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Supplementary materials for

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1 Derivation of Eq. (21)

$$\begin{aligned}\left|\mathbb{E}\left\{\sum_{m=1}^{M}\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{g}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k}\right\}\right|^2 &= \left|\sum_{m=1}^{M}\sqrt{p_{m,k}}\iota_{m,k}\mathbb{E}\left\{\left(\hat{\boldsymbol{g}}_{m,k}\right)^{\mathrm{H}}\boldsymbol{w}_{m,k}\right\}\right|^2\\ &= \left|\sum_{m=1}^{M}\sqrt{p_{m,k}}\iota_{m,k}\mathbb{E}\left\{\left(\hat{\boldsymbol{g}}_{m}^{\iota}\boldsymbol{e}_k\right)^{\mathrm{H}}\sqrt{\lambda_{m,k}}\hat{\boldsymbol{g}}_{m}^{\iota}\left(\left(\hat{\boldsymbol{g}}_{m}^{\iota}\right)^{\mathrm{H}}\hat{\boldsymbol{g}}_{m}^{\iota}\right)^{-1}\boldsymbol{e}_k\right\}\right|^2\\ &= \left|\sum_{m=1}^{M}\sqrt{p_{m,k}\lambda_{m,k}}\iota_{m,k}\right|^2.\end{aligned}\tag{S1}$$

$$\begin{aligned}&\sum_{k'=1}^{K}\mathbb{E}\left\{\left|\sum_{m=1}^{M}\sqrt{p_{m,k'}}\iota_{m,k'}\boldsymbol{g}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right|^2\right\}\\ &=\sum_{k'=1}^{K}\mathbb{E}\left\{\left|\sum_{m=1}^{M}\left(\sqrt{p_{m,k'}}\iota_{m,k'}\hat{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}+\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right)\right|^2\right\}\\ &=\sum_{k'=1}^{K}\mathbb{E}\left\{\underbrace{\left|\sum_{m=1}^{M}\left(\sqrt{p_{m,k'}}\iota_{m,k'}\hat{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right)\right|^2}_{(1)}+\underbrace{\left|\sum_{m=1}^{M}\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right|^2}_{(2)}\right\}\\ &=\sum_{k'=1}^{K}\mathbb{E}\left\{\underbrace{\left(\sum_{m=1}^{M}\left(\sqrt{p_{m',k'}}\iota_{m',k'}\hat{\boldsymbol{g}}_{m',k}^{\mathrm{H}}\boldsymbol{w}_{m',k'}\right)\right)^2}_{(1)}+\underbrace{\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\sqrt{p_{m',k'}}\iota_{m',k'}\boldsymbol{w}_{m',k'}^{\mathrm{H}}\tilde{\boldsymbol{g}}_{m',k}}_{(2)}\right\},\end{aligned}\tag{S2}$$

where term (1) is

$$\begin{aligned}\left|\sum_{m=1}^{M}\left(\sqrt{p_{m,k'}}\iota_{m,k'}\hat{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right)\right|^2 &\overset{k'=k}{=}\left|\sum_{m=1}^{M}\left(\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{e}_k^{\mathrm{H}}\hat{\boldsymbol{g}}_{m}^{\iota}\sqrt{\lambda_{m,k}}\hat{\boldsymbol{g}}_{m}^{\iota}\left(\left(\hat{\boldsymbol{g}}_{m}^{\iota}\right)^{\mathrm{H}}\hat{\boldsymbol{g}}_{m}^{\iota}\right)^{-1}\boldsymbol{e}_k\right)\right|^2\\ &=\left(\sum_{m=1}^{M}\left(\sqrt{p_{m,k}}\iota_{m,k}\sqrt{\lambda_{m,k}}\right)\right)^2,\end{aligned}\tag{S3}$$

and term (2) is

$$\begin{aligned}\mathbb{E}\left|\sum_{m=1}^{M}\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right|^{2} &\overset{m'=m}{=}\mathbb{E}\left\{\sum_{m=1}^{M}\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\boldsymbol{w}_{m,k'}^{\mathrm{H}}\sqrt{p_{m,k'}}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}\right\}\\ &=\mathbb{E}\left\{\sum_{m=1}^{M}p_{m,k'}\iota_{m,k'}\tilde{\boldsymbol{g}}_{m,k}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\boldsymbol{w}_{m,k'}^{\mathrm{H}}\tilde{\boldsymbol{g}}_{m,k}\right\}\\ &=\sum_{m=1}^{M}p_{m,k'}\iota_{m,k'}\left(\beta_{m,k}-r_{m,k}\right).\end{aligned}\tag{S4}$$

Finally, substituting Eqs. (S1)–(S4) into Eq. (16) yields Eq. (21).

