Electronic Supplementary Materials

For https://doi.org/10.1631/jzus.A2200158

Optimum insulation thickness of external walls by integrating indoor moisture buffering effect: a case study in the HSCW zone of China

Yan-hao FENG[1], Zi-tao YU[1,2]✉, Jiang LU[3], Xu XU[4]

[1]*Institute of Thermal Science and Power Systems, Zhejiang University, Hangzhou 310027, China*
[2]*State Key Laboratory of Clean Energy Utilization, Zhejiang University, Hangzhou 310027, China*
[3]*School of Civil Engineering and Architecture, Zhejiang University of Science and Technology, Hangzhou 310023, China*
[4]*Institute of Energy Engineering, China Jiliang University, Hangzhou 310018, China*

S1 Detailed derived steps of HAM model

- **Derive of moisture transfer equation and air transfer equation**

Based on the moisture transfer balance equation $\frac{\partial(w_\mathrm{v}+w_\mathrm{l})}{\partial t}+\frac{\partial(g_\mathrm{l}+g_\mathrm{v}+g_\mathrm{a})}{\partial x}=0$, Fick's law for water vapour diffusion g_v and Darcy's law for liquid water diffusion g_l, the expression can be written as:

$$\frac{\partial(w_\mathrm{v}+w_\mathrm{l})}{\partial t}+\frac{\partial}{\partial x}\left(\delta_\mathrm{l}\frac{\partial P_\mathrm{c}}{\partial x}\right)-\frac{\partial}{\partial x}\left(\delta_\mathrm{v}\frac{\partial P_\mathrm{v}}{\partial x}\right)+\frac{\partial g_\mathrm{a}}{\partial x}=0, \quad \text{(S1)}$$

where w_v and w_l are moisture content in water vapour and liquid water phases, respectively; δ_v and δ_l are permeabilities of water vapour and liquid water, respectively; P_v and P_c are vapour pressure in pores and capillary pressure, respectively.

From the water retention curve, it is known that the moisture content w in the material relates to relative humidity and temperature, and that w_v and w_l are in a coexistence state, so the unsteady term in Eq. (S1) can be rewritten as:

$$\frac{\partial(w_\mathrm{v}+w_\mathrm{l})}{\partial t}=\frac{\partial w}{\partial t}=\frac{\partial w}{\partial\varphi}\frac{\partial\varphi}{\partial t}+\frac{\partial w}{\partial T}\frac{\partial T}{\partial t}, \qquad \text{(S2)}$$

where φ is the relative humidity.

Since the moisture content varies less with temperature than with relative humidity, the second term in the right part of Eq. (S2) can be ignored. The liquid water diffusion term in Eq. (S1) can be replaced by using Kelvin's law:

$$\begin{aligned}\frac{\partial}{\partial x}\left(\delta_\mathrm{l}\frac{\partial P_\mathrm{c}}{\partial x}\right)&=-\frac{\partial}{\partial x}\left[\delta_\mathrm{l}\frac{\partial(\rho_\mathrm{l}\mathrm{R}T\ln\varphi)}{\partial x}\right]=\\&-\rho_\mathrm{l}\mathrm{R}\frac{\partial}{\partial x}\left\{\delta_\mathrm{l}\left[\ln\varphi\frac{\partial T}{\partial x}+\frac{T}{\varphi}\frac{\partial\varphi}{\partial x}\right]\right\},\end{aligned} \qquad \text{(S3)}$$

and the water vapour pressure P_v in Eq. (S1) is then replaced by the relative humidity as φP_bv, where P_bv is the saturated water vapour pressure.

