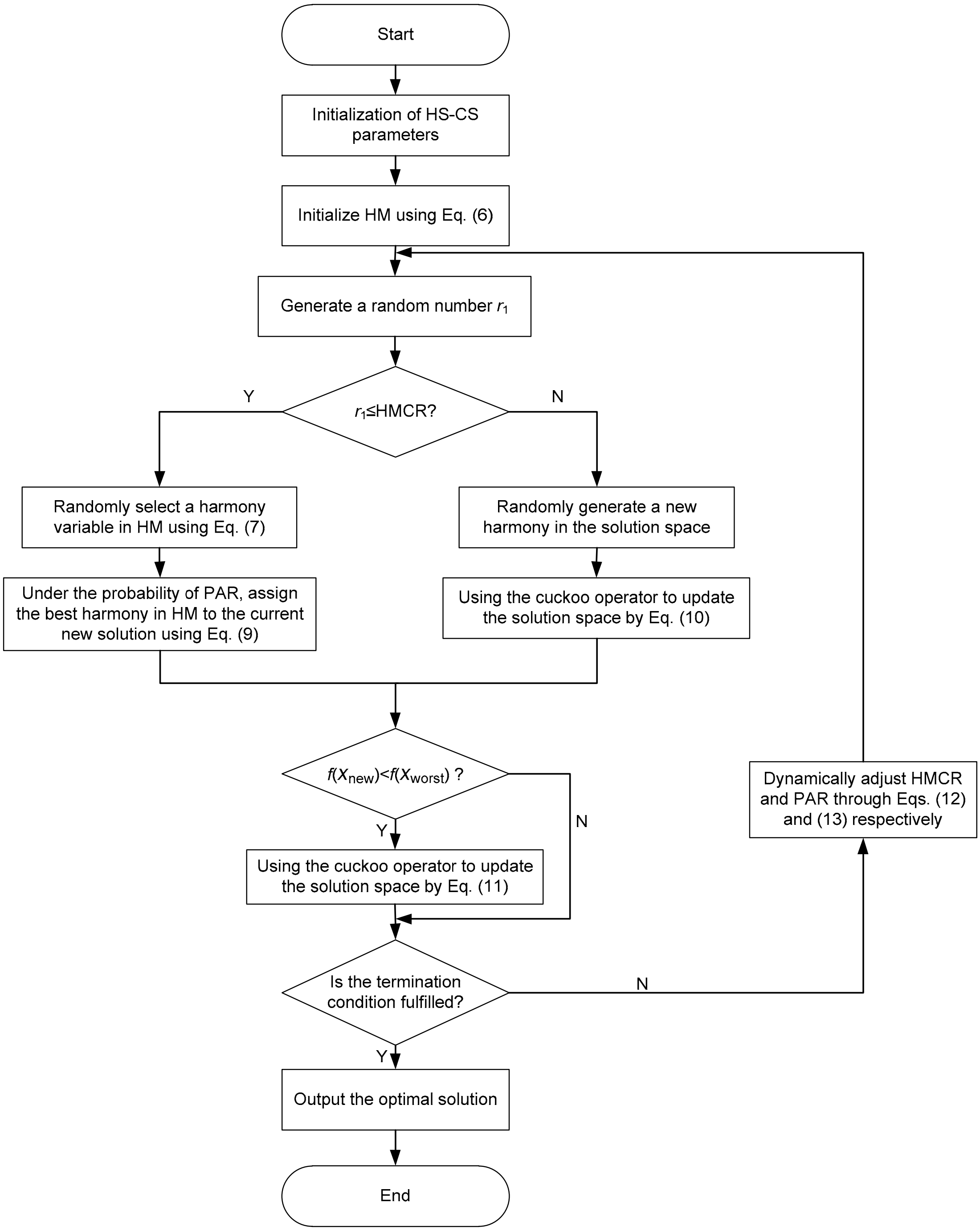
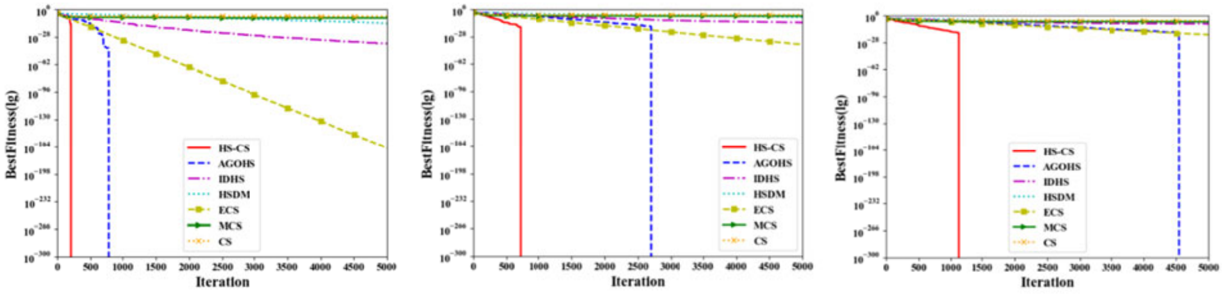
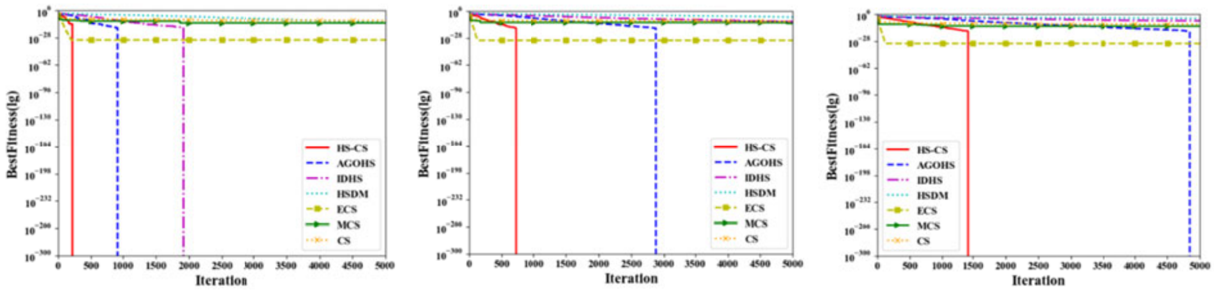


