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Adaptive fault-tolerant control of high-speed maglev train suspension system with partial actuator failure: design and experiments

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Section S1 Electrical and electromagnetic force equations of the suspension system

It is assumed that the electromagnetic attraction between the electromagnet and the track is F_e, the number of turns of electromagnet coil is N_e, effective polar area of an electromagnet is A_e, air permeability is μ_0, the current in the solenoid coil and the voltage at both ends are respectively i and u, the resistance of electromagnet coil is R, the magnetic flux within the polar area is $\Phi_e(i,\delta)$ and the air gap flux linkage is $\psi_e(i,\delta)$.

According to Maxwell equation and Biot-Savart theorem it follows that

$$F_e = \frac{\int_0^t \psi_e(i,\delta)dt}{\partial\delta} \tag{S1}$$

From Kirchhoff's law for magnetic circuits, it follows that

$$\psi_e(i,\delta) = N_e \frac{N_e i}{R} \tag{S2}$$

$$R = \frac{2\delta}{\mu_0 A_e} \tag{S3}$$

Substituting Eqs. (S2) and (S3) into Eq. (S1), the electromagnetic force equation for the suspension system can be obtained as

$$F_e = -\frac{\mu_0 N_e^{\ 2} A_e}{4}(\frac{i}{\delta})^2 \tag{S4}$$

The relationship between the voltage and current of the solenoid coil and the magnetic flux is that

$$u = Ri + N_e \dot{\Phi}_e \tag{S5}$$

$$\Phi_e = \frac{L_e}{N_e} i \tag{S6}$$

where $L_e = \dfrac{\mu_0 N_e^{\,2} A_e}{2\delta}$.

Substituting L_e into Eq. (S6) and deriving it with respect to time gives

$$\dot{\Phi}_e = \frac{\mu_0 N_e A_e}{2\delta} \dot{i} - \frac{\mu_0 N_e A_e i}{2\delta^2} \dot{\delta} \tag{S7}$$

Substituting equation Eq. (S7) into Eq. (S5) yields the electrical equation of the electromagnet as

$$u = Ri + \frac{\mu_0 N_e^{\,2} A_e}{2\delta} \dot{i} - \frac{\mu_0 N_e^{\,2} A_e i}{2\delta^2} \dot{\delta} \tag{S8}$$

The resulting electrical equations for the left and right electromagnets are

$$u_1 = R_1 i_1 + \frac{\mu_0 N_{e1}^{\,2} A_{e1}}{2\delta_1} \dot{i}_1 - \frac{\mu_0 N_{e1}^{\,2} A_{e1} i_1}{2\delta_1^{\,2}} \dot{\delta}_1 \tag{S9}$$

$$u_2 = R_2 i_2 + \frac{\mu_0 N_{e2}^{\,2} A_{e2}}{2\delta_2} \dot{i}_2 - \frac{\mu_0 N_{e2}^{\,2} A_{e2} i_2}{2\delta_2^{\,2}} \dot{\delta}_2 \tag{S10}$$

Section S2 Dynamic equations for suspension system based on join-structure

From 2.3 the electromagnetic force on the electromagnet is given by the formula below

$$F_{e1} = -\frac{\mu_0 N_{e1}^{\,2} A_{e1}}{4} \frac{i_1^{\,2}}{\delta_1^{\,2}} \tag{S11}$$

$$F_{e2} = -\frac{\mu_0 N_{e2}^{\,2} A_{e2}}{4} \frac{i_2^{\,2}}{\delta_2^{\,2}} \tag{S12}$$

The force on the electromagnet by the spring dampers is given by

$$F_{s1} = K_1 (H - z_1) + \xi_1 (\dot{H} - \dot{z}_1) \tag{S13}$$

$$F_{s2} = K_2 (H - z_2) + \xi_2 (\dot{H} - \dot{z}_2) \tag{S14}$$

Thus, for the electromagnets the kinetic equations for the left and right electromagnets can be obtained as

$$M_1 \ddot{z}_1 = M_1 g - \frac{\mu_0 N_{e1}^{\,2} A_{e1}}{4} \frac{i_1^{\,2}}{\delta_1^{\,2}} + K_1 (H - z_1) + \xi_1 (\dot{H} - \dot{z}_1) \tag{S15}$$

$$M_2 \ddot{z}_2 = M_2 g - \frac{\mu_0 N_{e2}^{\,2} A_{e2}}{4} \frac{i_2^{\,2}}{\delta_2^{\,2}} + K_2 (H - z_2) + \xi_2 (\dot{H} - \dot{z}_2) \tag{S16}$$