Because the indoor environment was embedded in the HAM-IEM, the air transfer through the wall should also be considered. Based on Poiseuille's law, the air transfer under a difference of air pressure can be expressed as $-\frac{\partial}{\partial x}\left(\delta_\mathrm{a}\frac{\partial P_\mathrm{a}}{\partial x}\right)=0$, and thus the air diffusion term (the air convection term) in Eq. (S1) can be obtained by the definition of specific humidity:

$$\begin{aligned}\frac{\partial g_\mathrm{a}}{\partial x}&=-\frac{\partial}{\partial x}\left(\frac{0.622\rho_\mathrm{a}U_\mathrm{a}P_\mathrm{v}}{P_\mathrm{atm}}\right)=\\&\rho_\mathrm{a}\frac{k_\mathrm{a}}{\mu_\mathrm{a}}\frac{0.622}{P_\mathrm{atm}}\frac{\partial}{\partial x}\left(\varphi P_\mathrm{bv}\frac{\partial P_\mathrm{a}}{\partial x}\right),\end{aligned} \qquad \text{(S4)}$$

where P_atm and P_a are atmospheric and air pressure, respectively; k_a, ρ_a, and μ_a are airflow coefficient, air density and air dynamic viscosity, respectively; δ_a is the air permeability; $U_\mathrm{a}=-\frac{k_\mathrm{a}}{\mu_\mathrm{a}}\frac{\mathrm{d}P_\mathrm{a}}{\mathrm{d}x}$ is the air velocity vertical to the wall surface.

Substitute Eqs. (S2)–(S4) into Eq. (S1), the final form of moisture transfer equation can be obtained.

- **Derive of energy transfer equation**

Based on the heat balance equation $\frac{\partial H}{\partial t}+\frac{\partial(q_\mathrm{cond}+Q_\mathrm{sensible}+Q_\mathrm{latent})}{\partial x}=0$, Fourier's law q_cond, sensible enthalpy transfer Q_sensible for liquid, vapour and air phases, and the latent enthalpy change Q_latent for vapour and air phases, the expression is written as:

$$\begin{aligned}&(\rho_\mathrm{m}C_{p\mathrm{m}}+wC_{p\mathrm{l}})\frac{\partial T}{\partial t}+\frac{\partial}{\partial x}\left(-\lambda_\mathrm{eff}\frac{\partial T}{\partial x}\right)+g_\mathrm{l}C_{p\mathrm{l}}\frac{\partial T}{\partial x}-\\&L\left(\frac{\partial g_\mathrm{v}}{\partial x}+\frac{\partial g_\mathrm{a}}{\partial x}\right)+\rho_\mathrm{a}(C_{p\mathrm{a}}+e_\omega C_{p\mathrm{v}})\frac{\partial(U_\mathrm{a}T)}{\partial x}=0,\end{aligned} \qquad \text{(S5)}$$

where subscript m is the porous matrix; L is the latent heat; $e_\omega=\frac{0.622P_\mathrm{v}}{P_\mathrm{atm}-P_\mathrm{v}}$ is the absolute air humidity; $C_{p\mathrm{l}}$, $C_{p\mathrm{a}}$ and $C_{p\mathrm{v}}$ are the specific heats of liquid water, air and water vapour, respectively.

S2 Figures for the validation of HAM model

The HAM model proposed in this study is val-idated with Heat, Air and Moisture STAnDardization (HAMSTAD) benchmarks #4 and #5. The detailed information about HAMSTAD benchmarks #4 and #5 can be referred to elsewhere (Hagentoft et al., 2004). Fig. S1 shows that the moisture contents at the exterior and interior surface of the wall vary over five days, which is the condition of calculation in HAMSTAD benchmark #4. Fig. S2 demonstrates the partial distribution profile of moisture content at 60 d, which is in accordance with HAMSTAD benchmark #5. The red curve in Fig. S1 represents the results of the HAM model in this study, and the black curve in Fig. S2 shows the average results provided by other institutes in the HAMSTAD project.

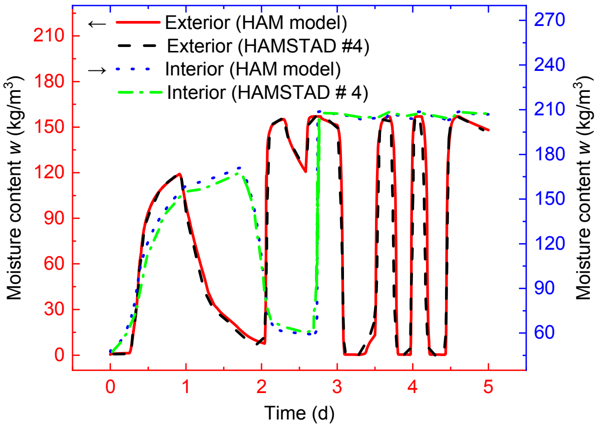


Fig. S1 Moisture content at exterior and interior surfaces of the wall (validation with HAMSTAD benchmark #4)