Fig. S1 Flowchart of the HS-CS algorithm



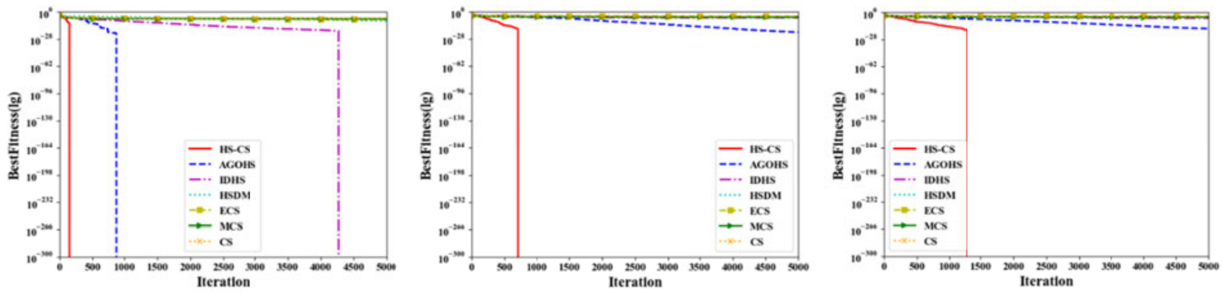
(a) (b) (c)

Fig. S2 Optimizing the convergence curve of F1 (Sphere) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



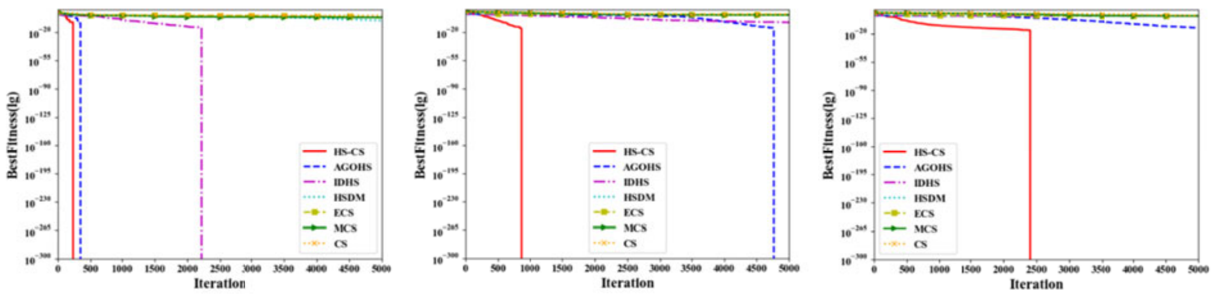
(a) (b) (c)

Fig. S3 Optimizing the convergence curve of F2 (LevyN13) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



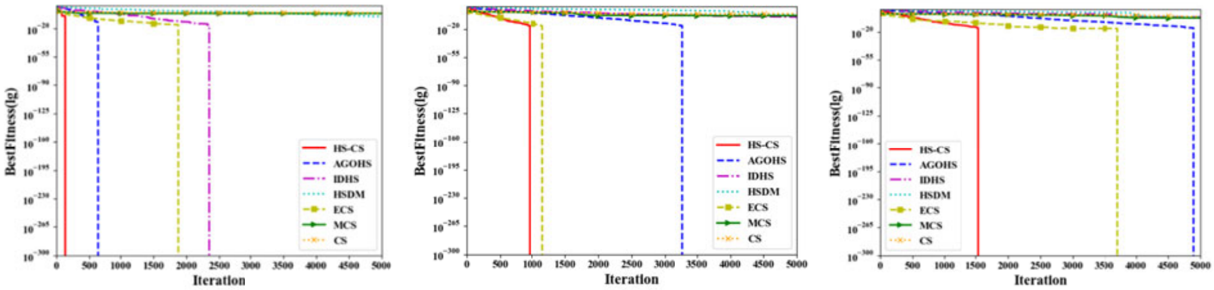
(a) (b) (c)

Fig. S4 Optimizing the convergence curve of F3 (Alpine) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



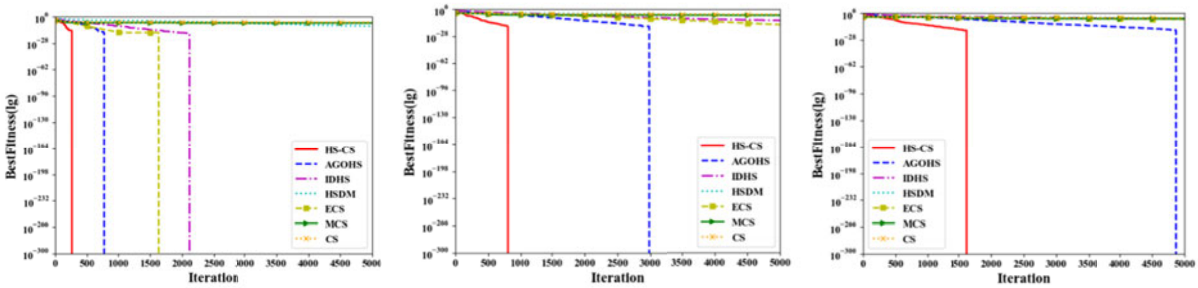
(a) (b) (c)

Fig. S5 Optimizing the convergence curve of F4 (Rastrigin) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



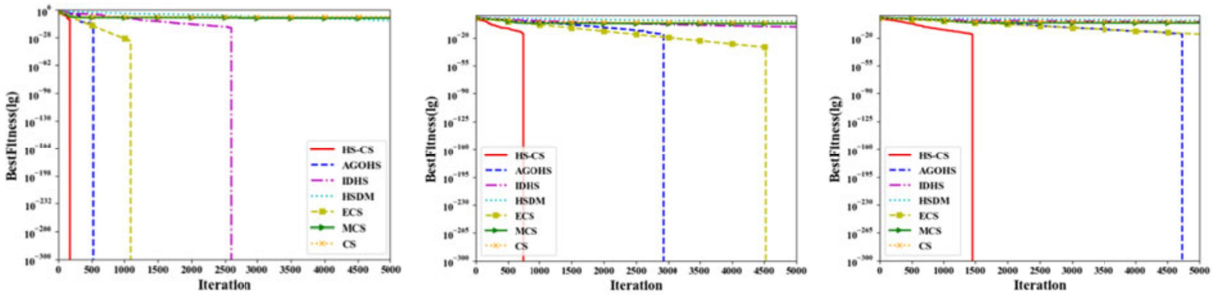
(a) (b) (c)

