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Permeability of structured porous media: numerical simulations and microfluidic models

Shaokai NIE[1,2,3], Pengfei LIU[1,2,3], Kexin CHEN[1,2,3], Wenyuan WANG[1,2,3], Yunmin CHEN[1,2,3], Bate BATE[1,2,3]

[1]*Institute of Geotechnical Engineering, College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058, China*
[2]*Hypergravity Research Center, College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058, China*
[3]*MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering, Zhejiang University, Hangzhou 310058, China*

S1 Comparison of streamlines obtained by numerical simulation and microscope

Comparison of streamlines obtained by numerical simulation and monodisperse fluorescent microspheres microscope images with different micropillar arrangements at the velocity of 0.033 m/s was shown in Fig. S1. It visually indicated that streamline patterns from numerical simulation corresponded to experimental measurements. The tortuosity was obtained by taking the average length of streamlines with numerical simulations.

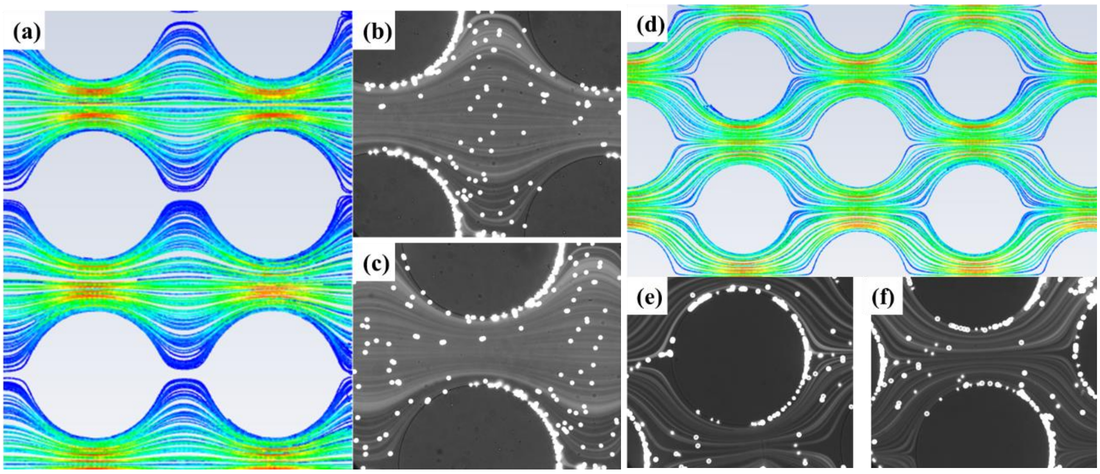


Fig. S1 Comparison of streamlines obtained by numerical simulation and microscope at the velocity of 0.033 m/s. (a) numerical simulation and (b), (c) experimental images of Asq-0.60-500; (d) numerical simulation and (e), (f) experimental images of Ast-0.60-500.

S2 Mathematical model for 2D circular-based soil tortuosity under different particle arrangements

To simplify the calculation of mathematical model, two assumptions are made: (a) two-dimensional circulars of identical size are presented; (b) laminar flow is in pore. As shown in Fig. S2a, L_H and L_V are the distances between two soil particles in the horizontal and vertical flow directions, respectively. The particle diameter is defined as *D*. An anisotropic parameter is defined as *m* to describe the ratio between L_V and L_H. The retarding parameter α is the horizontal angle between two particles, which ranges from 0 to $\arctan(\frac{L_V}{2L_H})$. As shown in Fig. S2b, the special case when α equals 0 is defined as the lower limit arrangement (LA). The normal case is defined as the upper limit arrangement (UA) when α equals $\arctan(\frac{L_V}{2L_H})$, as shown in Fig. S2c.

