

Supplementary materials for

Weifang HUANG, Lijian YANG, Xuan ZHAN, Ziyang FU, Ya JIA, 2023. Synchronization transition of a modular neural network containing subnetworks of different scales. *Front Inform Technol Electron Eng*, 23(10):1458-1470. <https://doi.org/10.1631/FITEE.2300008>

1 Results and discussions

1.1 Effect of coupling strength on the synchronization of networks

Coupling strength is an important factor affecting the synchronization of networks. To investigate the effect of coupling strength on the synchronization of networks, spatio-temporal firing raster plots of modular neural networks with different coupling strengths at time delay $\tau=10$ are plotted in Fig. S1. Five nodes on the left side of each subfigure represent neurons in the small-scale subnetwork and 50 nodes on the right side represent neurons in the large-scale subnetwork; the black dots are the moments of neuron firing. The results show that the synchronization of the networks becomes progressively better visually as the coupling strength increases. Coupling strength promotes network synchronization.

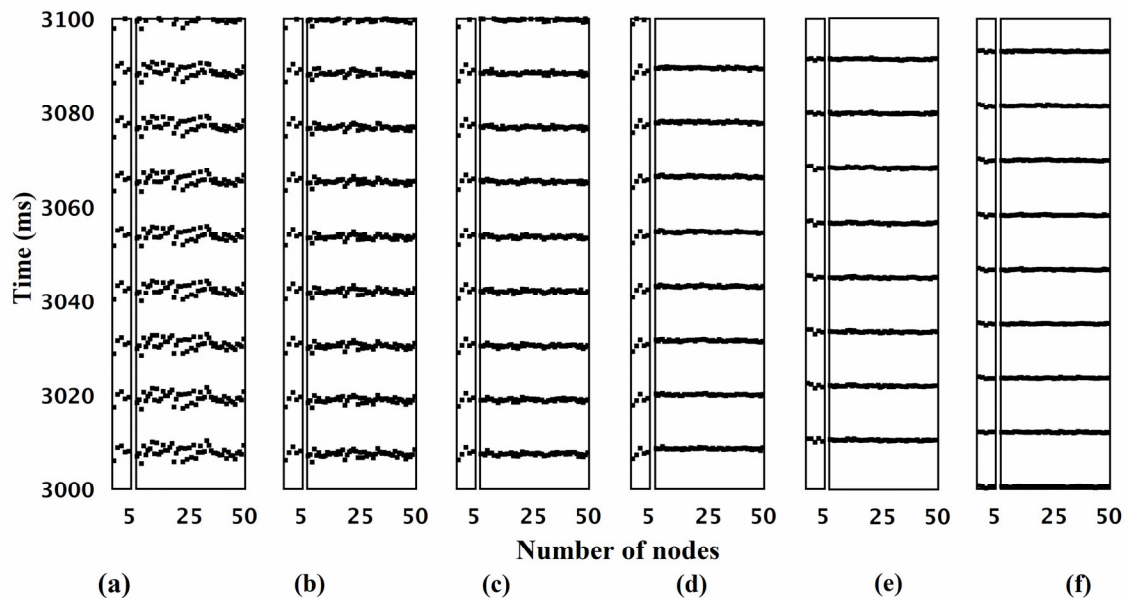


Fig. S1 Spatio-temporal firing raster plots of neuronal membrane potentials of the modular neural network at different coupling strengths g : (a) $g=0.001$; (b) $g=0.003$; (c) $g=0.005$; (d) $g=0.01$; (e) $g=0.05$; (f) $g=0.1$ ($\tau=10$ and $D=0.1$)

With an appropriate time delay, the increase of the coupling strength g can facilitate the synchronization transition of the neural network

1.2 Effect of time delay on the synchronization of modular neural networks

To further investigate the effect of time delay τ_2 on the synchronization of modular neural networks, spatio-temporal firing raster plots of neuronal membrane potentials of the modular neural network at different time delay τ_2 are plotted in Fig. S2. The synchronization of neurons within the two subnetworks remains good, while the firing of neurons within the large-scale subnetwork changes as τ_2 increases, making the phase difference of the firing of neurons between subnetworks vary periodically. This shows that it is the change in phase difference that causes the synchronization factor in the modular neural network to vary periodically.