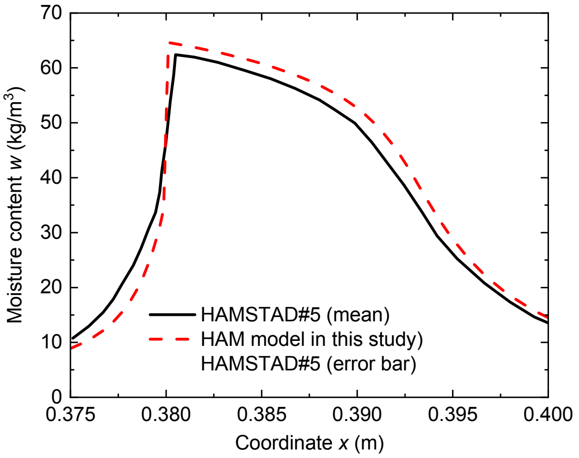


Fig. S2 Moisture content distribution along the wall (validation with HAMSTAD benchmark #5)

S3 Information about the building, wall configuration and hygro-thermal properties of the materials

Tables S1 and S2 are the detailed values of building parameters in the case study of Section 3 in the original paper. Table S1 shows the geometry and the information of heating, ventilating, and air conditioning (HVAC) system, while Table S2 is the hygrothermal properties which are used in the calculation of HAM model.

Table S1 Values of building parameters in the study

Parameter	Values
Total wall area (m^2)	113
Baseline of Air change per hour (ACH)	1
Total volume of the room (m^3)	196
Total air volume of HVAC systems (m^3/s)	1
Maximum cooling power of HVAC systems (W)	14000
Maximum heating power of HVAC systems (W)	15500
Direct radiation through windows (W)	0^*

* Direct radiation through windows is not considered.

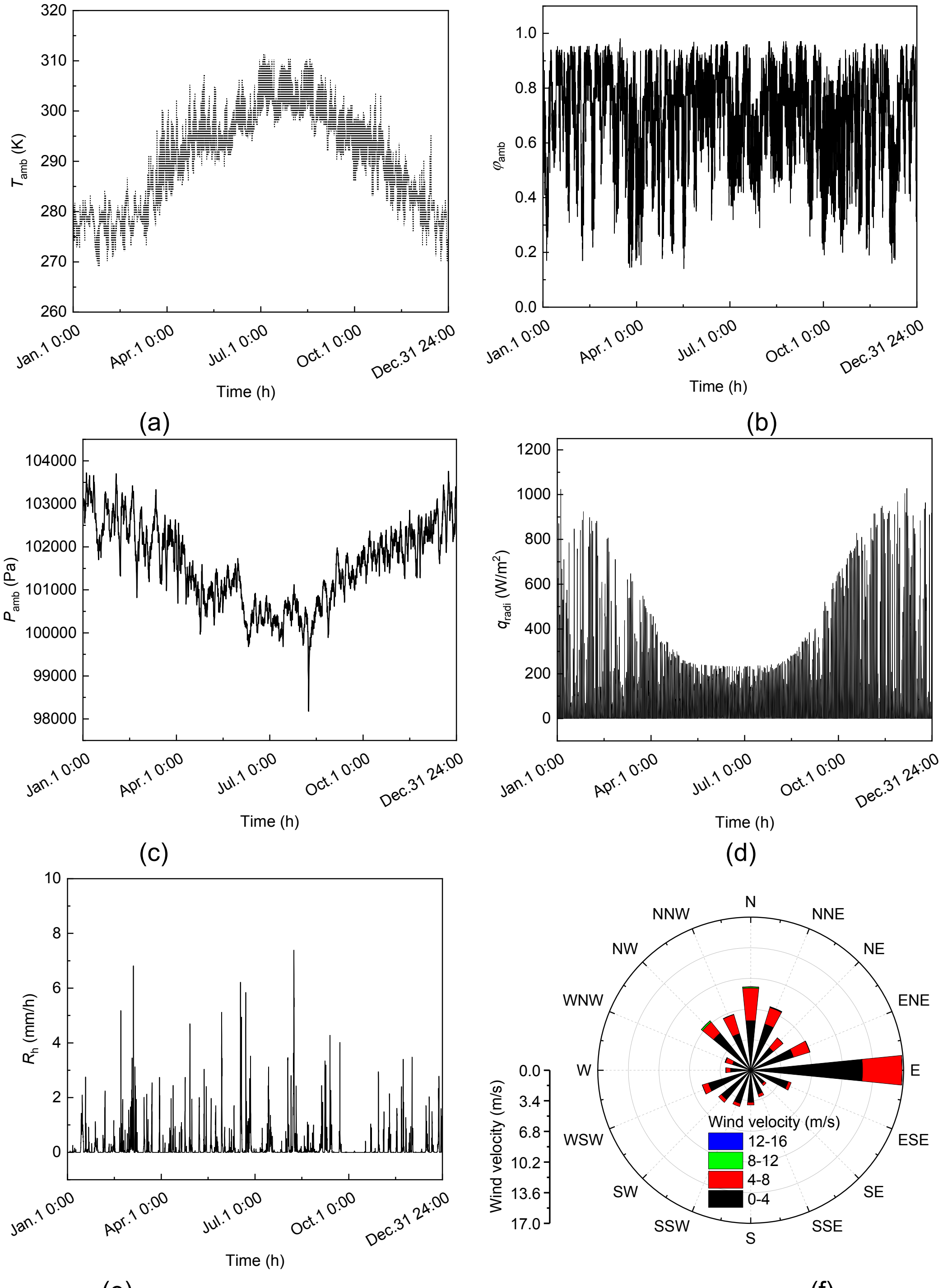
Table S2 Values of hygrothermal properties in the study (Kumaran, 1996; Liu et al., 2015)