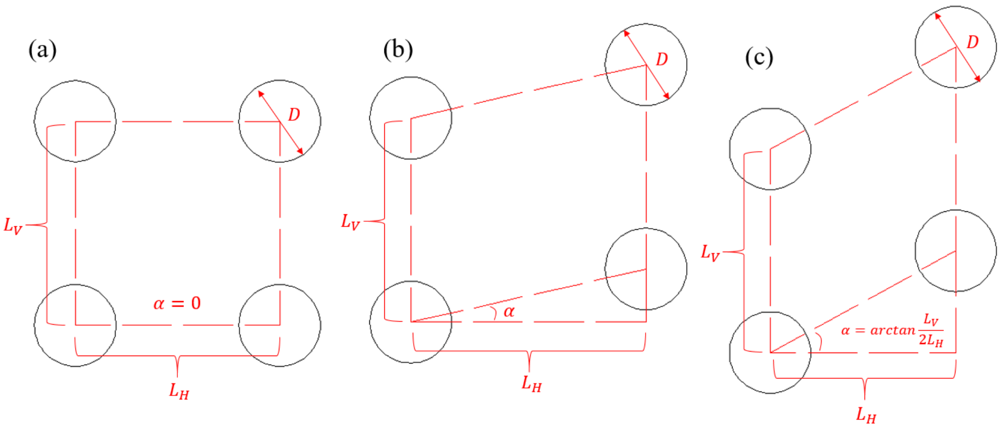


Fig. S2 Schematic diagram of unit particle arrangement

Model porosity ε can be determined by Eq. (S1):

$$\varepsilon = \frac{L_H * L_V - \frac{\pi D^2}{4}}{L_H * L_V} \tag{S1}$$

It can be transformed into Eq. (S2):

$$L_H * L_V = \frac{\pi D^2}{4(1-\varepsilon)} \tag{S2}$$

Since *m* is equal to L_V/L_H, and substitute it to Eq. (S2):

$$\frac{D}{L_H} = \sqrt{\frac{4m(1-\varepsilon)}{\pi}} \tag{S3}$$

It is assumed that *D* must be smaller than L_H and L_V, so $\sqrt{4m(1-\varepsilon)/\pi} \leq 1$ and $\sqrt{\pi m/4(1-\varepsilon)} \geq 1$. When ε is determined, the range of *m* is from $4(1-\varepsilon)/\pi$ to $\pi/4(1-\varepsilon)$.

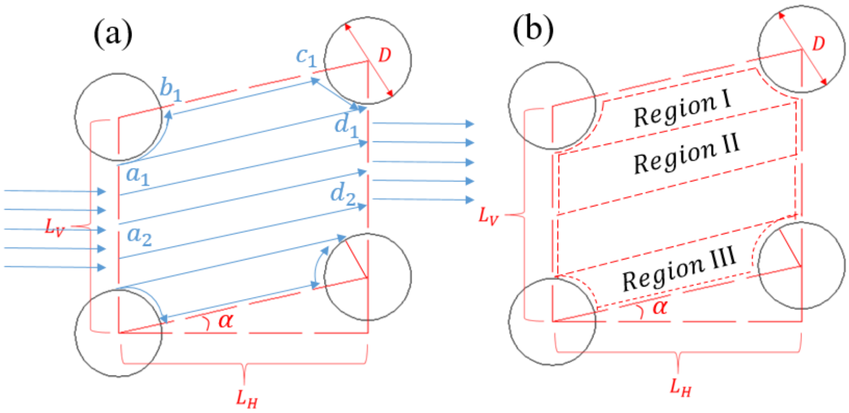


Fig. S3 Schematic diagram of flow path (a) and regions (b)

As shown in Fig. S3, the assumed laminar flow is described as the flow path characterized by many parallel lines and three regions are defined by flow regions.