For the hover frame, it is subject to the spring forces on both sides, its own gravity and the downward force generated by the body of the vehicle through the air springs, so that the kinetic equation is

$$M_h \ddot{H} = M_h g - F_{s1} - F_{s2} + M_c g \tag{S17}$$

Section S3 Fault tolerant controller design

Control object

The desired airgap can be denoted by x_d. The control object of the suspension system of high-speed maglev train is to make the air gap δ_i maintain at target regardless of partial actuator failure in the process of running. In order to quantitatively describe the control objective, the error signals are defined as follows:

$$\begin{cases} e_1 = x_1 - x_d \\ e_2 = x_3 - x_d \end{cases}, \begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{x}_d \\ \dot{e}_2 = \dot{x}_3 - \dot{x}_d \end{cases}, \begin{cases} \ddot{e}_1 = \ddot{x}_1 - \ddot{x}_d \\ \ddot{e}_2 = \ddot{x}_3 - \ddot{x}_d \end{cases} \tag{S18}$$

Assumption 1: x_d, $\dot{x}_d$ and $\ddot{x}_d$ satisfy these conditions: x_d, $\dot{x}_d$, $\ddot{x}_d \in L_\infty$, $|\ddot{x}_d| \le g$. $g \in \mathbb{R}^+$ denotes the gravity constant.

Based on Eq. (S18), the error systems of left and right electromagnets can be derived as follows:

Left:

$$M_1 \ddot{e}_1 = M_1 g + \frac{K_s}{2}(x_3 - x_1) + \frac{(M_h + M_c)}{2} g - F_{e1} - \ddot{x}_d \tag{S19}$$

Right:

$$M_2 \ddot{e}_2 = M_2 g + \frac{K_s}{2}(x_1 - x_3) + \frac{(M_h + M_c)}{2} g - F_{e2} - \ddot{x}_d \tag{S20}$$

where, $F_{e1} = \frac{\mu_0 N_e^{\,2} A_e}{4M_1} \frac{i_1^2}{\delta_1^2}$, $F_{e2} = \frac{\mu_0 N_e^{\,2} A_e}{4M_1} \frac{i_2^{\,2}}{\delta_2^{\,2}}$. These are controllable electromagnetic forces, i.e., the actual output of actuator. i_1, i_2 are the fault-tolerant suspension controllers of left and right suspension electromagnets that need to be designed, respectively. Consider the above second failure mode of actuator, i.e.:

$$F_{e1} = \varpi_1 F_{ce1} \tag{S21}$$

$$F_{e2} = \varpi_2 F_{ce2} \tag{S22}$$

where, F_{e1}, F_{e2} are the actual outputs of the left and right actuators, respectively. F_{ce1}, F_{ce2} are the ideal control input of left and right actuators. $\varpi_1, \varpi_2 \in (0,1]$ denote the failure degree of the left and right actuators, respectively.

First, the suspension controller of the left electromagnet module is designed. The sliding surface of the left electromagnet is designed as follows:

$$s_1 = c_1 e_1 + \dot{e}_1 \tag{S23}$$

where, $c_1 > 0$.