Fig. S6 Optimizing the convergence curve of F5 (Griewank) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



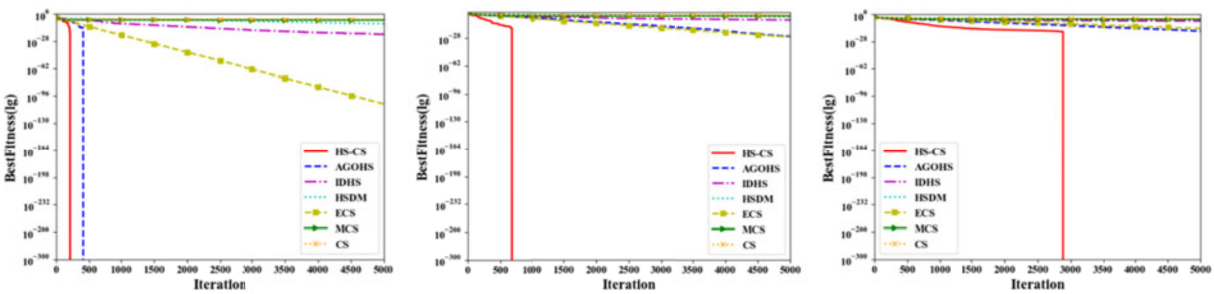
(a) (b) (c)

Fig. S7 Optimizing the convergence curve of F6 (Ackley) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



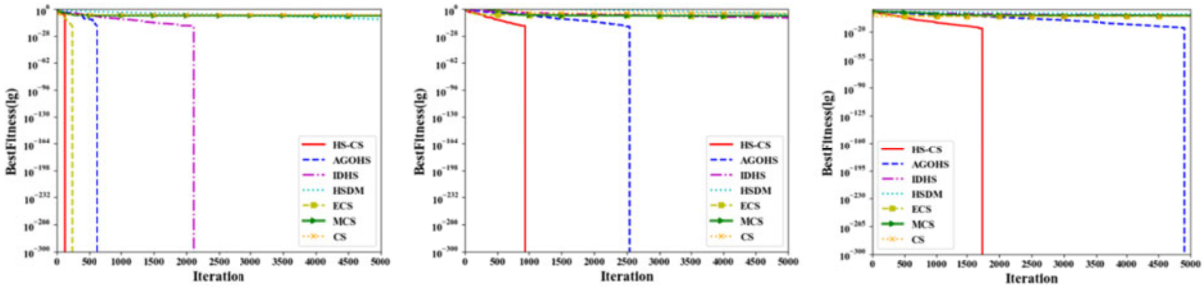
(a) (b) (c)

Fig. S8 Optimizing the convergence curve of F7 (Step) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



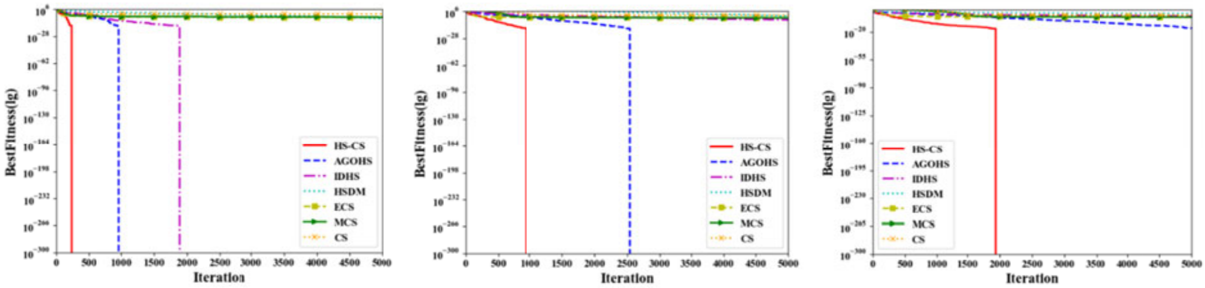
(a) (b) (c)

Fig. S9 Optimizing the convergence curve of F8 (Schwefel 2.22) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



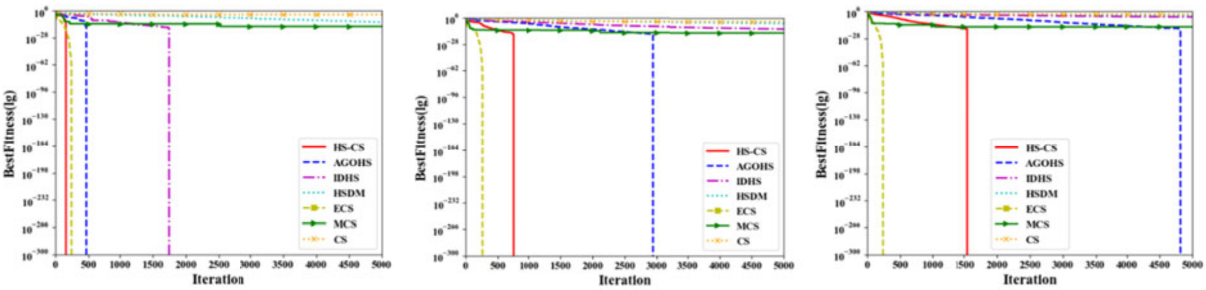
(a) (b) (c)

Fig. S10 Optimizing the convergence curve of F9 (Bohachevsky) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



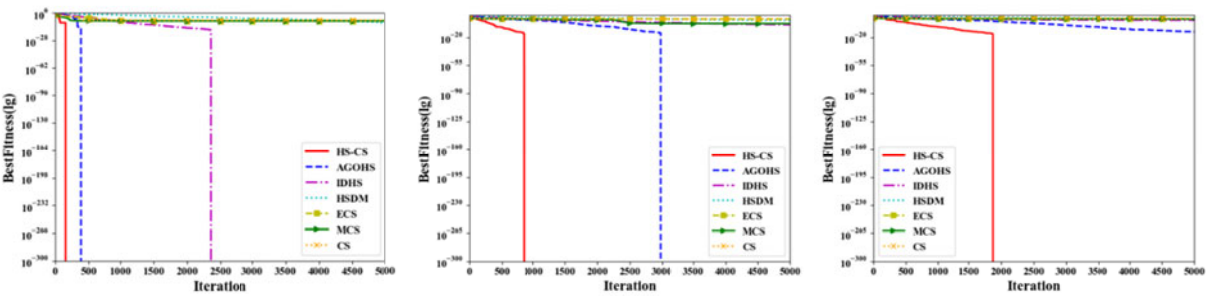
(a) (b) (c)

Fig. S11 Optimizing the convergence curve of F10 (Quartic) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



(a) (b) (c)

Fig. S12 Optimizing the convergence curve of F11 (Rosenbrock) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional



(a) (b) (c)

Fig. S13 Optimizing the convergence curve of F12 (Schwefel) in different dimensions: (a) 10-dimensional; (b) 30-dimensional; (c) 50-dimensional

Also, with the increase of the dimension, the HS-CS algorithm can handle the high-dimensional problems quickly.