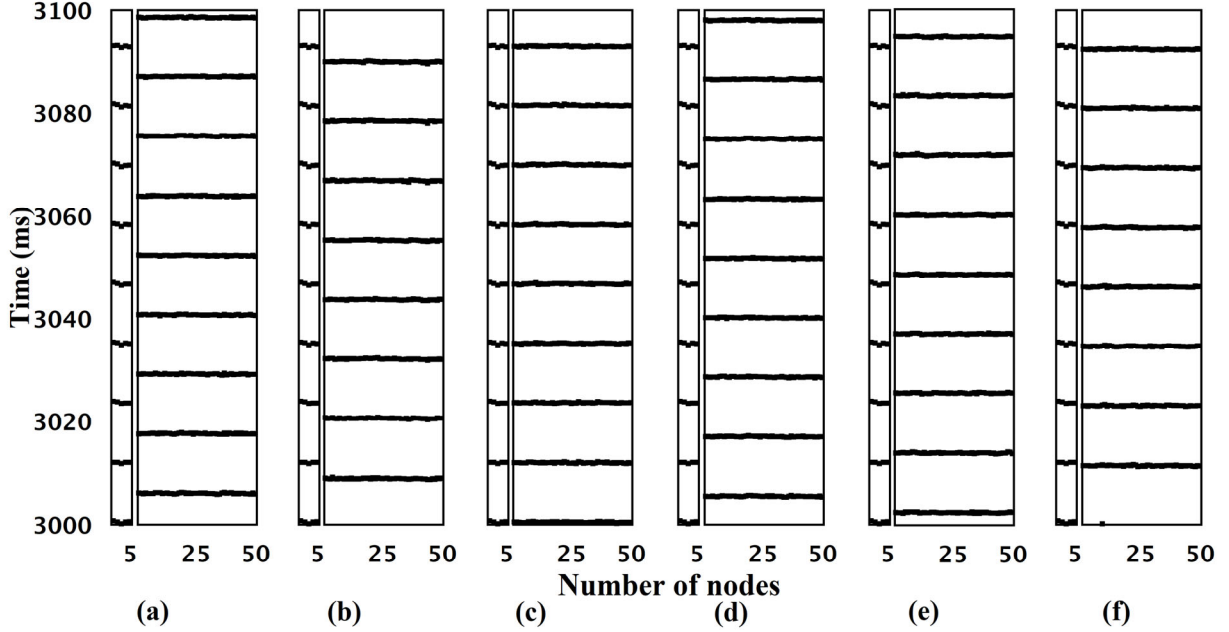


Fig. S2 Spatio-temporal firing raster plots of neuronal membrane potentials of the modular neural network at different time delays τ_2 : (a) $\tau_2=4$; (b) $\tau_2=7$; (c) $\tau_2=10$; (d) $\tau_2=12$; (e) $\tau_2=15$; (f) $\tau_2=21$ ($\tau_1=10$, $\tau_3=10$, $g=0.1$, and $D=0.1$)

When the two subnetworks are well synchronized internally, the phase difference between the two subnetworks is observed to decrease and increase periodically with the increase of τ_2

1.3 Effect of different parameters of modular neural networks on the synchronization of the network

To validate the generality of the study, more parameters of the modular neural network vary, i.e., the random connection probability p_1 within the small-scale subnetwork and the average degree $\langle k \rangle$ within the large-scale subnetwork, to observe the changing pattern of synchronization factors in the modular neural network. As shown in Fig. S3, (a) and (c) show the effect of time delay τ_3 on the synchronization of the modular network when the input small-scale network is well synchronized internally, where $\tau_1=10$, $\tau_2=5$; (b) and (d) show the effect of time delay τ_2 on modular network synchronization when both subnetworks are well synchronized internally, where $\tau_1=10$, $\tau_3=10$. In the study, p_1 takes values of 0.1, 0.3, 0.7, and 1.0, and $\langle k \rangle$ takes values of 1, 2, 3, and 4. When one of the parameter values is investigated, the other parameter values of the modular neural network remain unchanged.

The results show that the synchronization factor of the modular neural network still has a periodic distribution with increasing time delay over a wide range of parameters of the network. The period of all these synchronization transitions is approximately an integer multiple of the firing period of a single neuron. Our results are stable.

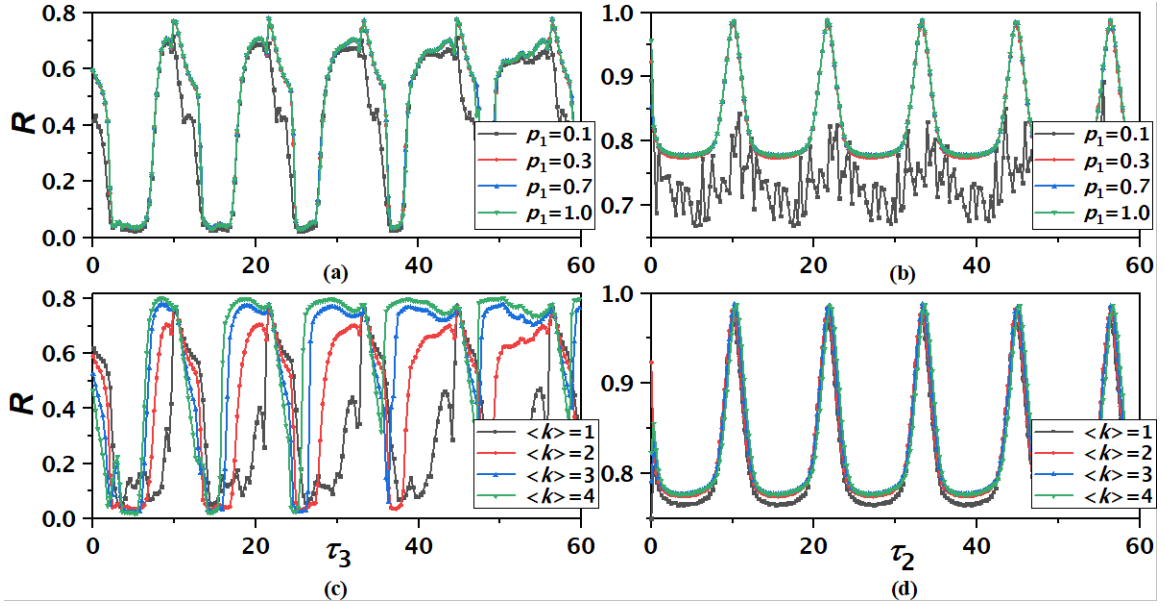


Fig. S3 Distribution of synchronization factors of modular neural networks over different network parameters with increasing time delays

(a) and (c) show the distributions of the synchronization factors with τ_3 when p_1 and $\langle k \rangle$ have different values, respectively, where $\tau_1=10$ and $\tau_2=5$. (b) and (d) show the distributions of the synchronization factors with τ_2 when p_1 and $\langle k \rangle$ have different values, respectively, where $\tau_1=10$ and $\tau_3=10$. Other parameters are $g=0.1$ and $D=0.1$. The different network parameters of the modular neural network have little effect on the regularity of the synchronization transition of the network