

Supplementary materials for

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1 Introduction to diffusion-tensor images (DTIs)

With the continuous development of medical imaging technology, medical images have become an important tool for doctors to diagnose patients's conditions. Currently, magnetic resonance imaging (MRI) is the mainstream of medical imaging for clinical testing and diagnosis. Diffusion MRI (dMRI) is a new type of MRI technology developed in recent years. dMRI uses the different diffusion properties of water molecules in various organ tissues under an applied magnetic field with different diffusion gradients. The difference between dMRI and traditional MRI images is that each voxel has not only gray values but also includes a high-dimensional tensor to describe the diffusion characteristics of water molecules in organs. In clinical applications, diffusion-weighted imaging (DWI) is used to acquire an image through dMRI; in the meantime, DTIs are calculated according to the corresponding DWI images to conduct fiber tracking to form a DTI spectral model of the tissue structure for clinical diagnosis. The MRI sequences are collected, which commonly include the T1-weighted (T1w) image, the T2-weighted image, DWI image, fluid-attenuated inversion recovery (FLAIR), and so on. The DTIs discussed in this paper are calculated by the corresponding DWI images(Kingsley, 2006a,b,c).

Basser et al.(Howard et al., 2017) first proposed the DTI, which is an MRI technique developed on the basis of the DWI technique to quantitatively analyze the diffusion characteristics of water molecules in tissues in three-dimensional (3D) space. The voxel in DTI is represented mathematically as a tensor. It is a second-order 3D tensor, which is mathematically represented as a symmetric positive definite matrix in Eq. (S1). Diffusion tensors are symmetric positive matrices \mathbf{D} that can be decomposed into three real and positive eigenvalues and three corresponding eigenvectors by using singular value decomposition (SVD) in Eq. (S2).

Each of the relationships between the diffusion ellipsoid and the diffusion tensor was introduced mathematically by Kingsley et al. (2018) in the mathematical introduction to diffusion tensor imaging(Hu et al., 2018). The voxel of a DTI defines an ellipsoid. Three eigenvalues λ_1 , λ_2 , and λ_3 describe the lengths of the ellipsoid in the three directions, and three eigenvectors e_1 , e_2 , and e_3 describe the directions of the ellipsoid on the three axes. The specific tensor display is shown in Fig. S1. The DTIs have a 5D data form. The data representation of the DTI is shown in Eq. (refDti). \mathbf{T} is a tensor data point; x, y, and z denote the height, width, and length of the tensor data. So, the DTI is a high-dimensional image composed of ellipses, as shown in Fig. S2.

$$\mathbf{T} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix} \quad (\text{S1})$$

$$\mathbf{T} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]^T \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3] \quad (\text{S2})$$