HS-CS can jump out of the local extremum with a higher convergence speed and higher convergence accuracy in the case of a 30-dimensional problem compared to the six other algorithms (Figs. S2b–S13b). From Figs. S10b and S12b, ECS indicates a strong competitiveness on two unimodal functions (F9, F11), while the convergence performance of HS-CS on those functions are slightly weaker than that of ECS, but far superior to the five other optimization algorithms. Since the search mechanism of Levy flight of CS was adopted by ECS and HS-CS, the convergence efficiency of these algorithms was similar. In addition, AGOHS has the best search accuracy on 10 out of all 12 test instances on functions F1, F2, F4–F7, and F9–F12, but it does not guide the direction of "pitch adjusting and selecting the best" in the final optimization process, which causes the algorithm to have low convergence efficiency and speed. However, with the function dimension increasing from 10 to 30, it is shown in Figs. S2b–S13b that IDHS cannot overcome the impact of the increase of dimension well, which reduces the local optimization capability of the algorithm in the latter period, resulting in the reduction of the search precision of obtaining 12 test functions. As shown in Figs. S3b and S11b–S13b, MCS and HSDM have similar convergence efficiency and accuracy in optimizing F2 and F10–F12 functions and the convergence accuracy is much better than that of CS. Moreover, the HS-CS algorithm integrated with Levy flight broadens the search scope of the population, enhances the position update strategy, and improves the global exploration ability, and the population diversity is improved compared with that of AGOHS, IDHS, and HSDM.

HS-CS can carry out the global optimization process continuously and find the best solution while dealing with functions such as F1–F12 of 50 dimensions, and it also exhibits great advantages on stability. However, by analyzing Figs. S2c–S13c, HSDM, IDHS, MCS, and CS cannot adaptively determine the current algorithm execution state or adjust the algorithm search flexibly when optimizing and solving 12 functions, which makes these algorithms unable to jump out of the local extremum, leading to poor searching results.

Figs. S2c–S13c show that AGOHS has obtained 7 best results in optimizing 12 different high-dimensional test cases. Among them, AGOHS has a good effect on most unimodal functions in the process of searching the optimal solution, except the F8 and F10 functions. However, due to its low convergence efficiency compared with the proposed algorithm, AGOHS needs to consume a lot of resources to find the optimal solution in the process of solving high-dimensional problems. At the same time, Figs. S4c, S5c, and S13c show that AGOHS is prone to falling into local optima when addressing the high-dimensional complex multimodal functions (F3, F4, F12), which leads to low search efficiency and inability to find the global optimal solution.

With respect to unimodal function F7, ECS could nearly arrive at the theoretical optimal value within 500 iterations with a cliff-jumping converge rate from Fig. S12c, where the convergence speed of ECS is three times that of HS-CS and nine times that of AGOHS. In addition, MCS appears "inflection point" in 500 iterations, but it reaches the stagnation state prematurely, which causes the algorithm to yield poor searching results. Moreover, IDHS, HSDM, and CS cannot obtain a high-precision solution within 5000 iterations. However, ECS could also achieve the best value when optimizing multimodal function F5, for which the convergence rate is much lower than that of HS-CS, but faster than that of AGOHS.

From Figs. S2c–S13c, we can see that the HS-CS algorithm can stably and quickly converge to the global optimal solution by optimizing both high-dimensional unimodal functions and multimodal functions. Particularly, for multimodal functions F3 and F12, the six other algorithms fall into the state of local extremum, but HS-CS still maintains the highest convergence speed and better optimization accuracy. In the later search process, HS-CS indicates the direction of "pitch adjusting and selecting the best." The Levy flight in the CS operator is adopted to find candidate individuals when updating the HM, which enriches the number of alternative solutions and strengthens the disturbance to avoid falling into stagnation prematurely in the searching process. Therefore, HS-CS exhibits good performance in dealing with high-dimensional optimization problems, proving that it has obvious better convergence performance and self-adaptive ability. It possesses the excellent capability of jumping out of local optima as an improvement of the problem dimension.

From the convergence results of these seven algorithms to optimize the high-dimensional function problem, HS-CS and AGOHS have the same convergence accuracy in some cases. However, in terms of population density and convergence speed, the global search ability of AGOHS is not strong, and it cannot jump out of the local extremum quickly, which leads to low population density and low convergence speed. Compared to AGOHS, IDHS, HSDM, ECS, MCS, and CS, the proposed HS-CS has stronger ability to deal with high-dimensional problems, and higher convergence speed and accuracy.

2.2 Further numerical analysis

The numerical analysis of the used six swarm intelligence algorithms is implemented from the following aspects.

First, the Wilcoxon rank-sum test is used to compare the superiority of the algorithm performance. In this analysis, the symbol "+" implies that the performance of the reference algorithm is better in the current dimension, the symbol "–" implies that the performance of the reference algorithm is poor in the current dimension, and the symbol "=" implies that there is no significant difference in the performance of the comparison algorithm in the current dimension.

Second, the mean value (MEAN) and the standard deviation (SD), calculated as shown by Eqs. (S1) and (S2), are used to judge the superiority of the performance of the improved HS algorithm:

$$\mathrm{MEAN} = \frac{1}{\mathrm{Num}} \sum_{i=1}^{\mathrm{Num}} f_i^{\mathrm{best}}, \tag{S1}$$

$$\mathrm{SD} = \sqrt{\frac{\sum_{i=1}^{\mathrm{Num}} \left(f_i^{\mathrm{best}} - \mathrm{MEAN} \right)}{\mathrm{Num}}}, \tag{S2}$$

where f_i^{best} is the optimal fitness value for each independent run of the algorithm.