In Regions I, the longest flow path is $a_1 \rightarrow b_1 \rightarrow c_1 \rightarrow d_1$, the shortest flow path is $a_1 \rightarrow d_1$:

$$L_{max} = \frac{1}{2}\pi D + \frac{L_H}{cos\alpha} - D \tag{S4}$$

$$L_{min} = \frac{L_H}{cos\alpha} \tag{S5}$$

when the porosity increases, the flow path becomes smoother (Zhang et al., 2020). Eqs. (S4) and (S5) can be transformed into:

$$L_{max} = \frac{1}{2}\pi D + \frac{L_H}{\cos[(1-\varepsilon)^{1/3}\alpha]} - D \tag{S6}$$

$$L_{min} = \frac{L_H}{\cos[(1-\varepsilon)^{1/3}\alpha]} \tag{S7}$$

According to Eqs. (S6) and (S7), the averaged flow path in Regions I is:

$$\overline{L_I} = \frac{L_{max}+L_{min}}{2} = \frac{L_H}{\cos\left[(1-\varepsilon)^{\frac{1}{3}}\alpha\right]} + (\frac{\pi}{4} - \frac{1}{2})D \tag{S8}$$

The average path in region II is:

$$\overline{L_2} = \frac{L_H}{\cos[(1-\varepsilon)^{1/3}\alpha]} \tag{S9}$$

The soil tortuosity τ_1 can be computed as the ration of $\overline{L_I}$ to L_H:

$$\tau_1 = \frac{\overline{L_I}}{L_H} = \frac{1}{\cos\left[(1-\varepsilon)^{\frac{1}{3}}\alpha\right]} + (\frac{\pi}{4} - \frac{1}{2})\sqrt{\frac{4m(1-\varepsilon)}{\pi}} \tag{S10}$$

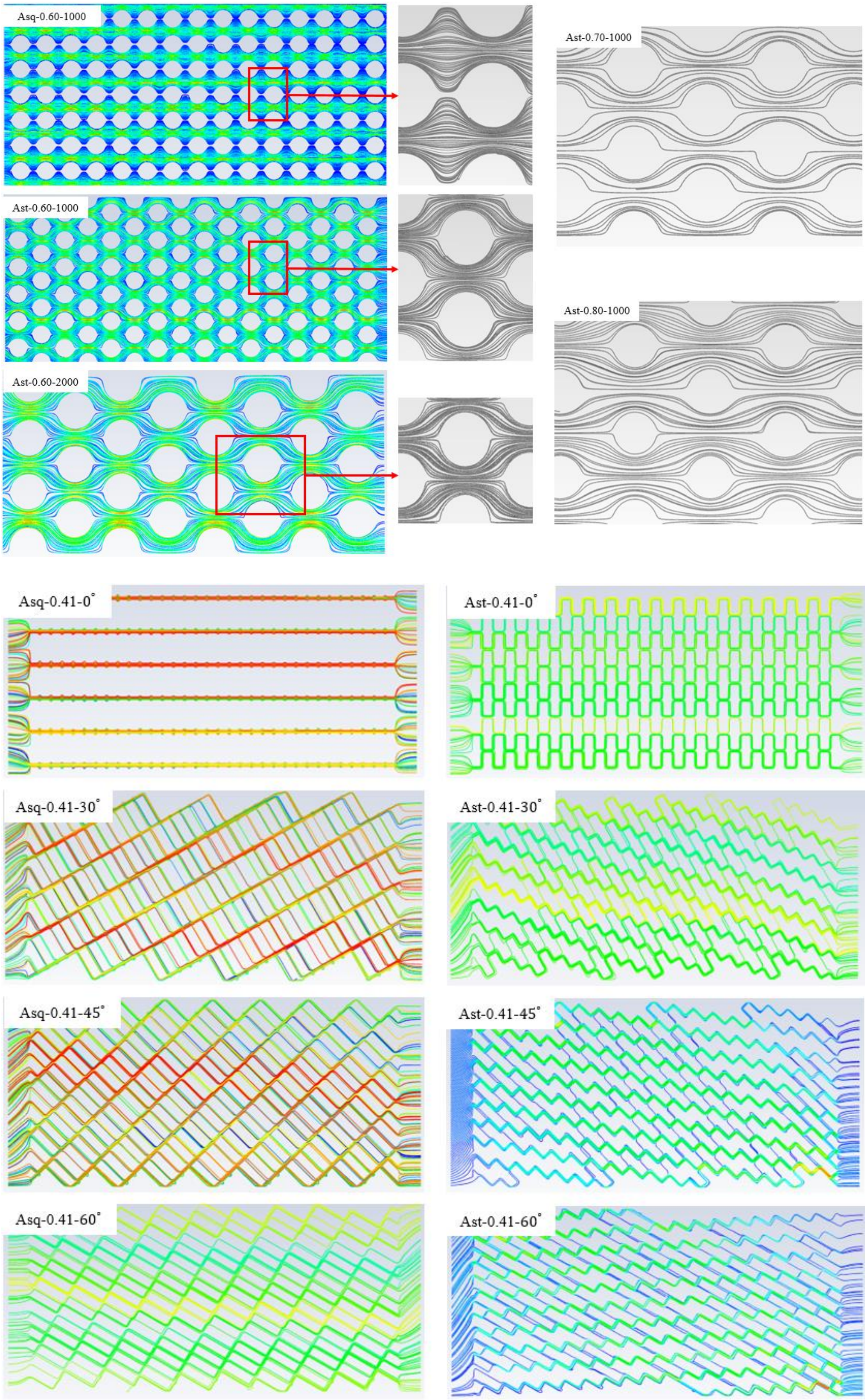
Tortuosity τ_2 in region II can be computed as the ration of $\overline{L_2}$ to L_H:

$$\tau_1 = \frac{\overline{L_2}}{L_H} = \frac{1}{\cos\left[(1-\varepsilon)^{\frac{1}{3}}\alpha\right]} \tag{S11}$$

Regions III is the same as Regions I. According to the streamline distribution obtained by numerical simulation in Fig. S4 and the area weighted assumption (Li et al., 2022), the tortuosity of model is the following expression:

$$\tau = \tau_1\frac{D}{L_V} + \tau_2\frac{L_V - D}{L_V} = \frac{1}{\cos\left[(1-\varepsilon)^{\frac{1}{3}}\alpha\right]} + (\frac{\pi}{4} - \frac{1}{2})\ \frac{4(1-\varepsilon)}{\pi} \tag{S12}$$

S3 Streamlines of microfluidic models obtained by numerical simulation



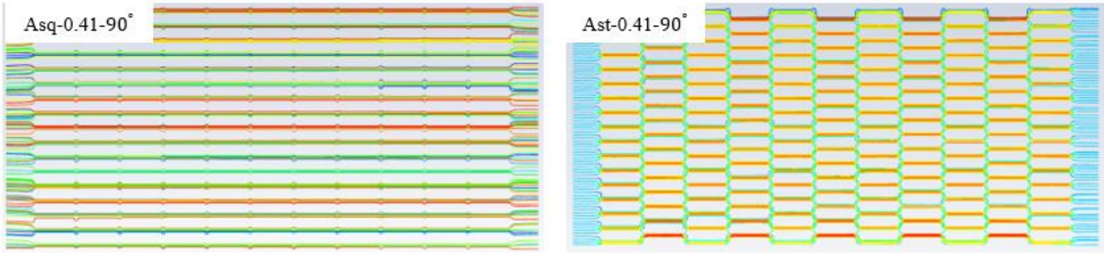


Fig. S4 Streamlines of microfluidic models obtained by numerical simulation

S4 Analytical tortuosity model for 2D fabric particle

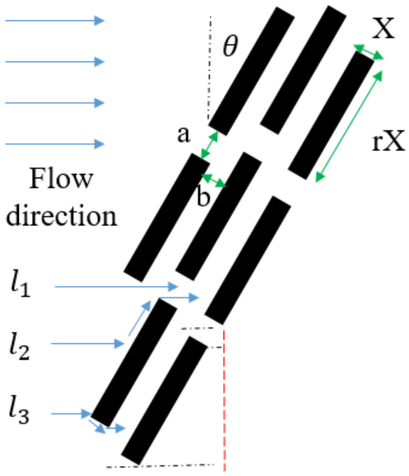


Fig. S5 Representative of fabric particle and parameters used in the model

Due to the high aspect ratios (ratio of diameter to length) of clay mineral particles, an idealized rectangular micropillar arrangement model with width *X* and length *rX* is shown in Fig. S5, where *r* is the aspect ratio. The rectangular particles are separated by a distance *a* between the edges of adjacent particles and a distance *b* between rows in a layers. In addition, the inclined angle with respect to vertical is defined as θ. Before a horizontal fluid particle encounters the layer immediately above, it travels an average distance of $\frac{b}{2cos\theta}$. At this point, three possible flow paths are assumed, as shown in Fig. S5. Flow path l_1 remains in free pore space. Flow path l_2 encounters the large face of a particle and must traverse the face before continuing upward. Flow path l_3 encounters the edge of a particle and must traverse the edge before continuing upward. These three possible flow paths could be calculated in the following expressions:

$$l_1 = \frac{b}{2cos\theta} + \frac{b+X}{cos\theta} = \frac{1.5b+X}{cos\theta} \tag{S13}$$

$$l_2 = \frac{b}{2cos\theta} + 0.5rX + \frac{b+X}{cos\theta} \tag{S14}$$

$$l_3 = \frac{b}{2cos\theta} + \frac{b}{cos\theta} + Xtan\theta + X \tag{S15}$$

