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Supplementary materials for

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1 Algorithm 1 (D-DCC algorithm)

In summary, by means of the abovementioned three compensation schemes, our resilient average consensus algorithm under the deterministic scenario is proposed as a deterministic detection compensation based consensus (D-DCC) algorithm, which is summarized in Algorithm 1. For simplicity, the proposed D-DCC algorithm shows only the execution of agent $j \in \mathcal{N}_i$ for a misbehaving agent i. Specifically, agent j first performs detection strategies I and II, and then compensation schemes I and II in order (see steps 4–8 in Algorithm 1). Then, agent j checks if the error of agent i exceeds the bound $\alpha_j \rho_j^k$. Subsequently, agent j identifies if agent i is to be isolated at this time. If so, agent j calculates $\eta_j^{i(3)}(k+1)$ by compensation scheme III and updates the new $|\mathcal{N}_i|$ to its remaining neighbors. In the sequel, agent j updates the error compensator $\eta_j(k+1)$. Eventually, agent j selects $\varepsilon_j(k+1)$ and renews its state $x_j(k+2)$ with the designed $\varepsilon_j(k+1)$. Note that the selection of the compensation input $\varepsilon_j(k+1)$ could be arbitrary, as long as it guarantees the security of the state and the non-increasing property of $|\eta_j(k)|$.

Algorithm 1 D-DCC algorithm

1: **Initialize**: The initial state of agent i ($x_i(0)$) and the number of neighboring agents of the agent i ($|\mathcal{N}_i|$) are initialized and exchanged between agents. Set $\eta_j(0) = 0$ and parameters ρ_j, α_j for all $j \in \mathcal{N}_i$.
2: **for** $k = 0$: Max_time **do**
3:   **for** $j \in \mathcal{N}_i$ (agent i is not isolated & $\pi_i(k) = 0$) **do**
4:     $\varepsilon_i^{j(1)}(k+1) = w_{ij}(x_j^{(i)}(k) - x_j(k))$
5:     $\varepsilon_i^{(2)}(k) = x_i(k+1) - \sum_{j \in N_i} w_{ij} x_j^{(i)}(k)$
6:     **if** $\varepsilon_i^{j(1)}(k) \neq 0$ or $\varepsilon_i^{(2)}(k) \neq 0$ **then**
7:       Calculate $\eta_j^{i(1)}(k+1)$ and $\eta_j^{i(2)}(k+1)$ by Eqs. (8) and (9)
8:     **end if**
9:     **if** $|\varepsilon_i^{j(1)}(k) + \varepsilon_i^{(2)}(k)| > \alpha_j \rho_j^k$ **then**
10:       Agent j cuts off the communication with agent i
11:     **end if**
12:     **if** agent i is isolated at this time **then**
13:       $\eta_j^{i(3)}(k+1) = (x_i(k+1) - x_i(0))/|\mathcal{N}_i|$
14:     **end if**
15:     Update the error compensator $\eta_j(k+1) = \eta_j(k) - \varepsilon_j(k) + \eta_j^{i(1)}(k+1) + \eta_j^{i(2)}(k+1) + \eta_j^{i(3)}(k+1)$
16:   **end for**
17:   Renew $x_j(k+2)$ according to Eq. (5)
18: **end for**

2 Illustrating example of the D-DCC algorithm

For ease of understanding, we provide an example here to illustrate the effectiveness of the proposed D-DCC detection method.

Illustrating example: We consider a ring network with one misbehaving agent for illustration as shown in Fig. S1, which is rather special among the network topologies considered in this paper, and helps illustrate the effectiveness of the method. Specifically, at time k, the states of agents 1, 2, 3, and n are set to be $x_1(k)=2, x_2(k)=6, x_3(k)=4$, and $x_n(k)=8$. The state updating of the misbehaving agent 2 is given by

$$x_2(k+1)=\frac{1}{4}x_1(k)+\frac{1}{2}x_2(k)+\frac{1}{4}x_3(k),$$

which is known by agents 1 and 3. Note that agents 1 and 3 are normal and they will receive the same information set from the misbehaving agent 2. At time $k+1$, the misbehaving agent 2 starts conducting misbehaviors and sends the following three kinds of information set to neighbors (i.e., agents 1 and 3) according to the misbehaviors described in Section 2.3, which will all be effectively detected by detection strategies I and II.

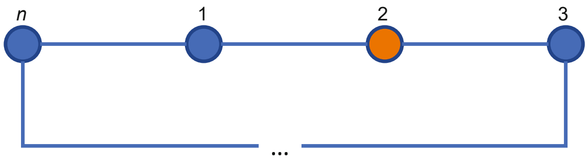

Fig. S1 Example: a ring network with a single misbehaving agent 2

Case 1: The information set sent by agent 2 is given by

$$\Psi_2(k+1)=\Big\{i=2, x_2(k+1)=4, \pi_2(k+1)=0, \varepsilon_2(k)=0, \{x_1^{(2)}(k)=2, x_3^{(2)}(k)=2\}\Big\}.$$

In this case, the misbehaving agent 2 does not inject false data to mislead the state update, but modifies the state information of agent 3. Therefore, agent 3 will detect that $x_3^{(1)}(k)\neq x_3(k)$ according to detection strategy I, then agent 3 can detect agent 2 as a misbehaving agent, and compensation will be added based on compensation scheme I at time $k+1$. Note that agent 1 is not able to detect agent 2 in this case.