The performance of seven optimization algorithms is further analyzed by using statistics such as mean and standard deviation in Table S1. The experimental statistics of mean and standard deviation show the function optimization results by the six variants under different dimensions after running 30 times independently. Based on the characteristics of both mean and standard deviation, the robustness of these algorithms is determined. If the current calculation is surrounded by more values in the global optimal solution, it means that the higher the accuracy of global convergence, the stronger the stability of the algorithm.

It is easy to find in the related results from Table S1 that the performance of the six other algorithms is weaker than that of HS-CS while optimizing the six unimodal functions (F1, F7–F11) and six multimodal functions. However, in the 10-dimensional problems, AGOHS achieves the same mean and standard deviation results as HS-CS, and most of its solutions are distributed near the theoretical optimal solution. At the same time, IDHS and ECS have reached the optimal state in obtaining the mean and standard deviation of some functions, while CS, MCS, and HSDM can obtain good solutions but the precision of these solutions is not high in general.

In the optimization of multimodal functions F2–F6 and F12, HS-CS has an excellent capability to deal with high-dimensional problems (30 and 50 dimensions). The mean and standard deviation obtained by HS-CS are smaller than that of the six other algorithms, which reveals that the proposed algorithm is better than the other meta-heuristics for stability. This gives full role to the global optimization ability; it jumps out of the local optimum state quickly and obtains high convergence accuracy. However, with the increase of dimension from 30 to 50, AGOHS and ECS have weakened their search ability to optimize multimodal functions with a widespread number of complex local extreme points, which are prone to stagnation state, and they obtain the optimal fitness with a large fluctuations resulting in high values of mean and standard deviation, indicating that AGOHS and ECS have weak stability in the case of high-dimensional problems, except that AGOHS and ECS achieve the optimal solution for optimizing function F3. In addition, the average value and standard deviation of MCS are smaller than that of HSDM, IDHS, and CS, and that MCS is the second when compared with ECS for stability.

As for the unimodal functions (F1, F7–F11) in the 30- and 50-dimensional cases, AGOHS can obtain slightly higher search precision on function F1, yet AGOHS has a great effect on dimensions; so, it is impossible to obtain the optimal solutions in the optimization of functions F7–F11. At the same time, ECS only obtains the optimal solution for optimizing function F11 on the 30- and 50-dimensional cases, and it could achieve the best accuracy for optimizing functions F7 and F9 on the 30-dimensional case, but it obtains poor results on the other functions. Furthermore, the mean and standard deviation obtained by IDHS are better than that of MCS, HSDM, and CS, and these four algorithms can be expressed as IDHS>MCS>HSDM>CS in terms of stability. In addition, HS-CS outperforms the six other optimization methods on the high dimension of unimodal functions, and its mean value has a great advantage when compared with AGOHS, ECS, MCS, IDHS, HSDM, and CS.

In the process of data analysis such as the mean and standard deviation of high-dimensional multimodal functions and unimodal functions, HS-CS is not affected by dimensions, and it could accurately identify the position of the optimal solution and achieve the highest convergence to the global optimum. It can be shown that HS-CS has strong robustness in dealing with high-dimensional problems through the numerical analysis and convergence curves.

According to the mean data, the Wilcoxon rank-sum test is used to distinguish the differences in performance of the seven variants. The results of the six different swarm intelligence optimization algorithms' Wilcoxon rank-sum are compared for reference to HS-CS in Tables S2–S4. In these tables, the final ranking of the optimized function performance of each algorithm is obtained according to the average ranking.

From the statistical analysis of the data in Table S2, it can be found that HS-CS and AGOHS are more suitable to optimize the 12 functions in the 10-dimensional case, and that there is no significant difference between those in performance optimization. The ranking sum test W^{+}/W^{-} was represented by NA. In addition, HSDM, MCS, and CS are much worse than HS-CS under the 10-dimensional condition. Relatively, IDHS and ECS were very similar to the proposed method on some functions in performance, yet poor on the other functions, and the final ranking is the third and fourth, respectively.

As the dimensionality increases, the data in Tables S3 and S4 reflect that HS-CS has gradually shown a significant improvement in the ability to address the high-dimensional test cases. By contrast, ECS and AGOHS have a significant decline in their ability to deal with high-dimensional problems. Moreover, there is no significant difference between ECS and AGOHS in performance from the existence of F4 and F9 functions to the existence of F1 and F2 functions compared with HS-CS. Particularly, the optimization performance of AGOHS decreases significantly under the condition of high-dimensional problems, and W^{+}/W^{-} increases from 6/0 to 55/0. The results indicate that the stability of AGOHS decays rapidly, which is significantly different from that of the proposed algorithm. Furthermore, the overall performance of MCS has been improved from 30 to 50 dimensions, which moves its rank from the fifth to fourth. In addition, the performance of HSDM has been in an inferior position without significant change at the sixth place.

Furthermore, the Wilcoxon rank-sum test is used to verify the performance comparison ranking results of the seven algorithms under different dimensions. Under 10-dimensional condition, AGOHS and HS-CS are the first in parallel. However, in the process of upgrading from 30 to 50 dimensions, AGOHS shows that the ability to optimize for high-dimensional test cases is insufficient, and its final ranking is the second. The overall performance of HS-CS ranks the first compared with the six other algorithms. Additionally, except for IDHS falling from fourth to fifth and MCS rising from fifth to fourth in the final ranking, ECS and HSDM remained unchanged in the third and sixth places, respectively. In other words, the improvement of dimension has a certain impact on the optimization performance of the six other algorithms, resulting in a decrease in the accuracy of the six other algorithms for the majority of the 12 classical benchmark problems, but the proposed HS-CS method has better optimization accuracy that not affected by dimensions, which indicates that HS-CS integrated with the CS operator, “pitch adjusting and selecting the best” and CS operator perturbation update, has strong robustness and adaptive ability.

According to the statistical analysis of the data in Table S2, it can be found that HS-CS and AGOHS are more suitable for optimizing the 12 functions in the 10-dimensional problem. From the statistical analysis of the variable W^{+}/W^{-}, there is no obvious difference in the optimized performance of these two algorithms. Relatively, the performance of IDHS and HSDM is far worse than that of HS-CS under this condition. With the continuous increase of dimension, it can be found from Tables S3 and S4 that HS-CS gradually shows a significant improvement in its ability to deal with high-dimensional function problems. By contrast, AGOHS and IDHS have a significant decline in their ability to deal with high-dimensional problems, while the performance of HSDM continues in an inferior position without significant change.