The flow path directly in the flow direction is:

$$l = \frac{1.5b+X}{cos\theta} \tag{S16}$$

The grains are assumed to do not touch, so these tortuosities of the three different flow paths are obtained by taking the ratio of the average flow path length to the shortest distance.

$$\tau_1 = 1 \tag{S17}$$

$$\tau_2 = 1 + \frac{0.5rXcos\theta}{1.5b+X} \tag{S18}$$

$$\tau_3 = \frac{Xsin\theta+Xcos\theta+1.5b}{1.5b+X} \tag{S19}$$

By determining the probability of a fluid particle encountering free space, a particle face, or a particle edge, the model tortuosity can be obtained by normalizing τ_1, τ_2, and τ_3 by this probability. The probabilities can be computed as ratios of areas projected onto a plane normal to the direction of flow (Daigle and Dugan, 2011), which are $acos\theta$, $rXcos\theta$ and $Xsin\theta$, respectively. The Δ is defined as $acos\theta + mXcos + Xsin\theta$. So the tortuosity of model can be written as:

$$\tau = \frac{acos\theta}{\Delta}\tau_1 + \frac{rXcos\theta}{\Delta}\tau_2 + \frac{Xsin\theta}{\Delta}\tau_3$$

$$= \frac{acos\theta}{\Delta} + \frac{rXcos\theta}{\Delta} + \frac{rXcos\theta}{\Delta} \times \frac{0.5rXcos\theta}{1.5b+X} + \frac{Xsin\theta}{\Delta}\tau_3 \tag{S20}$$

As for engineering clay grains, the distance between grains in a layer is much less than the length of the grains, that is a、$b \ll rX$. Thus, Eq. (S20) can simplify to:

$$\tau = 1 + \frac{rXcos\theta}{\Delta} \times \frac{0.5rXcos\theta}{1.5b+X} \approx 1 + \frac{rcos\theta}{rcos\theta+sin\theta} \times \frac{0.5rXcos\theta}{1.5b+X} \tag{S21}$$

Based on the assumed particle geometry, the porosity can be calculated as:

$$\varepsilon = 1 - \frac{X*rX}{(X+b)*(rX+a)} \approx 1 - \frac{X*rX}{(X+b)*rX} = 1 - \frac{X}{X+b} \tag{S22}$$

Eq. (S22) can be rearranged:

$$b = \frac{\varepsilon X}{1-\varepsilon} \tag{S23}$$

Substituting Eq. (S23) to Eq. (S21), yielding:

$$\tau = 1 + \frac{rcos\theta}{rcos\theta+sin\theta} \times \frac{0.5rXcos\theta}{1.5\frac{\varepsilon X}{1-\varepsilon}+X} = 1 + \frac{r^2cos^2\theta}{(\frac{3\varepsilon}{1-\varepsilon}+2)(rcos\theta+sin\theta)} \tag{S24}$$