Case 2: The information set sent by agent 2 is given by

$$\Psi_2(k+1)=\Big\{i=2, x_2(k+1)=3, \pi_2(k+1)=0, \varepsilon_2(k)=0, \{x_1^{(2)}(k)=2, x_3^{(2)}(k)=4\}\Big\}.$$

Here, the misbehaving agent 2 does not follow the update rule based on the information set. Hence, agents 1 and 3 can detect the misbehavior of agent 2 by detection strategy II at time $k+1$, and compensation will be added based on compensation scheme II by agents 1 and 3.

Case 3: The information set sent by agent 2 is given by

$$\Psi_2(k+1)=\Big\{i=2, x_2(k+1)=3, \pi_2(k+1)=0, \varepsilon_2(k)=0, \{x_1^{(2)}(k)=2, x_3^{(2)}(k)=2\}\Big\},$$

where both $x_2(k+1)$ and $x_3^{(2)}(k)$ are modified by agent 2. Therefore, agent 3 will perform detection strategies I and II and compensation schemes I and II, while agent 1 will effectively execute detection strategy II and compensation scheme II. Through the abovementioned three cases, all kinds of misbehaviors will be detected and compensated for by our proposed schemes.

3 Algorithm 2 (S-DCC algorithm)

The details of the stochastic detection compensation based consensus (S-DCC) algorithm are summarized in Algorithm 2. Note that at each time slot of detection, a newly detected error will be added to the error set, and a new estimate of the undetected error will be utilized to replace the former one. Hence, at step 11 of S-DCC, the former compensation by compensation scheme IV will be removed and updated.

Algorithm 2 S-DCC algorithm

Initialize: Set the number of detection times $m_j = 0$, and the other parameters are the same as those in Algorithm 1.
for $k = 0$: Max_time **do**
  if detection is enabled **then**
    for $j \in \mathcal{N}_i$ (agent i is not isolated & $\pi_i(k) = 0$) **do**
      Execute steps 4–11 in Algorithm 1
      if agent i is misbehaving **then**
        Store $\varepsilon_i^j(k) = \varepsilon_i^{j(1)}(k) + \varepsilon_i^{(2)}(k)/|\mathcal{N}_i|$ in $\Omega_j^{(i)}(k)$
        Calculate $\eta_j^{i(4)}(k+1)$ by Eq. (11)
      end if
      Execute steps 12–14 in Algorithm 1
      $\eta_j(k+1) = \eta_j(k) - \varepsilon_j(k) + \eta_j^{i(1)}(k+1) + \eta_j^{i(2)}(k+1) + \eta_j^{i(3)}(k+1) + \eta_j^{i(4)}(k+1) - \eta_j^{i(4)}(k)$
    end for
  end if
  Execute step 17 in Algorithm 1
end for

4 Proof of Theorem 3

Using the absolute value inequality, we have

$$\begin{aligned} R(\varepsilon_i(k), \overline{\varepsilon}_i(k)) &= \int_{-\infty}^{\infty} |F_{\varepsilon_i(k)}(x) - F_{\overline{\varepsilon}_i}(x)| \mathrm{d}x \\ &\leq \int_{-\infty}^{\infty} |F_{\varepsilon_i(k)}(x) - F_{Y_i}(x)| \mathrm{d}x + \int_{-\infty}^{\infty} |F_{\overline{\varepsilon}_i}(x) - F_{Y_i}(x)| \mathrm{d}x. \end{aligned}$$

According to Eq. (20), we have

$$\begin{aligned} &\int_{-\infty}^{\infty} |F_{\varepsilon_i(k)}(x) - F_{Y_i}(x)| \mathrm{d}x \\ &= \int_{-\infty}^{0} |(1-\theta_i) F_{Y_i}(x)| \mathrm{d}x + \int_{0}^{\infty} |(1-\theta_i)(1 - F_{Y_i}(x))| \mathrm{d}x \\ &= (1-\theta_i) \left[\int_{-\infty}^{0} F_{Y_i}(x) \mathrm{d}x + \int_{0}^{\infty} (1 - F_{Y_i}(x)) \mathrm{d}x \right] \\ &= (1-\theta_i) \mathbb{E}[|Y_i|]. \end{aligned}$$

Additionally, according to the Wasserstein distance between two normal distributions, we have

$$\begin{aligned} &\int_{-\infty}^{\infty} |F_{\overline{\varepsilon}_i}(x) - F_{Y_i}(x)| \mathrm{d}x \\ &\leq \sum_{l=1}^{N_l} a_l \left[\int_{-\infty}^{\infty} |F_{\overline{\varepsilon}_i}(x) - \Phi(\frac{x - \mu_l}{\sigma_l})| \mathrm{d}x \right] \\ &\leq \sum_{l=1}^{N_l} a_l (|\theta_i \mu_i - \mu_l| + |\sigma_{\varepsilon_i} / \sqrt{M_i} - \sigma_l|). \end{aligned}$$

Combining the above, we complete the proof.