The Wilcoxon rank-sum test is also used to verify the performance comparison ranking results of the improved HS algorithm under different dimensions; the overall performance of HS-CS ranked the first compared with the three other variants. Hence, the performance of HS-CS is significantly better than that of the three other improved algorithms. Thus, the proposed HS-CS algorithm has strong adaptive ability and robustness.

Through the above comparative analysis, HS-CS shows good robustness in solving the function optimization problems under high dimensions due to the effective mechanisms. The strategy of “pitch adjusting and selecting the best” is used in the improvisation stage. The optimal solution generated in HM is used to point out the direction for the algorithm to find the optimal solution in the later iteration stage. This reduces the blindness of the local optimization and enhances the adaptive ability of the algorithm, improving the probability of finding the global optimal solution. On the other hand, the search ability of HS-CS is enhanced due to the characteristics of the CS operator. The density of the population is also improved. The CS operator expands the number of alternative solutions by finding candidate solutions. The risk of premature convergence of the HS-CS algorithm and falling into a local optimal is avoided.

3 Entire weighted fuzzy production rules extracted using HS-CS

The weighted fuzzy production rules could be automaticly extracted from the importance index matrix. The whole generated local weighted fuzzy production rules are listed below.

The first row of the matrix produces 18 classification rules as Iris-setosa (Table 4). The second row of the matrix produces 18 classification rules as Iris-versicolor (Table S5). The third row of the matrix produces 18 classification rules as Iris-virginica (Table S6).

Table S1 Experimental statistics of mean and standard deviation

Function	Dim	Mean±Std						
		CS	ECS	MCS	AGOHS	IDHS	HSDM	HS-CS
F1	10	4.36E-03±1.04E-03	2.75E-164±0.00E+00	1.46E-04±3.47E-05	**0.00E+00±0.00E+00**	4.84E-15±1.45E-14	1.50E-11±9.68E-12	**0.00E+00±0.00E+00**
	30	2.96E-01±2.77E-02	6.26E-39±3.08E-39	1.00E-03±2.58E-04	**0.00E+00±0.00E+00**	1.37E-10±8.77E-11	3.38E-04±1.11E-04	**0.00E+00±0.00E+00**
	50	1.38E+00±8.42E-02	9.49E-19±2.22E-19	3.68E-02±2.72E-03	**0.00E+00±0.00E+00**	4.38E-04±3.71E-04	5.33E-02±1.50E-02	**0.00E+00±0.00E+00**
F2	10	1.10E-07±9.77E-08	1.35E-31±6.57E-47	1.67E-08±1.52E-08	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	2.37E-09±1.89E-09	**0.00E+00±0.00E+00**
	30	9.08E-08±6.29E-08	1.35E-31±6.57E-47	1.05E-08±1.09E-08	**0.00E+00±0.00E+00**	6.13E-08±5.87E-15	7.02E-02±2.20E-02	**0.00E+00±0.00E+00**
	50	1.31E-07±1.13E-07	1.35E-31±6.57E-47	1.14E-08±1.29E-08	**0.00E+00±0.00E+00**	6.76E-02±7.37E-02	7.68E+00±1.46E+00	**0.00E+00±0.00E+00**
F3	10	8.29E-03±9.06E-04	4.10E-04±1.20E-04	2.74E-03±3.87E-04	**0.00E+00±0.00E+00**	3.76E-02±1.13E-01	1.47E-05±1.37E-05	**0.00E+00±0.00E+00**
	30	2.39E+00±8.17E-01	3.14E+00±4.83E-01	6.44E-01±2.33E-01	5.16E-01±1.55E+00	5.37E+00±6.49E+00	6.77E-02±5.12E-02	**0.00E+00±0.00E+00**
	50	1.30E+01±2.02E+00	1.61E+01±1.41E+00	2.57E+00±5.33E-01	2.33E+00±7.00E+00	2.06E+01±1.18E+01	2.42E+00±9.29E-01	**0.00E+00±0.00E+00**
F4	10	3.13E+00±7.01E-01	1.99E+00±7.43E-01	3.82E-02±8.48E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	1.73E-05±1.91E-05	**0.00E+00±0.00E+00**
	30	9.40E+01±9.11E+00	6.83E+01±8.57E+00	2.22E+01±3.39E+00	1.52E+01±2.49E+01	7.84E+01±3.27E+01	2.65E+01±3.32E+00	**0.00E+00±0.00E+00**
	50	3.97E+02±7.28E+01	2.39E+02±1.37E+01	1.70E+02±2.34E+01	6.46E+01±7.03E+01	2.34E+02±3.22E+01	1.14E+02±7.30E+00	**0.00E+00±0.00E+00**
F5	10	5.90E-02±2.67E-02	**0.00E+00±0.00E+00**	5.70E-02±2.30E-02	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	6.15E-06±2.82E-11	**0.00E+00±0.00E+00**
	30	1.72E-02±1.19E-02	**0.00E+00±0.00E+00**	8.39E-03±1.15E-02	**0.00E+00±0.00E+00**	2.93E-04±3.33E-07	3.25E-02±6.41E-03	**0.00E+00±0.00E+00**
	50	9.40E-02±1.04E-01	2.07E-16±7.07E-16	8.78E-03±1.37E-04	1.53E-13±1.24E-25	6.89E-02±1.08E-01	3.38E-02±4.89E-02	**0.00E+00±0.00E+00**
F6	10	1.06E-01±1.34E-02	2.57E-15±1.59E-15	1.63E-02±2.36E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	7.03E-07±6.55E-07	**0.00E+00±0.00E+00**
	30	8.33E-01±6.95E-02	6.54E-14±8.40E-14	6.47E-02±3.37E-03	**0.00E+00±0.00E+00**	1.49E-06±2.29E-06	1.97E-01±2.11E-02	**0.00E+00±0.00E+00**
	50	1.71E+00±6.12E-02	2.29E-02±9.39E-03	1.13E-01±4.65E-03	1.31E-15±5.81E-15	2.41E+00±3.31E+00	1.76E+00±1.92E-01	**0.00E+00±0.00E+00**
F7	10	2.65E-03±6.29E-04	**0.00E+00±0.00E+00**	1.84E-04±4.42E-05	**0.00E+00±0.00E+00**	3.88E-08±4.22E-14	1.43E-07±1.82E-14	**0.00E+00±0.00E+00**
	30	2.65E-03±3.71E-02	**0.00E+00±0.00E+00**	7.98E-03±9.25E-07	**0.00E+00±0.00E+00**	6.09E-07±9.86E-07	5.02E+00±1.38E+00	**0.00E+00±0.00E+00**
	50	1.34E+00±1.27E-01	3.62E-16±1.13E-16	3.84E-02±2.84E-03	4.32E-15±8.19E-29	6.14E-02±2.98E-02	8.97E+01±1.53E+02	**0.00E+00±0.00E+00**

