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Geometrical transition properties of vortex cavitation and associated flow-choking characteristics in poppet valves

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S1 Complete form of the momentum equations for 2D axisymmetric geometries

Under the axisymmetric assumption, there are no circumferential gradients in the flow, but circumferential velocities are permitted.

$$\frac{\partial}{\partial t}(\rho_{\mathrm{m}}u)+\frac{1}{r}\frac{\partial}{\partial z}(r\rho_{\mathrm{m}}u^2)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho_{\mathrm{m}}uv)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial z}(2r\mu\frac{\partial u}{\partial z})+\frac{1}{r}\frac{\partial}{\partial r}[r\mu(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z})], \tag{S1}$$

$$\frac{\partial}{\partial t}(\rho_{\mathrm{m}}v)+\frac{1}{r}\frac{\partial}{\partial z}(r\rho_{\mathrm{m}}uv)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho_{\mathrm{m}}v^2)=-\frac{\partial p}{\partial r}+\frac{1}{r}\frac{\partial}{\partial z}[r\mu(\frac{\partial v}{\partial z}+\frac{\partial u}{\partial r})]+\frac{1}{r}\frac{\partial}{\partial r}(2r\mu\frac{\partial v}{\partial r})-2\mu\frac{v}{r^2}+\rho_{\mathrm{m}}\frac{u^2}{r}, \tag{S2}$$

$$\frac{\partial}{\partial t}(\rho_{\mathrm{m}}w)+\frac{1}{r}\frac{\partial}{\partial z}(r\rho_{\mathrm{m}}uw)+\frac{1}{r}\frac{\partial}{\partial r}(r\rho_{\mathrm{m}}vw)=\frac{1}{r}\frac{\partial}{\partial z}(r\mu\frac{\partial w}{\partial z})+\frac{1}{r^2}\frac{\partial}{\partial r}[r^3\mu\frac{\partial}{\partial r}(\frac{w}{r})]-\rho_{\mathrm{m}}\frac{vw}{r}, \tag{S3}$$

where t is the time, z represents the axial coordinates, r represents the radial coordinates of the symmetry axis, ρ_{m} is the mixture density, μ represents the molecular viscosity, u, v and w denotes the axial, radial and swirl velocity components, respectively.

S2 Complete formulas of the Wall-Adapting Local Eddy Viscosity (WALE) model

The rate-of-strain tensor is calculated by:

$$S_{ij}=\frac{1}{2}(\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial\bar{u}_j}{\partial x_i}). \tag{S4}$$

The eddy viscosity is modeled as:

$$\mu_{\mathrm{t}}=\rho_{\mathrm{m}}L_{\mathrm{s}}^2\frac{(S_{ij}^{\mathrm{d}}S_{ij}^{\mathrm{d}})^{\frac{3}{2}}}{(\bar{S}_{ij}\bar{S}_{ij})^{\frac{5}{2}}+(S_{ij}^{\mathrm{d}}S_{ij}^{\mathrm{d}})^{\frac{5}{4}}}, \tag{S5}$$

$$S_{ij}^{\mathrm{d}}=\frac{1}{2}(\bar{g}_{ij}^2+\bar{g}_{ji}^2)-\frac{1}{3}\delta_{ij}\bar{g}_{kk}^2, \tag{S6}$$

$$\overline{g}_{ij}=\frac{\partial\overline{u}_i}{\partial x_j},\tag{S7}$$

$$L_{\mathrm{s}}=\min(\kappa d,C_{\mathrm{w}}V^{\frac{1}{3}}),\tag{S8}$$

where $\overline{g}_{ij}$ denotes the velocity gradient tensor; δ_{ij} denotes the Kronecker symbol; d is the length of the point from the closed wall; V is the calculated grid volume; κ is the Von Karman's constant of 0.41; C_{w} is defined as the WALE constant of 0.325; S_{ij}^{d} and L_{s} represent model variable defined by Eqs. (S6) and (S8), respectively.

S3 Complete formulas of the Schnerr-Sauer model

The mass transport equation could be described as:

$$\frac{\partial(\rho_{\mathrm{v}}\alpha_{\mathrm{v}})}{\partial t}+\frac{\partial(\rho_{\mathrm{v}}\alpha_{\mathrm{v}}u_j)}{\partial x_j}=\dot{m}^{+}-\dot{m}^{-},\tag{S9}$$

where, the source term $\dot{m}^{+}$ represents evaporation and $\dot{m}^{-}$ represents condensation. In the bubble dynamics, the two terms are calculated through the Rayleigh-Plesset equation:

$$\begin{cases}\dot{m}^{+}=\dfrac{\rho_{\mathrm{v}}\rho_{\mathrm{l}}}{\rho}\alpha_{\mathrm{v}}(1-\alpha_{\mathrm{v}})\dfrac{3}{R_{\mathrm{b}}}\sqrt{\dfrac{2}{3}\dfrac{(p_{\mathrm{v}}-p)}{\rho_{\mathrm{l}}}}, & p\le p_{\mathrm{v}},\\ \dot{m}^{-}=\dfrac{\rho_{\mathrm{v}}\rho_{\mathrm{l}}}{\rho}\alpha_{\mathrm{v}}(1-\alpha_{\mathrm{v}})\dfrac{3}{R_{\mathrm{b}}}\sqrt{\dfrac{2}{3}\dfrac{(p-p_{\mathrm{v}})}{\rho_{\mathrm{l}}}}, & p\ge p_{\mathrm{v}},\end{cases}\tag{S10}$$

The bubble radius R_{b}, vapor volume fraction α_{v} and the bubble number density N_{b} are related, can be represented as:

$$R_{\mathrm{b}}=\left(\frac{\alpha_{\mathrm{v}}}{1-\alpha_{\mathrm{v}}}\cdot\frac{3}{4\pi}\cdot\frac{1}{N_{\mathrm{b}}}\right)^{\frac{1}{3}}.\tag{S11}$$

In current work the bubble number density N_{b} is defined as a constant value of 10^{13}.