Set $\varphi_1 = \dfrac{\varpi_1}{M_1}$, $\xi_1 = \dfrac{1}{\varphi_1}$. The fault-tolerant control law of the left electromagnet module is designed as follows:

$$i_1(t)=\sqrt{\frac{1}{\kappa_1}\delta_1^2\hat{\xi}_l(k_{t1}s_1 + c_1\dot{e}_1 + g + \frac{K_s(x_3 - x_1)}{2M_1} + \frac{(M_h + M_c)}{2M_1}g - \ddot{x}_d)} \tag{S24}$$

where, $\kappa_1 = \dfrac{\mu_0 N_{e1}{}^2 A_{e1}}{4}$, $k_{t1} > 0$ are the fault tolerance control coefficients. $\hat{\xi}_1$ is an adaptive estimate value containing fault information for the left module, which can be determined by the following adaptive update law:

$$\dot{\hat{\xi}}_1 = \lambda_1 s_1 (k_{t1}s_1 + c_1\dot{e}_1 + g + \frac{K_s(x_3 - x_1)}{2M_1} + \frac{(M_h + M_c)}{2M_1}g - \ddot{x}_d)\operatorname{sgn}(\varphi_1) \tag{S25}$$

where $\lambda_1 > 0$ denotes adaptive gain.

Easy to know $\operatorname{sgn}(\varphi_1) = \operatorname{sgn}(\dfrac{1}{M_1})$.

Meanwhile, the sliding surface of the right electromagnet is designed as follows:

$$s_2 = c_2 e_2 + \dot{e}_2 \tag{S26}$$

where, $c_2 > 0$.

Set $\varphi_2 = \dfrac{\varpi_2}{M_2}$, $\xi_2 = \dfrac{1}{\varphi_2}$, The fault-tolerant control law of the right electromagnet module is designed as follows:

$$i_2(t)=\sqrt{\frac{1}{\kappa_2}\delta_2^2\hat{\xi}_2(k_{t2}s_2 + c_2\dot{e}_2 + g + \frac{K_s(x_1 - x_3)}{2M_2} + \frac{(M_h + M_c)}{2M_2}g - \ddot{x}_d)} \tag{S27}$$

where , $\kappa_2 = \dfrac{\mu_0 N_{e2}{}^2 A_{e2}}{4}$, $k_{t2} > 0$ are the fault tolerance control coefficients. $\hat{\xi}_2$ is an adaptive estimate value containing fault information for the right module, which can be determined by the following adaptive update law:

$$\dot{\hat{\xi}}_2 = \lambda_2 s_2 (k_{t2} s_2 + c_2 \dot{e}_2 + g + \frac{K_s (x_1 - x_3)}{2M_2} + \frac{(M_h + M_c)}{2M_2} g - \ddot{x}_d) \operatorname{sgn}(\varphi_2) \tag{S28}$$

where $\lambda_2 > 0$ denotes adaptive gain.

Closed-loop stability analysis

Before proceeding to the stability analysis, the extended Barbalat lemma is presented firstly, which will be utilized in the following analysis.

Extended Barbalat lemma: $x:[0,\infty) \to \mathbf{R}$ is square integrable , i.e.: $\lim_{t\to\infty} \int_0^t x^2(\tau) d\tau < \infty$. If $x(t)$ is uniform continuous, then $\lim_{t\to\infty} x(t) = 0$.

The closed-loop stability analysis of the proposed control law under partial actuator failure is carried out below. For the suspension system based on join-structure with two suspension electromagnets, we obtain:

Theorem 1: The designed adaptive suspension controller Eqs. (S24) and (S27), together with the proposed update mechanism Eqs. (S25) and (S28), can also achieve the control objective even if the left actuator has a partial failure in Eqs. (S21) and (S22) in the sense that

$$\lim_{t\to\infty} e_1(t) = 0 \text{ , } \lim_{t\to\infty} \dot{e}_1(t) = 0 \text{ , } \lim_{t\to\infty} e_2(t) = 0 \text{ and } \lim_{t\to\infty} \dot{e}_2(t) = 0 \tag{S29}$$

Proof: The nonnegative scalar Lyapunov function is constructed as follows:

$$V = \frac{1}{2} s_1^2 + \frac{|\varphi_1|}{2\lambda_1} \tilde{\xi}_1^2 + \frac{1}{2} s_2^2 + \frac{|\varphi_2|}{2\lambda_2} \tilde{\xi}_2^2 \tag{S30}$$

where $s_1 = c_1 e_1 + \dot{e}_1$, $\tilde{\xi}_1 = \hat{\xi}_1 - \xi_1$, $\lambda_1 > 0$, $s_2 = c_2 e_2 + \dot{e}_2$, $\tilde{\xi}_2 = \hat{\xi}_2 - \xi_2$, $\lambda_2 > 0$.

