Electronic supplementary materials

for https://doi.org/10.1631/jzus.A2300026

Bifurcation control of solid angle car-following model through a time-delay feedback method

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Eqs. (S1)–(S13)

Section S1 Specific solution process of the controlled SAM

Considering the periodic boundary, Eq. (13) is simplified to the matrix form:

$$\begin{pmatrix} \dot{\eta} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} \delta \boldsymbol{I} & -\beta \boldsymbol{A} \\ \boldsymbol{I} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \eta \\ \xi \end{pmatrix} + \begin{pmatrix} -M\varepsilon \boldsymbol{I} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \eta(t-\tau) \\ \xi(t-\tau) \end{pmatrix} \tag{S1}$$

where $\eta = \left(\eta_1(t), \eta_2(t), \cdots, \eta_N(t)\right)^{\mathrm{T}}$, $\xi = \left(\xi_1(t), \xi_2(t), \cdots, \xi_N(t)\right)^{\mathrm{T}}$; $\boldsymbol{O}$ is the zero matrix of $N \times N$. The form of matrix $\boldsymbol{A}$ is as follows:

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ -1 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \tag{S2}$$

The corresponding characteristic equation of Eq. (20) can be derived as follows:

$$\left[\lambda^2 - \delta\lambda + M\varepsilon \mathrm{e}^{-\tau\lambda}\lambda + \beta\right]^N - \left[\beta\right]^N = 0 \tag{S3}$$

Therefore, we can obtain:

$$\left[\frac{\lambda^2 - \delta\lambda + M\varepsilon \mathrm{e}^{-\tau\lambda}\lambda + \beta}{\beta}\right]^N = 1 \tag{S4}$$

The transformation of Eq. (23) can obtain:

$$\lambda^2 - \delta\lambda + M\varepsilon e^{-\tau\lambda}\lambda + \beta = \beta\left(\cos\frac{2k\pi}{N} + i\sin\frac{2k\pi}{N}\right) \tag{S5}$$

where $k \in \{1, 2, \cdots N\}$ is the wave number of the oscillation.

Inserting $\lambda = \mu + i\omega$ into Eq. (24):

$$\begin{cases} \mu^2 - \omega^2 - \delta\mu + M\varepsilon e^{-\tau\mu}(\mu c_\tau + \omega s_\tau) + \beta = \beta c_k \\ 2\mu\omega - \varepsilon\omega + M\varepsilon e^{-\tau\mu}(\omega c_\tau - \mu s_\tau) + \beta = \beta s_k \end{cases} \tag{S6}$$

where $c_k = \cos\frac{2k\pi}{N}$, $s_k = \sin\frac{2k\pi}{N}$, $c_\tau = \cos\tau_2\omega$, $s_\tau = \sin\tau_2\omega$.

Section S2 Specific improvement procedures of definite integral method

Step 1 Inserting $\lambda = i\omega$ into Eq. (17) and separating the real part of $i^{-n} f(i\omega)$ yields:

$$R(\omega) = \mathrm{Re}\left(i^{-n} f(i\omega)\right) = \omega^2 - M\varepsilon\omega s_\tau - \beta + \beta c_k \tag{S7}$$

where n represents the highest degree of the characteristic equation Eq. (24).

Step 2 According to Eq. (26), all positive roots $W = \{\omega_1, \omega_2, \ldots, \omega_m\}$ of $R(\omega) = 0$ exist. Take the maximum value of the set W as $\omega_{\max}$, and choose any $T_0 > \omega_{\max}$ as the upper limit of the definite integral.

Step 3 Considering $\Re(\omega) = \mathrm{Re}\left(f'(i\omega)/f(i\omega)\right)$, the integrand $\Re(\omega)$ is transformed into:

$$\begin{aligned} \Re(\omega) &= \mathrm{Re}\left(\frac{f'(i\omega)}{f(i\omega)}\right) \\ &= \frac{\mathrm{Re}(f'(i\omega))\mathrm{Re}(f(i\omega)) + \mathrm{Im}(f'(i\omega))\mathrm{Im}(f(i\omega))}{\left[\mathrm{Re}(f(i\omega))\right]^2 + \left[\mathrm{Im}(f(i\omega))\right]^2} \end{aligned} \tag{S8}$$

where the specific expression of the real part $\mathrm{Re}(f'(i\omega))$ and imaginary part $\mathrm{Im}(f'(i\omega))$ of $f'(i\omega)$ are as follows:

$$\mathrm{Re}\left(f'(i\omega)\right) = -\delta + M\varepsilon c_\tau - M\varepsilon\tau s_\tau \tag{S9}$$

$$\mathrm{Im}\left(f'(i\omega)\right) = 2\omega - M\varepsilon s_\tau - M\varepsilon\tau\omega c_\tau \tag{S10}$$

where the specific expression of the real part $\mathrm{Re}(f'(i\omega))$ and imaginary part $\mathrm{Im}(f'(i\omega))$ of $f(i\omega)$ are as follows:

$$\mathrm{Re}(f(i\omega)) = -\omega^2 + M\varepsilon\omega s_\tau + \beta - \beta c_k \tag{S11}$$

$$\mathrm{Im}(f(i\omega)) = -\delta\omega + M\varepsilon\omega c_\tau - \beta s_k \tag{S12}$$

Step 4 The following definite integrals are introduced into the product function T_0 in step 2 and the product function $\Re(\omega)$ in step 3, respectively, and yield:

$$F(0,T_0)_k = \int_0^{T_0} \Re(\omega)\mathrm{d}\omega \tag{S13}$$

For every k and $k \neq N$, the total number $\Pi_k(\tau)$ of eigenvalues with positive real part is:

$$\Pi_k(\tau) = \mathrm{round}\left(\frac{n}{2} - \frac{F(0,T_0)_k}{\pi}\right) \tag{S14}$$

Step 5 In summary, if $\Pi(\tau) = 0$ is stable under the control of time delay τ. Otherwise, unstable oscillation of the system occurs.