To be continued

Table S1

Function	Dim	Mean±Std						
		CS	ECS	MCS	AGOHS	IDHS	HSDM	HS-CS
F8	10	1.66E-01±1.98E-02	5.56E-106±5.28E-106	3.12E-02±3.37E-03	**0.00E+00±0.00E+00**	1.93E-19±1.55E-19	1.94E-06±6.55E-07	**0.00E+00±0.00E+00**
	30	2.34E+00±1.64E-01	3.43E-27±7.18E-28	3.78E-01±1.96E-02	1.22E-22±2.30E-22	1.39E-06±2.17E-06	3.95E-01±7.72E-03	**0.00E+00±0.00E+00**
	50	6.65E+00±2.57E-01	3.04E-12±4.83E-13	1.06E+00±3.95E-02	1.32E-14±9.34E-15	1.30E+01±7.74E-03	7.02E-01±1.24E-01	**0.00E+00±0.00E+00**
F9	10	1.03E-01±1.81E-07	**0.00E+00±0.00E+00**	7.28E-03±1.84E-03	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	2.23E-07±1.07E-07	**0.00E+00±0.00E+00**
	30	1.03E-01±1.45E-07	**0.00E+00±0.00E+00**	3.79E-01±7.19E-04	**0.00E+00±0.00E+00**	8.90E-06±7.43E-06	4.83E+00±1.40E+00	**0.00E+00±0.00E+00**
	50	2.51E+01±2.73E+01	7.37E-01±9.10E-01	2.00E+00±2.13E-01	2.66E-16±3.95E-16	1.20E+01±6.64E+00	8.65E+01±1.17E+01	**0.00E+00±0.00E+00**
F10	10	1.46E+00±1.43E-01	1.68E-04±5.56E-05	3.78E-04±1.75E-04	**0.00E+00±0.00E+00**	2.68E-07±1.42E-06	8.09E-06±7.26E-06	**0.00E+00±0.00E+00**
	30	1.04E+01±5.16E-01	7.82E-03±2.22E-03	7.28E-03±2.25E-03	**0.00E+00±0.00E+00**	9.58E-06±2.08E-05	5.14E+00±3.11E+00	**0.00E+00±0.00E+00**
	50	2.48E+01±8.48E-01	3.84E-02±3.90E-03	3.86E-02±6.62E-03	2.78E-15±1.31E-14	7.47E+00±8.17E+00	9.49E+02±1.81E+02	**0.00E+00±0.00E+00**
F11	10	7.51E+00±5.30E+00	**0.00E+00±0.00E+00**	8.34E-12±8.34E-12	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	1.39E-08±9.41E-09	**0.00E+00±0.00E+00**
	30	7.83E+01±5.41E+01	**0.00E+00±0.00E+00**	1.08E-11±1.44E-11	**0.00E+00±0.00E+00**	4.31E-07±5.50E-07	5.47E-01±2.20E-01	**0.00E+00±0.00E+00**
	50	2.81E+02±1.27E+02	**0.00E+00±0.00E+00**	8.62E-12±9.48E-12	7.66E-18±4.05E-17	6.80E-01±7.09E-01	2.40E+01±2.86E+00	**0.00E+00±0.00E+00**
F12	10	7.05E-04±1.43E-04	1.27E-04±0.00E+00	1.48E-04±1.92E-05	**0.00E+00±0.00E+00**	**0.00E+00±0.00E+00**	4.26E-06±2.59E-06	**0.00E+00±0.00E+00**
	30	5.06E+02±2.34E+02	1.32E+03±1.41E+02	1.30E+01±3.59E+01	**0.00E+00±0.00E+00**	4.69E-04±9.63E-04	2.59E+01±3.87E+00	**0.00E+00±0.00E+00**
	50	4.25E+03±5.70E+02	3.30E+03±2.06E+03	1.31E+03±3.58E+02	7.10E-12±3.71E-11	4.16E+01±1.19E+01	1.44E+03±3.82E+02	**0.00E+00±0.00E+00**

Optimal values are shown in bold

Table S2 Wilcoxon rank-sum test results of six algorithms referring to HS-CS in the 10-dimensional case

Function	CS		ECS		MCS		AGOHS		IDHS		HSDM		HS-CS
	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank
F1	7	+	3	+	6	+	1	=	4	+	5	+	1
F2	6	+	3	+	5	+	1	=	1	=	4	+	1
F3	6	+	4	+	5	+	1	=	7	+	3	+	1
F4	6	+	5	+	4	+	1	=	1	=	3	+	1
F5	6	+	1	=	5	+	1	=	1	=	4	+	1
F6	6	+	3	+	5	+	1	=	1	=	4	+	1
F7	6	+	1	=	5	+	1	=	3	+	4	+	1
F8	7	+	3	+	6	+	1	=	4	+	5	+	1
F9	6	+	1	=	5	+	1	=	1	=	4	+	1
F10	7	+	5	+	6	+	1	=	3	+	4	+	1
F11	6	+	1	=	4	+	1	=	1	=	5	+	1
F12	6	+	4	+	5	+	1	=	1	=	3	+	1
Ave rank	6.25		2.83		5.08		1		2.33		4		1
Final rank	**7**		**4**		**6**		**1**		**3**		**5**		**1**
W^{+}/W^{-}		**78/0**		**36/0**		**78/0**		**NA**		**15/0**		**78/0**	
+/−/=		12/0/0		8/0/4		12/0/0		0/0/12		5/0/7		12/0/0	

W^{+} and W^{-} denote the rank sum of corresponding symbols; +/−/= denote the total rank number obtained by reference to the HS-CS algorithm; Ave rank and Final rank represent the average ranking and final ranking of each algorithm under the current dimension, respectively

Table S3 Wilcoxon rank-sum test results of six algorithms referring to HS-CS in the 30-dimensional case