By taking the time derivative of Eq. (S23), inserting Eq. (S19) into the resulting equation, and by taking the time derivative of Eq. (S26), inserting Eq. (S20) into the resulting equation, one has

$$\dot{s}_1 = c_1\dot{e}_1 + \ddot{e}_1 = c_1\dot{e}_1 + g + \frac{K_s(x_3 - x_1)}{2M_1} + \frac{(M_h + M_c)}{2M_1}g - \varphi_1\kappa_1\frac{i_l^{\,2}}{\delta_1^{\,2}} - \ddot{x}_d \tag{S31}$$

$$\dot{s}_2 = c_2\dot{e}_2 + \ddot{e}_2 = c_2\dot{e}_2 + g + \frac{K_s(x_1 - x_3)}{2M_2} + \frac{(M_h + M_c)}{2M_2}g - \varphi_2\kappa_2\frac{i_2^{\,2}}{\delta_2^{\,2}} - \ddot{x}_d \tag{S32}$$

Taking the derivative of both sides of Eq. (S30) with respect to time, and inserting Eqs. (S31) and (S32) into the resulting equation, we can obtain:

$$\begin{aligned}\dot{V} &= s_1\dot{s}_1 + \frac{|\varphi_1|}{\lambda_1}\tilde{\xi}_1\dot{\tilde{\xi}}_1 = s_1(c_1\dot{e}_1 + g + \frac{K_s(x_3 - x_1)}{2M_1} + \frac{(M_h + M_c)}{2M_1}g - \varphi_1\kappa_1\frac{i_1^{\,2}}{\delta_1^{\,2}} - \ddot{x}_d) + \frac{|\varphi_1|}{\lambda_1}\tilde{\xi}_1\dot{\hat{\xi}}_1 \\ &+ s_2\dot{s}_2 + \frac{|\varphi_2|}{\lambda_2}\tilde{\xi}_2\dot{\tilde{\xi}}_2 = s_2(c_2\dot{e}_2 + g + \frac{K_s(x_1 - x_3)}{2M_2} + \frac{(M_h + M_c)}{2M_2}g - \varphi_2\kappa_2\frac{i_2^{\,2}}{\delta_2^{\,2}} - \ddot{x}_d) + \frac{|\varphi_2|}{\lambda_2}\tilde{\xi}_2\dot{\hat{\xi}}_2\end{aligned} \tag{S33}$$

To facilitate subsequent analysis, set $\gamma_1 = k_{t1}s_1 + c_1\dot{e}_1 + g + \frac{K_s(x_3 - x_1)}{2M_1} + \frac{(M_h + M_c)}{2M_1}g - \ddot{x}_d$ and $\gamma_2 = k_{t2}s_2 + c_2\dot{e}_2 + g + \frac{K_s(x_1 - x_3)}{2M_2} + \frac{(M_h + M_c)}{2M_2}g - \ddot{x}_d$.

$$\begin{aligned}\dot{V} &= s_1(\gamma_1 - k_{t1}s_1 - \varphi_1\kappa_1\frac{i_1^{\,2}}{\delta_1^{\,2}}) + \frac{|\varphi_1|}{\lambda_1}\tilde{\xi}_1\lambda_1 s_1\gamma_1\,\mathrm{sgn}(\varphi_1) + s_2(\gamma_2 - k_{t2}s_2 - \varphi_2\kappa_2\frac{i_2^{\,2}}{\delta_2^{\,2}}) \\ &+ \frac{|\varphi_2|}{\lambda_r}\tilde{\xi}_2\lambda_2 s_2\gamma_2\,\mathrm{sgn}(\varphi_2)\end{aligned} \tag{S34}$$