2 Derivation of Eq. (24)

$$\begin{aligned}&\mathbb{E}\left\{\left|\sum_{m=1}^{M}\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k}\right|^{2}\right\}\\ &=\sum_{m=1}^{M}\left(\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k}\right)^{2}+\sum_{m=1}^{M}\sum_{n\neq m}\left(\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k}\right)\left(\sqrt{p_{n,k}}\iota_{n,k}\boldsymbol{g}_{n,e}^{\mathrm{H}}\boldsymbol{w}_{n,k}\right)\\ &=\mathbb{E}\left(\sqrt{p_{m,k}}\iota_{m,k}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k}\right)^{2}=p_{m,k}\iota_{m,k}\mathbb{E}\left|\boldsymbol{g}_{m,e}^{\mathrm{H}}\right|^{2}\mathbb{E}\left|\boldsymbol{w}_{m,k}\right|^{2}=p_{m,k}\iota_{m,k}\beta_{m,e}.\end{aligned}\tag{S5}$$

$$\sum_{k'\neq k}^{K}\mathbb{E}\left\{\left|\sum_{m=1}^{M}\sqrt{p_{m,k'}}\iota_{m,k'}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right|^{2}\right\}=\sum_{k'\neq k}^{K}\sum_{m=1}^{M}\mathbb{E}\left(\sqrt{p_{m,k'}}\iota_{m,k'}\boldsymbol{g}_{m,e}^{\mathrm{H}}\boldsymbol{w}_{m,k'}\right)^{2}=\sum_{k'\neq k}^{K}\sum_{m=1}^{M}p_{m,k'}\iota_{m,k'}\beta_{m,e}.\tag{S6}$$

$$\begin{aligned}-\log_2\left(1-\frac{|a|^2}{b}\right)&\geq-\left(f\left(\bar{a},\bar{b}\right)+\left(a-\bar{a}\right)f'_{a}\left(\bar{a},\bar{b}\right)+\left(b-\bar{b}\right)f'_{b}\left(\bar{a},\bar{b}\right)\right)\\ &=-\left(\log_2\left(1-\frac{|\bar{a}|^2}{\bar{b}}\right)+\left(a-\bar{a}\right)\frac{-\frac{2|\bar{a}|}{\bar{b}}}{(1-\frac{|\bar{a}|^2}{\bar{b}})\ln 2}+\left(b-\bar{b}\right)\frac{\frac{|\bar{a}|^2}{\bar{b}^2}}{(1-\frac{|\bar{a}|^2}{\bar{b}})\ln 2}\right)\\ &=-\log_2\left(1-\frac{|\bar{a}|^2}{\bar{b}}\right)+2\frac{a\left|\bar{a}\right|}{(\bar{b}-|\bar{a}|^2)\ln 2}-\frac{|\bar{a}|^2}{(\bar{b}-|\bar{a}|^2)\ln 2}-\frac{b|\bar{a}|^2}{\bar{b}(\bar{b}-|\bar{a}|^2)\ln 2}.\end{aligned}\tag{S7}$$

Finally, substituting Eqs. (S5)–(S7) into Eq. (18) yields Eq. (24).

3 Derivation of inequality (37)

It is known that $-\ln(1-x)$ is convex, and when $g\left(a,b\right)=|a|^2/b$ is convex, the composite function $f\left(g\left(a,b\right)\right)=-\ln\left(1-\frac{|a|^2}{b}\right)$ is convex.

$$\log_2\left(1+\frac{|S|^2}{I}\right)=-\log_2\left(1-\frac{|S|^2}{I+|S|^2}\right).\tag{S8}$$

$$b=I+|S|^2.\tag{S9}$$

$$\log_2\left(1+\frac{|S|^2}{I}\right)\geq\log_2\left(1+\frac{|\bar{S}|^2}{\bar{I}}\right)+2\frac{S\left|\bar{S}\right|}{\bar{I}\ln 2}-\frac{|\bar{S}|^2}{\bar{I}\ln 2}-\frac{\left(I+|S|^2\right)\left|\bar{S}\right|^2}{\left(\bar{I}+\left|\bar{S}\right|^2\right)\bar{I}\ln 2}.\tag{S10}$$