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Effect of segments of soil-water characteristic curves on the estimated permeability function using statistical methods

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Data S1 Statistical models and evaluation criterion

1 Statistical models

The SWCC or soil water retention curve defines the relationship between water content (can be expressed in the forms of gravimetric water content, volumetric water content, or degree of saturation) in the soil and the soil suction. SWCC is an effective interpretive model that uses the elementary capillary model to provide an understanding of the distribution of water in the voids (Fredlund and Rahardjo, 1993; Fredlund et al., 2012). In addition, Childs and Collis-George (1950), Zhai and Rahardjo (2015) and Zhai et al. (2018) recommended that SWCC be considered as analogous to the pore-size distribution function. Therefore, SWCC can provide information on pore-size distribution in soil (i.e. the pore-size distribution function).

Childs and Collis-George (1950) are considered to have been the first to have proposed a method for estimating the coefficient of permeability of unsaturated soil by adopting the concept of a pore-size distribution function. They considered the pores in soil as a series of capillaries that were randomly interconnected in a given cross section. Consequently, the coefficient of permeability of unsaturated soil was calculated from the effective area of the cross section, which can be computed using statistical theory. That model was further extended by Marshall (1958) and Kunze et al. (1968). Kunze et al. (1968) proposed dividing the SWCC into several segments to represent pores with different sizes by using the capillary law. They proposed evenly dividing the SWCC in the domain of the volumetric water content (Fig. S1). As a result, the pores in each group have the same distribution (i.e., the pore sizes follow a uniform distribution), which yields a simple equation to calculate the permeability function. However, during the division the suctions corresponding to the points on these SWCC segments need to be calculated from the volumetric water content. Bharat and Sharma (2012), Bharat (2014) and Rahimi et al. (2015) found that it was not always easy to calculate the suction from the volumetric water content because of the difficulty of converting the reverse function from the SWCC best fit equations.

To avoid calculating the suction from the volumetric water content, Zhai and Rahardjo (2015) proposed dividing the entire SWCC within the domain of matric suction (Fig. S2). During the division, the degree of saturation can be calculated directly from the matric suction using the SWCC best fit equation. However, it should be noted that the pores in each group have different distributions.

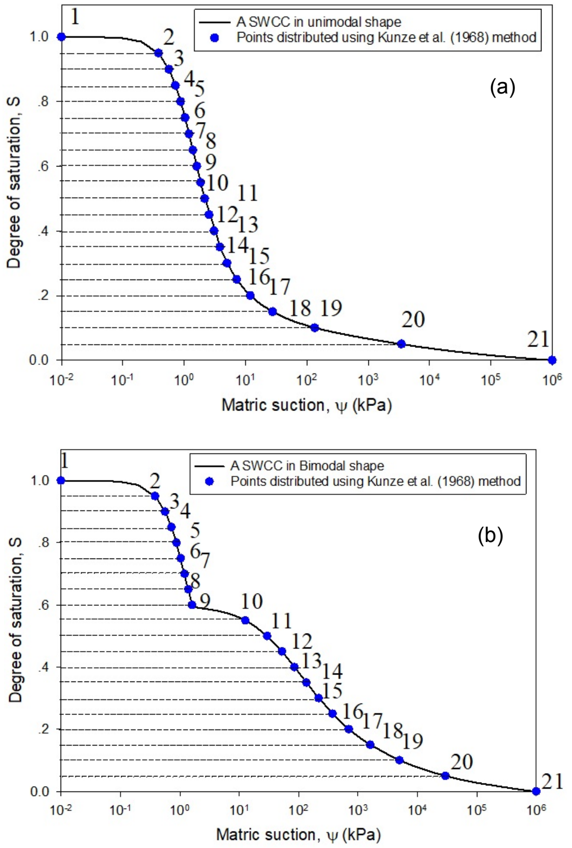

Fig. S1 Illustration of the division of SWCCs using the method of Kunze et al. (1968)
(a) Division of a unimodal SWCC in the domain of volumetric water content; (b) division of a bimodal SWCC in the domain of volumetric water content

In Kunze et al.'s (1968) model, pores in the soil are divided into groups with the same distributions, and the unsaturated coefficient of permeability can be calculated using Eq. (S1), as follows:

$$k_w(\theta_w)_i = \frac{k_s}{k_{sc}} A_d \sum_{j=i}^{m} \left\{ (2j+1-2i)(u_a-u_w)_j^{-2} \right\}, \tag{S1}$$
$$i = 1,\ 2,\ \cdots,\ m,$$

where $k_w(\theta_w)_i$ is the estimated unsaturated coefficient of permeability for a given volumetric water content; u_a is the pore-air pressure (kPa); u_w is the pore-water pressure (kPa); $(\theta_w)_i$ is the volumetric water content corresponding to the i^{th} interval; i is the interval number that increases as the volumetric water content decreases; j is a count from "i" to "m"; m is the total number of intervals between the saturated volumetric water content, θ_s, and the lowest volumetric water content, θ_L; k_s is the measured saturated coefficient of permeability (m/s); k_{sc} is the calculated saturated coefficient of permeability (m/s); and A_d is the adjusting constant.

