

Electronic Supplementary Materials

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Thermal strain response of saturated clays in 1D condition

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Theory

1 Thermoplasticity

One of the key thermal effects on the behavior of soils is to cause thermoplastic strain. Experimental results show that the yielding mechanism is sensitive to temperature change. It is not easy to accurately measure the amount of thermoplastic deformation. A more common method is a heating and cooling test that maintains constant stress. Based on the experiments, it is well-established in the literature that the normally consolidated clay tends to contract during drained heating, while the same clay under overconsolidated condition expands during drained heating. Figure S1 plots typical testing result on Boom clays at OCR equals to 12 experimented by Sultan et al. (2002). The result shows that there are conversions from expansion to contraction on the volume during drained heating. The observations show that Boom clay specimens with an OCR of 12 will expand until achieving an increment of temperature by 57 °C, after which it starts to contract for higher temperatures. Similar behavior has also been observed by Abuel-Naga et al. (2007), Graham et al. (2001), Cekerevac and Laloui (2004), Towhata et al. (1993). The difference in thermally induced volumetric deformation between heating and cooling for normally consolidated clay reveals an irreversible contraction after a heating-cooling cycle (Coccia et al., 2016). The irreversible contraction due to the thermal plastic strains will happen when heating above the transition temperature is applied to the soil.

During cooling, an approximated linear thermal contraction is typically found as shown in Fig.S1, even though thermal expansion during cooling has been found by Campanella and Mitchell (1968). To take this into account, a simple form has been adopted to express the volumetric elastic strain increment, $d\varepsilon_v^{te}$, associated with thermal loading as follows:

$$d\varepsilon_v^{te} = -3\alpha dT \quad (S1)$$

where α is the volumetric thermal elastic expansion coefficient in drained condition for clay; dT denotes the temperature increment. Considering simplify, α is supposed to be a constant value in this study. Thus, the elastic volumetric strains are assumed to vary linearly with temperature.

During heating, when the increased temperature is smaller than the transition temperature, only thermal elastic strain occurs. The thermoplastic strain will

induce when the increased temperature is larger than the transition temperature, and its increment can be calculated by total and elastic incremental strain, as

$$d\varepsilon_v^{tp} = d\varepsilon_v^t - d\varepsilon_v^{te} \quad (S2)$$

where $d\varepsilon_v^{tp}$ and $d\varepsilon_v^t$ are thermal-induced plastic and total strain increment, respectively. Thus, the thermoplastic strain can be measured by Eq.(S2) as shown in Fig.S1. Thereafter, the experimental results are used to analyze the relationship among thermoplastic strain, temperature, and stress conditions. Figure S2 presents the relationships between thermoplastic strain and temperatures for different soils at various stress conditions. Here, a simple equation with only one additional material parameter is proposed as

$$d\varepsilon_v^{tp} = \omega(T - T_0) \left(\frac{p}{p_c(T_0)} \right)^2 dT \quad (S3)$$

where ω is a thermal related parameter controlling the development of ε_v^{tp} and is assumed constant in this study; p is the effective mean stress; $p_c(T_0)$ is the preconsolidation pressure at reference (or room) temperature T_0 ; T is the current temperature. Figure S2 shows an excellent agreement between the measured results and predicted results by Eq.(S3), which directly verified its rationality. Eq.(S3) also indicates that thermal plastic strain associates with the stress and temperature conditions and the stress history which coincides with the experimental observations (Baldi et al., 1991; Abuel-Naga et al., 2007; Sultan et al., 2002; Demars et al., 1982). For normally consolidated states, Eq.(S3) is reduced to the following equation:

$$d\varepsilon_v^{tp} = \omega(T - T_0) dT \quad (S4)$$

Then, Eq.(S4) implies that the magnitude of the thermally induced plastic strain of normally consolidated clay is independent of the applied stress, which coincides with the observations (Abuel-Naga et al., 2007; Demars et al., 1982). Furthermore, compared to the expression by Cui et al. (2000) the proposed Eq.(S3) has a more concise form.