Components	ρ_m (kg/m^3)	C_{pm} ($J/(kg \cdot K)$)	δ_v ($\times 10^{-12}$ s)	ω (kg/m^3)	λ_{eff} ($W/(m \cdot K)$)	$D_w \times 10^{-9}$ (m^2/s)
Cement plaster	1807	840	54.67	$\varphi/(-0.022\varphi^2+0.025\varphi+0.0001)$	$0.854+4.5\times10^{-3}\omega$	$1.4\exp(0.027\omega)$
Red brick	1918	840	26.00	$\varphi/(0.02451\varphi^2-0.2362\varphi+0.273)$	$1.035+4.2\times10^{-3}\omega$	$7.4\exp(0.0316\omega)$
Lime plaster	1500	840	13.57	$\varphi/(-0.052\varphi^2+0.052\varphi+0.005)$	$0.526+3.1\times10^{-3}\omega$	$2.7\exp(0.0204\omega)$
Expanded polystyrene (EPS)	30	1470	11.00	$\varphi/(-0.5277\varphi^2+0.9647\varphi+0.07086)$	$0.0331+1.23\times10^{-3}\omega$	0

* The moisture diffusive $D_w=(\delta_l\,(\rho_l RT/\varphi)+\delta_v P_{bv})(\partial\varphi/\partial w)$

S4

A series of specific meteorological conditions of the outdoor atmosphere was shown in Fig. S3 (NOAA, 2001).



(a) (b)

(c) (d)

(e) (f)

Fig. S3 Meteorological conditions used in the case study: (a) temperature profile; (b) relative humidity profile; (c) atmospheric pressure profile; (d) received solar radiation profile on the southern wall; (e) rainfall intensity profile; (f) wind rose map (wind velocity and direction)

S5

Table S4 shows the detailed calculation results for different levels of airtightness and indoor heat sources of occupants. The Q_c and Q_h stand for the annual energy loads of a wall in heating and cooling season, respectively. The LCT and AES are the abbreviation of the total lifecycle cost which contains operation costs and investment costs and the annual energy saving per unit area by the insulation, respectively.