Since $\mathrm{sgn}(\frac{1}{M_1}) = \mathrm{sgn}(\varphi_1)$, substituting the proposed controllers Eqs. (S24) and (S25) into Eq. (S34) at the same time, and canceling out the common terms, and making some arrangements, the following results are obtained:

$$\begin{aligned}\dot{V} &= s_1(\gamma_1 - k_{t1}s_1 - \varphi_1\hat{\xi}_1\gamma_1) + \varphi_1\tilde{\xi}_1 s_1\gamma_1 + s_2(\gamma_2 - k_{t2}s_2 - \varphi_2\hat{\xi}_2\gamma_2) + \varphi_2\tilde{\xi}_2 s_2\gamma_2 \\ &= s_1(\gamma_1 - k_{t1}s_1 + \varphi_1\tilde{\xi}_1\gamma_1 - \varphi_1\hat{\xi}_1\gamma_1) + s_2(\gamma_2 - k_{t2}s_2 + \varphi_2\tilde{\xi}_2\gamma_2 - \varphi_2\hat{\xi}_2\gamma_2) \\ &= s_1(\gamma_1 - k_{t1}s_1 + \varphi_1\xi_1\gamma_1) + s_2(\gamma_2 - k_{t2}s_1 + \varphi_1\xi_1\gamma_1) \\ &= -k_{t1}s_1^{\,2} - k_{t2}s_2^{\,2} \le 0\end{aligned} \tag{S35}$$

Due to $V \ge 0$, $\dot{V} \le 0$, based on Lyapunov method, we can conclude that V is bounded.

It can be obtained from $\dot{V} = -k_{t1}s_1^{\,2} - k_{t2}s_2^{\,2}$ that:

$$\int_0^t \dot{V}dt = -k_{t1}\int_0^t s_1^{\,2}dt - k_{t2}\int_0^t s_2^{\,2}dt \tag{S36}$$

i.e., $V(\infty) - V(0) = -k_{t1}\int_0^\infty s_1^{\,2}dt - k_{t2}\int_0^t s_2^{\,2}dt$.

While $t \to \infty$, because $V(\infty)$ is bounded, then $\int_0^\infty s_1^{\,2}dt$ and $\int_0^t s_2^{\,2}dt$ are bounded respectively , i.e., s_1 and s_2 are square integrable respectively. And it is easy to know that s_1 and s_2 are uniform continuous. Based on the above extended Barbalat lemma , while $t \to \infty$,

$\lim_{t\to\infty} s_1 = 0$ and $\lim_{t\to\infty} s_2 = 0$. And since $s_1 = c_1 e_1 + \dot{e}_1$, $s_2 = c_2 e_2 + \dot{e}_2$, $c_1 \neq 0$ and $c_2 \neq 0$, thus: $\lim_{t\to\infty} e_1(t) = 0$, $\lim_{t\to\infty} \dot{e}_1(t) = 0$, $\lim_{t\to\infty} e_2(t) = 0$, $\lim_{t\to\infty} \dot{e}_2(t) = 0$. So, the closed-loop system with the proposed control law is asymptotic asymptotically stable. QED.

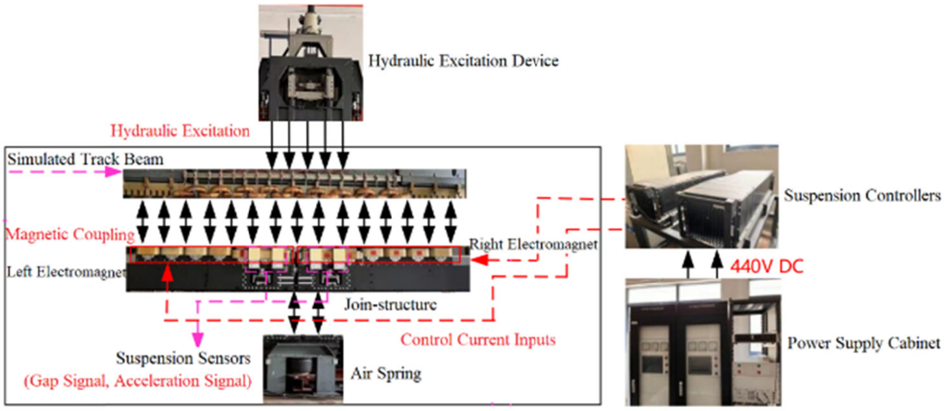

Fig. S1 High-speed maglev vehicle-rail magnetic coupling experiment platform