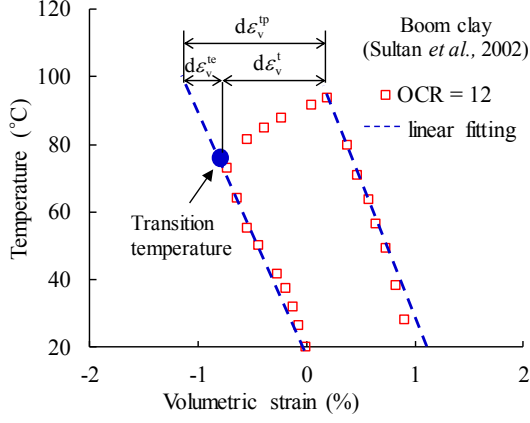


Fig.S1 Illustration of thermal volumetric strain during drained heating

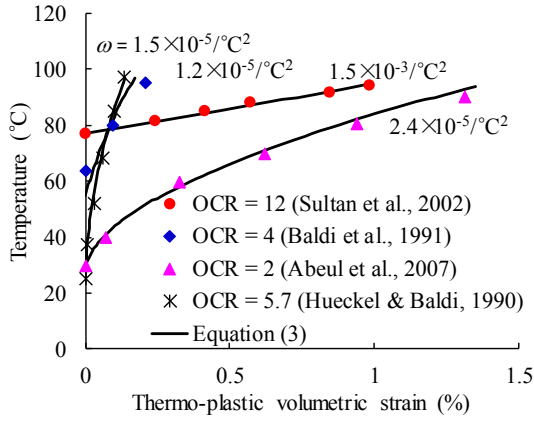


Fig.S2 Measured and predicted thermal plastic volumetric strains

Therefore, to accurately calculate the thermoelastic and thermoplastic deformation, it is very important to determine the transition temperature. Based on thermal volumetric tests under different stress conditions, Abuel-Naga et al. (2007) proposed an equation to express the relationship between the stress and transition temperature, as

$$\frac{p_c(T_0)}{p(T_T)} = \beta \sqrt{\ln \frac{T_T}{T_0}} + 1 \quad (S5)$$

where $p(T_T)$ is the thermal yielding stress corresponding to the transition temperature T_T ; β is a material parameter with initial value β_0 . Following the definition, Eq.(S5) is adopted to express the thermal yield equation (TY) in this study, as

$$f^{TY} = p \left(\beta \sqrt{\ln \frac{T}{T_0}} + 1 \right) - p_c(T_0) = 0 \quad (S6)$$

where f^{TY} is the expression of TY in (p, T) plane under non-isothermal condition.

2 Thermomechanical model

2.1 Thermomechanical yielding

Apart from the thermoplasticity, the mechanical plasticity would also simultaneously generate during the thermomechanical loading. One of the important influences of heating on the mechanical behavior of soil is the thermal dependency of the preconsolidation pressure. The expression firstly proposed by Moritz (1995) based the thermal oedometer tests is adopted as follows

$$p_c(T) = p_c(T_0) \left(\frac{T_0}{T} \right)^\theta \quad (S7)$$

where $p_c(T_0)$ is the preconsolidation pressure at the reference temperature T_0 ; θ is a thermal-related material parameter. The validity of Eq.(S7) is revisited on various clays and good fitting is observed (Wang et al., 2016). Considering that governing ability for the evolution of the preconsolidation pressure with temperature, Eq.(S7) is used to express the mechanical yield equation (MY) in this study, as

$$f^{MY} = p - p_c(T_0) \left(\frac{T_0}{T} \right)^\theta \quad (S8)$$

where f^{MY} is the expression of MY in (p, T) plane under non-isothermal condition.