Table S4 Detailed results for different levels of airtightness and internal heat sources caused by occupants

x_{ins}/mm	0	20	40	60	80	100	120	140	160	180	200
(a) For A_{CH}=1 ACH and 75% Q^{oc}_{iamb} in Fig. S3a											
Q_c (kW·h/m^2)	14.18	7.71	5.42	4.24	3.51	3.04	2.64	2.38	2.21	2.05	1.90
Q_h (kW·h/m^2)	33.48	18.15	13.11	10.38	8.59	7.38	6.51	5.84	5.32	4.92	4.57
LCT (\$/m^2)	27.94	16.16	12.88	11.59	**11.11**	**11.11**	11.37	11.83	12.42	13.09	13.79
AES (kW·h/m^2)	0.00	10.88	14.53	16.48	17.74	18.58	19.22	19.68	20.02	20.3	20.56
(b) For A_{CH}=1 ACH and 100% Q^{oc}_{iamb} in Fig. S3a											
Q_c (kW·h/m^2)	14.10	7.70	5.43	4.26	3.52	3.09	2.78	2.43	2.32	2.08	1.95
Q_h (kW·h/m^2)	33.57	18.13	13.07	10.32	8.52	7.29	6.42	5.74	5.23	4.81	4.47
LCT (\$/m^2)	27.96	16.14	12.85	11.56	**11.1**	**11.09**	11.39	11.85	12.42	13.03	13.76
AES (kW·h/m^2)	0.00	10.91	14.56	16.51	17.76	18.62	19.21	19.67	20.04	20.36	20.60
(c) For A_{CH}=1 ACH and 125% Q^{oc}_{iamb} in Fig. S3a											
Q_c (kW·h/m^2)	14.06	7.70	5.51	4.37	3.63	3.15	2.90	2.58	2.40	2.27	2.10
Q_h (kW·h/m^2)	33.59	18.12	13.00	10.26	8.45	7.22	6.33	5.66	5.13	4.70	4.37
LCT (\$/m^2)	27.95	16.14	12.85	11.58	**11.08**	**11.07**	11.39	11.82	12.39	13.07	13.77
AES (kW·h/m^2)	0.00	10.9	14.55	16.49	17.77	18.62	19.2	19.69	20.05	20.33	20.58
(d) For A_{CH}=2 ACH and 100% Q^{oc}_{iamb} in Fig. S3a											
Q_c (kW·h/m^2)	14.36	7.71	5.43	4.49	3.45	3.20	2.63	2.46	2.24	2.20	2.10
Q_h (kW·h/m^2)	34.35	18.99	13.32	10.33	8.57	7.37	6.53	5.81	5.30	4.93	4.60
LCT (\$/m^2)	28.57	16.68	13.00	11.68	**11.06**	11.19	11.38	11.85	12.42	13.17	13.91
AES (kW·h/m^2)	0.00	10.98	14.95	16.93	18.31	19.05	19.74	20.19	20.56	20.77	20.99

Reference

Hagentoft CE, Kalagasidis AS, Adl-Zarrabi B, et al., 2004. Assessment method of numerical prediction models for combined heat, air and moisture transfer in building components: benchmarks for one-dimensional cases. *Journal of Thermal Envelope and Building Science*, 27(4):327-352. https://doi.org/10.1177/1097196304042436

Kumaran MK, 1996. Heat, Air and Moisture Transfer in Insulated Envelope Parts: Final Report: Volume 3: Task 3: Material Properties. 90-75741-01-4, Energy Conservation in Buildings and Community Systems, International Energy Agency, Laboratorium Bouwfysica. Departement Burgerlijke Bouwkunde National Research Council of Canada. Institute for Research in Construction, Text.

Liu XW, Chen YM, Ge H, et al., 2015. Determination of optimum insulation thickness for building walls with moisture transfer in hot summer and cold winter zone of China. Energy and Buildings, 109:361-368. https://doi.org/10.1016/j.enbuild.2015.10.021

NOAA (National Oceanic and Atmospheric Administration), 2001. Global Hourly - Integrated Surface Database (ISD). Available from https://www.ncei.noaa.gov/products/land-based-station/integrated-surface-database [Accessed on Dec. 2021].