S4 Solution methods

Solution method	
Solver	Coupled
Gradient discretization method	Least Squares Cell Based
Pressure discretization method	PRESTO！
Momentum discretization method	Bounded Central Differencing
Volume fraction discretization method	QUICK
Transient formulation	Bounded Second Order Implicit

S5 Complete grid refinement method and GCI calculation formulas

Firstly, a coarse grid, Grid 1, was generated, and then the grid spacing was halved to obtain the finer grids, Grid 2 and Grid 3. The numerical calculation solutions, f_1, f_2 and f_3, can be expressed using the generalized Richardson extrapolation equation as:

$$\begin{cases} f_1 = f_{\mathrm{exa}} + ch_1^s + o(h_1^{s+1}), \\ f_2 = f_{\mathrm{exa}} + ch_2^s + o(h_2^{s+1}), \\ f_3 = f_{\mathrm{exa}} + ch_3^s + o(h_3^{s+1}), \end{cases} \tag{S12}$$

where, f_{exa} denotes the calculation solution when the grid spacing approaches zero, serving as the exact solution, c is a constant value, h_1, h_2 and h_3 represent grid spacings, and s denotes the order of convergence and dependents on the calculation method.

The grid refinement ratio r_{g} is defined as:

$$r_{\mathrm{g}} = \frac{h_1}{h_2} = \frac{h_2}{h_3}. \tag{S13}$$

Neglecting higher-order terms in Eq. (S12) and eliminating the f_{exa} and c:

$$\frac{f_1 - f_2}{f_2 - f_3} = \frac{h_1^s - h_2^s}{h_2^s - h_3^s}. \tag{S14}$$

Taking the logarithm of both sides of Eq. (S14):

$$s = \frac{\ln(\frac{f_1 - f_2}{f_2 - f_3})}{\ln r_{\mathrm{g}}}. \tag{S15}$$

Set $\varepsilon_1 = (f_1 - f_2)/f_1$, $\varepsilon_2 = (f_2 - f_3)/f_2$, the estimated fractional error could be expressed as,

$$\begin{cases} \dfrac{f_1 - f_{\mathrm{exa}}}{f_1} = \dfrac{f_1 - f_2}{f_1[1-(\frac{1}{r_{\mathrm{g}}})^s]} = \dfrac{\varepsilon_1 r_{\mathrm{g}}^s}{r_{\mathrm{g}}^s - 1}, \\ \dfrac{f_2 - f_{\mathrm{exa}}}{f_2} = \dfrac{f_1 - f_2}{f_2(r_{\mathrm{g}}^s - 1)} = \dfrac{\varepsilon_1}{r_{\mathrm{g}}^s - 1}, \\ \dfrac{f_3 - f_{\mathrm{exa}}}{f_3} = \dfrac{f_2 - f_3}{f_3(r_{\mathrm{g}}^s - 1)} = \dfrac{\varepsilon_2}{r_{\mathrm{g}}^s - 1}. \end{cases} \tag{S16}$$

The Grid Convergence Index (GCI) is defined as:

$$\mathrm{GCI} = F_{\mathrm{s}} \cdot \left| \frac{f - f_{\mathrm{exa}}}{f} \right|, \tag{S17}$$

where f represents the numerical calculation solution and F_s is a factor of safety, F_s=1.25. GCI indicates an error band of the deviation of the solution from the exact value. Therefore, it also reflects the variation of the solution as the grid is further refined.

S6 Nomenclature

α	throttling angle, rad
α_v	volume fraction number of vapor
μ_l	dynamic viscosity of liquid, kg/(m s)
μ_m	mixing dynamic viscosity, kg/(m s)
μ_t	eddy viscosity, kg/(m s)
μ_v	dynamic viscosity of vapor, kg/(m s)
ρ_l	density of liquid, kg/m ^{3}
ρ_m	mixing density, kg/m ^{3}
ρ_v	density of vapor, kg/m ^{3}
σ	Cavitation number
τ_{ij}	the sub-grid scale stress
L_s	sealing length, mm
L_x	valve opening, mm
$\dot{m}^+$	mass transfer source term connected to evaporation, kg/(m^3 s)
$\dot{m}^-$	mass transfer source term connected to condensation, kg/(m^3 s)
p	pressure, Pa
p_{in}	inlet pressure, Pa
p_{out}	outlet pressure, Pa
p_v	vapor pressure, Pa
r_a	inlet radius, mm
R_a	length-to-diameter ratio
S_{ij}	rate-of-strain tensor for the resolved scale, s^{-1}
t	time, s
u	velocity, m/s
x	Cartesian coordinate, m