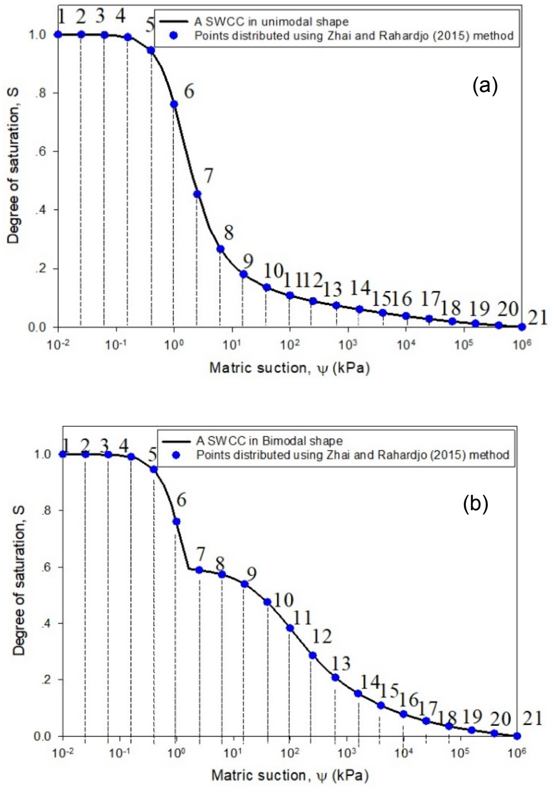

Fig. S2 Illustration of the division of SWCCs using the method of Zhai and Rahardjo (2015)

(a) Division of a unimodal SWCC in the domain of matric suction; (b) division of a bimodal SWCC in the domain of matric suction

In Zhai and Rahardjo's (2015) model, pores in the soil are divided into groups that have different distributions and the unsaturated coefficient of permeability can be calculated using Eq. (S2), as follows:

$$k(\psi_{m+i}) = k(\psi_m)\frac{{n_{m+i}}^2\left\{\begin{array}{l}\left(S(\psi_{m+i})-S(\psi_{m+i+1})\right)^2 r_{m+i}^2 + \\ \sum\limits_{j=m+i+1}^{N}\left[\begin{array}{l}\left(S(\psi_{m+i})-S(\psi_j)\right)^2 - \\ \left(S(\psi_{m+i})-S(\psi_{j-1})\right)^2\end{array}\right] r_j^2\end{array}\right\}}{{n_m}^2\left\{\begin{array}{l}\left(S(\psi_m)-S(\psi_{m+1})\right)^2 r_m^2 + \\ \sum\limits_{i=m+1}^{N}\left[\begin{array}{l}\left(S(\psi_m)-S(\psi_i)\right)^2 - \\ \left(S(\psi_m)-S(\psi_{i-1})\right)^2\end{array}\right] r_i^2\end{array}\right\}}, \tag{S2}$$

where $k(\psi_{m+i})$ is the unsaturated coefficient of permeability with respect to a suction of ψ_{m+i},; ψ_{m+i} is the suction state in the soil; and n_{m+i} is the porosity corresponding to suction ψ_{m+i}.

The form of Eq. (S2) is more complicated than that of Eq. (S1). However, note that Eqs. (S1) and (S2) are essentially the same, and both methods do not specify the best fit equation to be used. This indicates that both methods allow users to use different best fit equations in the estimation. It is difficult to calculate the matric suction from a given volumetric water content if Eq. (S1) is adopted, while it is easy to calculate the degree of saturation from a given matric suction using the SWCC best fit equation (e.g., the equation of Fredlund and Xing, 1994) if Eq. (S2) is selected. Fredlund (2016) indicated that suction could be considered as a stress state variable and the permeability function was commonly defined as the relationship between the unsaturated coefficient of permeability and soil suction. However, the first and last segments as shown in Figs. S1a and S1b cover a much wider range of suction than other segments, which may lead to a rough estimation of the permeability function in very low or very high suction ranges if Eq. (S1) is adopted.