The yield curves denoted MY and TY are controlled by the evolution of preconsolidation pressure $p_c(T_0)$ at the reference temperature T_0 , and two plastic volumetric strains are related to the evolution of MY and TY, respectively. For the thermoplastic strain increment $d\varepsilon_v^{ip}$, it is calculated by Eq.(S3). The mechanical plastic strain increment $d\varepsilon_v^{mp}$, obtained when the stress reaches the mechanical yield limit, has the following expression as in the common Cam Clay model:

$$d\varepsilon_v^{mp} = \frac{\lambda - \kappa dp}{1 + e_0 p} \quad (S9)$$

with λ and κ are the normal compression and recompression slopes in $e-\ln p$ plane. Besides, the mechanical loading induced elastic volumetric strain increment is estimated by

$$d\varepsilon_v^{me} = \frac{\kappa dp}{1 + e_0 p} \quad (S10)$$

where e_0 is the void ratio. Here, λ and κ are assumed

temperature independent.

2.2 Hardening laws

The thermal and mechanical yield limits are coupled through the hardening parameter $p_c(T_0)$, which hardens with the development of total volumetric plastic strains increment, $d\varepsilon_v^p$. That is

$$dp_c(T_0) = p_c(T_0) \frac{1+e_0}{\lambda - \kappa} d\varepsilon_v^p \quad (S11)$$

Initially, $p_c(T_0)$ equals to $p_{c0}(T_0)$, which denotes the initial preconsolidation pressure corresponding to the reference temperature T_0 ; $d\varepsilon_v^p$ is the sum of thermal and mechanical plastic strains calculated by Eq.(S3) and Eq.(S9) as follows

$$d\varepsilon_v^p = d\varepsilon_v^{tp} + d\varepsilon_v^{mp} \quad (S12)$$

Therefore, if $d\varepsilon_v^p$ increases due to either type of plastic strain, MY and TY will both move due to the increase of $p_c(T_0)$.

Parameter β controlling the evolution of TY will also harden when the thermal yielding is activated. As the thermal plastic strain is directly obtained by Eq.(S3), the evolution of β must be met with the consistency condition of TY, which can be expressed as

$$df^{TY} = \frac{\partial f^{TY}}{\partial p} dp + \frac{\partial f^{TY}}{\partial T} dT + \frac{\partial f^{TY}}{\partial \beta} d\beta + \frac{\partial f^{TY}}{\partial p_c(T_0)} dp_c(T_0) = 0 \quad (S13)$$

Then, β can directly be solved with just one unknown in the equation.

Using the above equations and an explicit integration method, the proposed model has been built at the stress point level. In the calculation, temperature increment, dT , and strain increment, $d\varepsilon$ are considered as prescribed inputs and stress increment, $d\sigma$ as output has to be obtained.

2.3 Model parameters and the determinations

The proposed model has 8 parameters (λ , κ , $p_{c0}(T_0)$, e_0 , α , β_0 , θ , ω). In the following, the role of each parameter and the methods for determinations are briefly discussed.

(1) The common constants: λ , κ , $p_{c0}(T_0)$, e_0 . λ and κ the slopes of normal compression and recompression line of the isotropic compression curve in e - $\ln p$ plane. $p_{c0}(T_0)$ represents the initial value of

$p_c(T_0)$, that is, the initial preconsolidation pressure at a reference temperature T_0 , which corresponds to the initial size of MY on p axis. e_0 is the initial void ratio.

(2) The thermal elastic coefficient α can be correlated from the cooling stage of a drained heating-cooling test on a normally consolidated sample or the heating stage of a drained heating test on a highly overconsolidated sample. For most practical purposes, this parameter is considered constant in the order of $10^{-5} \text{ } ^\circ\text{C}^{-1}$.

(3) θ controls the thermal dependency of the preconsolidation pressure which is considered as temperature independent here. This parameter can be estimated from yielding stresses obtained from two isotropic compression tests conducted at different temperatures with Eq.(S7).

(4) The initial TY is defined through $p_{c0}(T_0)$, and β_0 by Eq.(S6). At least two isotropic drained heating tests at different OCR conditions are required. The value of β_0 is then obtained from Eq.(S5) by besting fitting the yield values in the p - T plane.

(5) The parameter ω controls the development of thermal plastic strain. It can be correlated from one isotropic drained heating test on an intermediately overconsolidated sample.

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