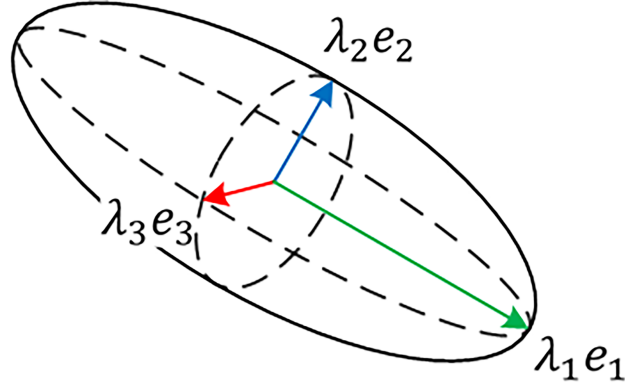


Fig. S1 Ellipsoidal representation of diffusion-tensor voxel

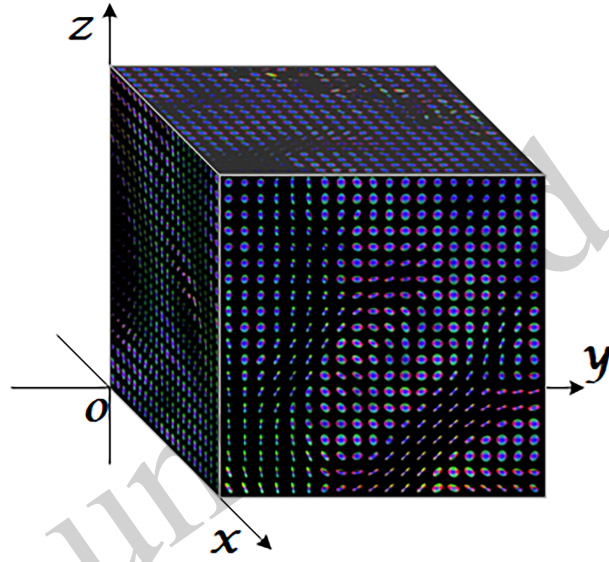


Fig. S2 DTI structure diagram

$$DTI = D(x, y, z, T) = D(x, y, z, 3, 3) \quad (S3)$$

The robust watermarking algorithm for DTI medical images has stricter requirements for image quality after embedding the watermark signals compared with the original images. Besides the peak signal-to-noise ratio (PSNR), the DTI-watermarking algorithm needs to meet the clinical metrics to prove that water molecules' diffusion characteristics and the tissue structure in the DTIs are unchanged. The specific clinical metrics are the mean diffusivity (MD), fractional anisotropy (FA), and main axis deflection angle (α_{AC}).

MD reflects the average diffusion capacity of water molecules within the tissue structure. If the MD value is larger, the diffusion capacity of water molecules is greater. The MD is defined by Eq. (S4), where λ_1 , λ_2 and λ_3 are the eigenvalues of the diffusion tensor \mathbf{T} .

$$MD = (\lambda_1 + \lambda_2 + \lambda_3)/3 \quad (S4)$$

The FA value reflects whether the diffusion direction of water molecules is consistent or not, which is defined by Eq. (S5). The value of FA is between zero and one. The larger the value, greater is the anisotropic and directional diffusion.

$$FA = \sqrt{\frac{3 * [(\lambda_1 - MD)^2 + (\lambda_2 - MD)^2 + (\lambda_3 - MD)^2]}{2 * (\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}} \quad (S5)$$

The diffusion of water molecules is anisotropic, the trajectory of which is like an ellipsoid. The long axis direction of the ellipsoid is the direction of the main axis. The long axis direction reflects the fiber orientation, which is an important indicator for physicians to diagnose fiber disease. The angle between the main axis direction and the XOY plane is defined as α_{AC} . To avoid affecting the doctor's diagnosis, the deflection angle of the spindle change α_{AC} must not exceed 5° . As shown in Eq. (S6), α_{AC} can be calculated from the difference between the original eigenvectors e_1, e_2 , and e_3 and the changed eigenvectors e'_1, e'_2 , and e'_3 .

$$\alpha_{AC} = \frac{e_1 \bullet e'_1}{\|e_1\| \|e'_1\|} \quad (S6)$$

In order to verify the superiority of this paper's algorithm in terms of image quality after embedding the watermark signals, quantitative and qualitative analyses of this paper's algorithm and the currently popular watermarking algorithm and DTI-reconstruction algorithm are studied.

In order to ensure the medical value of DTIs, the diffusion properties such as the FA, the MD, and the main axis deflection angle (α_{AC}) of the watermarked DTIs should not be destroyed too much after embedding the watermark signals. FA reflects that the diffusion direction of water molecules is consistent, MD reflects the average diffusion capacity of water molecules within the tissue structure, and the main axis deflection angle responds to the angle changed by the ellipse's long axis.

2 Complete watermarked DTI results

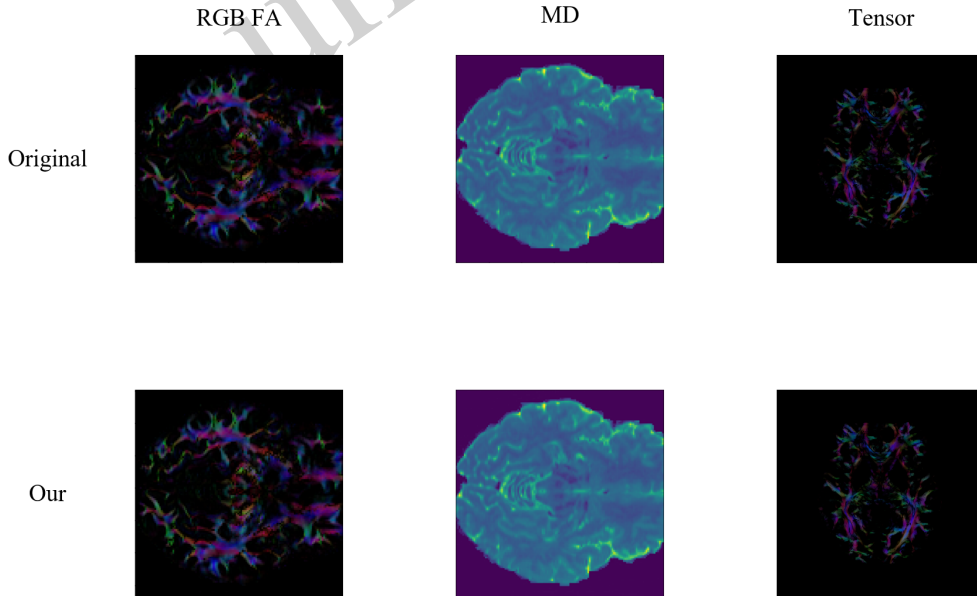


Fig. S3 A comparison between the complete watermarked image and the complete original image

In order to show the complete brain structure in the DTI, we obtain the display result by stitching, as shown in Fig. S3. The first row indicates the original DTI, and the second row indicates our algorithm's watermarked DTI.