Function	CS		ECS		MCS		AGOHS		IDHS		HSDM		HS-CS
	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank
F1	7	+	3	+	6	+	1	=	4	+	5	+	1
F2	7	+	3	+	4	+	1	=	5	+	6	+	1
F3	6	+	5	+	4	+	3	+	7	+	2	+	1
F4	7	+	5	+	3	+	2	+	6	+	4	+	1
F5	6	+	1	=	4	+	1	=	3	+	5	+	1
F6	7	+	3	+	5	+	1	=	4	+	6	+	1
F7	4	+	1	=	5	+	1	=	3	+	6	+	1
F8	7	+	2	+	5	+	3	+	4	+	6	+	1
F9	6	+	1	=	4	+	1	=	3	+	5	+	1
F10	7	+	5	+	4	+	1	=	3	+	6	+	1
F11	6	+	1	=	3	+	1	=	4	+	5	+	1
F12	6	+	7	+	4	+	1	=	3	+	5	+	1
Ave rank	6.33		3.08		4.25		1.42		4.08		5.08		1
Final rank	**7**		**3**		**5**		**2**		**4**		**6**		**1**
W^+/W^-		**78/0**		**36/0**		**78/0**		**6/0**		**78/0**		**78/0**	
+/−/=		12/0/0		8/0/4		12/0/0		3/0/9		12/0/0		12/0/0	

W^+ and W^- denote the rank sum of corresponding symbols; +/−/= denote the total rank number obtained by reference to the HS-CS algorithm; Ave rank and Final rank represent the average ranking and final ranking of each algorithm under the current dimension, respectively

Table S4 Wilcoxon rank-sum test results of six algorithms referring to HS-CS in the 50-dimensional case

Function	CS		ECS		MCS		AGOHS		IDHS		HSDM		HS-CS
	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank	Wilcoxon-test	Rank
F1	7	+	3	+	5	+	1	=	4	+	6	+	1
F2	5	+	3	+	4	+	1	=	6	+	7	+	1
F3	6	+	5	+	3	+	2	+	7	+	4	+	1
F4	7	+	6	+	4	+	2	+	5	+	3	+	1
F5	7	+	3	+	4	+	2	+	6	+	5	+	1
F6	6	+	3	+	4	+	2	+	7	+	5	+	1
F7	6	+	2	+	4	+	3	+	5	+	7	+	1
F8	6	+	3	+	5	+	2	+	7	+	4	+	1
F9	6	+	3	+	4	+	2	+	5	+	7	+	1
F10	6	+	3	+	4	+	2	+	5	+	7	+	1
F11	7	+	1	=	4	+	3	+	5	+	6	+	1
F12	7	+	6	+	4	+	2	+	3	+	5	+	1
Ave rank	6.33		3.42		4.08		2		5.42		5.5		1
Final rank	**7**		**3**		**4**		**2**		**5**		**6**		**1**
W^+/W^-		**78/0**		**66/0**		**78/0**		**55/0**		**78/0**		**78/0**	
+/−/=		12/0/0		11/0/1		12/0/0		10/0/2		12/0/0		12/0/0	

W^+ and W^- denote the rank sum of corresponding symbols; +/−/= denote the total rank number obtained by reference to the HS-CS algorithm; Ave rank and Final rank represent the average ranking and final ranking of each algorithm under the current dimension, respectively

Table S5 Classification rules as Iris-versicolor

No.	IF
1	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT SM [12.89], and PW is NOT SM [0.66]
2	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT SM [12.89], and PW is MED [29.8]
3	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT SM [12.89], and PW is NOT LGR [12.79]
4	SL is NOT MED [6.53], SW is LGR [4.0], PL is MED [22.29], and PW is NOT SM [0.66]
5	SL is NOT MED [6.53], SW is LGR [4.0], PL is MED [22.29], and PW is MED [29.8]
6	SL is NOT MED [6.53], SW is LGR [4.0], PL is MED [22.29], and PW is NOT LGR [12.79]
7	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is NOT SM [0.66]
8	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is MED [29.8]
9	SL is NOT MED [6.53], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is NOT LGR [12.79]
10	SL is LGR [5.95], SW is LGR [4.0], PL is NOT SM [12.89], and PW is NOT SM [0.66]
11	SL is LGR [5.95], SW is LGR [4.0], PL is NOT SM [12.89], and PW is MED [29.8]
12	SL is LGR [5.95], SW is LGR [4.0], PL is NOT SM [12.89], and PW is NOT LGR [12.79]
13	SL is LGR [5.95], SW is LGR [4.0], PL is MED [22.29], and PW is NOT SM [0.66]
14	SL is LGR [5.95], SW is LGR [4.0], PL is MED [22.29], and PW is MED [29.8]
15	SL is LGR [5.95], SW is LGR [4.0], PL is MED [22.29], and PW is NOT LGR [12.79]
16	SL is LGR [5.95], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is NOT SM [0.66]
17	SL is LGR [5.95], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is MED [29.8]
18	SL is LGR [5.95], SW is LGR [4.0], PL is NOT LGR [22.77], and PW is NOT LGR [12.79]

SL: sepal length; SW: sepal width; PL: petal length; PW: petal width; LGR: large; MED: medium; SM: small

Table S6 Classification rules as Iris-virginica

No.	IF
1	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is NOT SM [4.11]
2	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is NOT MED [16.48]
3	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is LGR [11.26]
4	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is NOT SM [4.11]
5	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is NOT MED [16.48]
6	SL is MED [5.75], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is LGR [11.26]
7	SL is MED [5.75], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is NOT SM [4.11]
8	SL is MED [5.75], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is NOT MED [16.48]
9	SL is MED [5.75], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is LGR [11.26]
10	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is NOT SM [4.11]
11	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is NOT MED [16.48]
12	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT SM [16.26], and PW is LGR [11.26]
13	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is NOT SM [4.11]
14	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is NOT MED [16.48]
15	SL is LGR [9.82], SW is NOT LGR [3.52], PL is NOT MED [14.82], and PW is LGR [11.26]
16	SL is LGR [9.82], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is NOT SM [4.11]
17	SL is LGR [9.82], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is NOT MED [16.48]
18	SL is LGR [9.82], SW is NOT LGR [3.52], PL is LGR [34.18], and PW is LGR [11.26]

SL: sepal length; SW: sepal width; PL: petal length; PW: petal width; LGR: large; MED: medium; SM: small