As Fredlund and Xing's (1994) equation is one of most popular SWCC best fit equations and is commonly used by researchers, it was adopted to represent the SWCC of the soil. In their original equation, the water amount in soil is expressed in the form of the volumetric water content. In studies by Zhai and Rahardjo (2015) and Zhai et al. (2018, 2019), the SWCC was adopted to represent the pore-size distribution function of soil in the statistical model, and the SWCC in the form of degree of saturation was most analogous to the pore-size distribution function. Therefore, Fredlund and Xing's (1994) equation was revised in the form of degree of saturation, as illustrated in the following equation.

$$S = \left[1 - \frac{\ln\left(1+\dfrac{\psi}{C_r}\right)}{\ln\left(1+\dfrac{10^6}{C_r}\right)}\right]\frac{1}{\left\{\ln\left[e+\left(\dfrac{\psi}{a_f}\right)^{n_f}\right]\right\}^{m_f}}, \tag{S3}$$

where a_f, n_f and m_f are fitting parameters and C_r is an input value, which is a rough estimation of the residual suction.

As a result, the relative coefficient of permeability, k_r, which defines the ratio between the unsaturated coefficient of permeability and the saturated coefficient of permeability of soil, can be calculated directly from the fitting parameters in Fredlund and Xing's (1994) equation by substituting Eq. (S3) into Eq. (S2) as follows:

$$k_r = \frac{1}{A_{FX}} \left\{ \begin{array}{l} \left(\dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_m}{a_f} \right)^{n_f} \right] \right\}^{m_f}} - \dfrac{C(\psi_{m+1})}{\left\{ \ln\left[e + \left(\dfrac{\psi_{m+1}}{a_f} \right)^{n_f} \right] \right\}^{m_f}} \right)^2 \dfrac{1}{\psi_m^2} + \\ \displaystyle\sum_{j=m+1}^{N} \left\{ \begin{array}{l} \left\{ \dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_{mf}}{a_f} \right)^{n_f} \right] \right\}^{m_f}} - \dfrac{C(\psi_j)}{\left\{ \ln\left[e + \left(\dfrac{\psi_j}{a_f} \right)^{n_f} \right] \right\}^{m_f}} \right\}^2 - \\ \left\{ \dfrac{C(\psi_m)}{\left\{ \ln\left[e + \left(\dfrac{\psi_{mf}}{a_f} \right)^{n_f} \right] \right\}^{m_f}} - \dfrac{C(\psi_{j-1})}{\left\{ \ln\left[e + \left(\dfrac{\psi_{j-1}}{a_f} \right)^{n_f} \right] \right\}^{m_f}} \right\}^2 \end{array} \right\} \dfrac{1}{\psi_j^2} \end{array} \right\}, \tag{S4}$$

where

$$A_{FX} = \sum_{i=1}^{N} \left\{ \begin{array}{l} \left\{ 1 - \dfrac{C(\psi_i)}{\left\{ \ln\left[e + \left(\dfrac{\psi_i}{a_f} \right)^{n_f} \right] \right\}^{m_f}} \right\}^2 \\ - \left\{ 1 - \dfrac{C(\psi_{i-1})}{\left\{ \ln\left[e + \left(\dfrac{\psi_{i-1}}{a_f} \right)^{n_f} \right] \right\}^{m_f}} \right\}^2 \end{array} \right\} \frac{1}{\psi_i^2}. \tag{S5}$$

For the estimation, a total of m pieces of SWCC segments are adopted in Eq. (S1), and N pieces in Eq. (S2). In this paper, the effects of the numbers of SWCC segments (i.e., the value of m or N) on the estimated relative coefficient of permeability are investigated and discussed.

2 Evaluation criterion

The coefficient of determination (R^2), as illustrated in Eq. (S6), was used for comparing and understanding the predictive capabilities of Eqs. (S1) and (S2) after adopting different numbers of SWCC segments. This criterion tests mainly the degree of curve matching.

$$R^2 = 1 - \frac{\text{SSE}}{\text{SST}}, \tag{S6}$$

$$\text{SST} = \text{SSE} + \text{SSR}, \tag{S7}$$

$$\text{SSE} = \sum_{i=1}^{N} \left(y_i - \hat{y}_i \right)^2, \tag{S8}$$

$$\text{SSR} = \sum_{i=1}^{N} \left(y_i - \bar{y} \right)^2, \tag{S9}$$

where R^2 is the coefficient of determination; SSE is the error sum of squares; SST is the total corrected sum of squares; SSR is the regression sum of squares; y_i is the measured data; $\hat{y}_i$ is the estimated result; $\bar{y}$ is the average value of the measured data; and N is the total number of measured data.

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