As seen from the figures, even stitching the watermarked DTI obtained by our algorithm can be highly similar to the original image. The changes in FA and tensor of DTI are so subtle that they are unnoticeable to the human eye.

3 Attacking the network

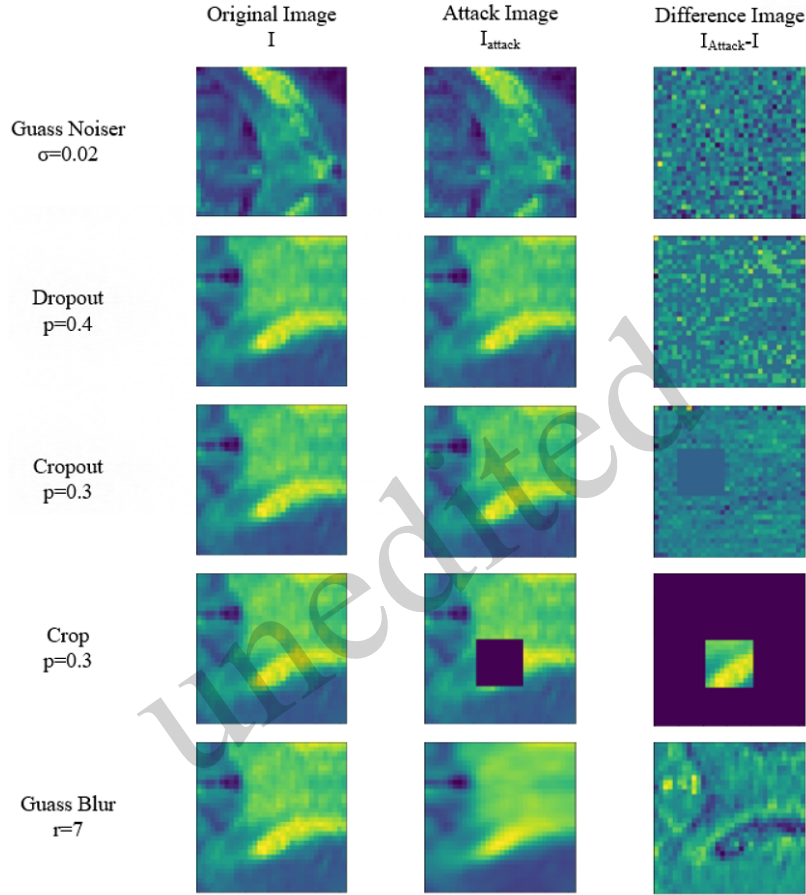


Fig. S4 Attack renderings

To ensure the robustness of the watermarked DTIs, the proposed algorithm includes a variety of attacks for DTIs in the attack layer of the network training so that the network can be trained to extract the watermark signals adaptively according to these attacks in advance. These attacks are mostly intentional or unintentional attack methods for DTI high-dimensional data, which include but are not limited to the following: Crop, Cropout, Dropout, Gaussian Blur, and Gaussian noise. Crop attack uses a randomly cropped area of the watermarked DTI as the attacked image, and the ratio p between the attacked image and the original image indicates the crop attack strength. Cropout is to first randomly crop on the watermarked DTI and then fill the empty pixels of the watermarked DTI crop with pixels at the corresponding positions in the original image, and the ratio p is the number of replaced pixels to the number of pixels of the entire image.

Dropout is the random use of pixels in the original image to replace pixels at corresponding positions in the image containing the watermarked DTI, and the ratio p of the number of replaced pixel points to the number of pixel points in the entire image is the replacement intensity. Gaussian Blur involves the process of each pixel in the DTI with a Gaussian low-pass filter. The size r of the Gaussian low-pass filter represents

the intensity of the attack.

Gaussian noise is generated random noise, which matches the Gaussian distribution. The size of the noise is the same as the watermarked DTI. These random noises are added directly to the pixel values of the watermarked DTI, where the magnitude of the standard deviation σ indicates the intensity of the attack. The experimental results are shown in Figure S4.

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