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Multi-directional wind energy harvesting based on the coupling effect between a piezoelectric beam and an elastic supported sphere

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Abstract: A new piezoelectric energy harvester is proposed which employs the coupling effect between a piezoelectric beam and an elastic supported sphere to capture wind energy from multiple directions. As wind flows across the sphere, it induces vortical vibrations that transfer to the piezoelectric beam, converting wind energy into electricity. A nonlinear coupled dynamic theoretical model based on the Euler-Lagrange equation is developed to study the interactions between the sphere and beam vibrations. The vortex-induced force acting on the sphere is determined, and the dynamic model of the coupled system is validated through experiments. The results show that in order to reach convergence, at least four modes are required in the Galerkin discretization. Moreover, the output performance of the energy harvester strongly depends on the frequency ratio between the sphere and piezoelectric beam. We find that at a frequency ratio of approximately 1.34, the harvester achieves a maximum average power of 0.19 mW at a wind speed of 3.9 m/s, with the lock-in region between 2.63 m/s and 5.25 m/s. Subsequently, the impact of wind flow direction on the electrical performance of the energy harvester is investigated in a wind tunnel, by adjusting the angle between the harvester and incoming flows ranging from 0 ° to 360 °. The findings indicate that the harvester maintains strong and consistent performance across variable wind flow directions and speeds. Particularly within the lock-in region, the output voltage fluctuations are below 5.5%, showcasing the robustness of the design. This result points to the potential utility of this novel harvester in complex environments. Our study also provides a theoretical basis for the development of small-scale offshore wind energy harvesting technologies.

Key words: Wind energy harvesting; Vortex-induced vibration; Piezoelectric effect; Lock-in region

1 Introduction

Civil infrastructure, such as buildings, bridges, and tunnels, inevitably encounters challenges from damage and aging. Structural Health Monitoring (SHM) is essential for the operation and maintenance of such structures, as it enables rapid detection of structural changes (Xie et al., 2020; Zhou et al., 2020). Recent advancements in wireless and low-power electronics have led to the adoption of Structural Health Monitoring Systems (SHMS). However, most Wireless Sensor Network (WSN) nodes, powered by batteries or power lines, face issues such as the frequent need for battery replacement and complex wiring requirements. These challenges are especially significant in inaccessible areas, harsh conditions, and embedded environments, creating an urgent need for reliable power sources for WSN nodes (Wang et al., 2021).

Energy harvesting involves converting energy from the environment into electrical power using various transduction technologies (Tao and Hu, 2016; Su et al., 2023). This process can charge the battery of a device or even eliminate the need for batteries entirely. Micro-energy harvesting technologies, with output power levels ranging from microwatts to milliwatts, offer a practical solution for powering low-power electronic devices and developing self-sufficient energy. This technology can replace

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batteries and simplify maintenance in applications where battery replacement is inconvenient, costly, or hazardous.

Among these technologies, piezoelectric energy harvesters stand out for their compatibility with MEMS (micro-electromechanical systems) devices, the fact that they don't need an initial power supply, and their simple structure. Early research into piezoelectric energy harvesting focused on converting wasted mechanical vibrations into electricity. For instance, Xia et al. (2023) developed theoretical models to improve the performance and efficiency of electromechanical energy harvesters. Similarly, He et al. (2022), Wu and Xu (2022), and Machado et al. (2021) enhanced piezoelectric energy harvesters through innovative designs and optimization strategies. These advancements enabled devices to adapt to various environmental conditions, thereby increasing their energy harvesting efficiency.

Although piezoelectric energy harvesters that leverage base excitation are widely used (Huang et al., 2023), they often face limitations, such as the need for specific frequency tuning, and limited energy capturing from random or broadband vibrations. In contrast, piezoelectric devices designed to capture energy from flow-induced vibrations offer distinct advantages, such as receptiveness to a broader range of frequencies and greater adaptability to varying environmental conditions (Huang et al., 2023; Wang et al., 2022). Vortex-induced vibration (VIV) is a phenomenon commonly observed in fluid flow. It induces periodic oscillations in structures, as driven by periodic vortical shedding forces. The lock-in phenomenon refers to the synchronization of a structure's vibration frequency with the frequency of the incoming flow, which occurs within a specific range of wind speeds known as the lock-in range. This synchronization enhances the energy conversion efficiency of the harvester, as it maximizes the interaction between the fluid flow and the vibrating structure. However, beyond this range, the efficiency of energy extraction diminishes due to the loss of this resonance condition. In energy harvesting research, the characteristics of VIV are often used to capture fluid energy through fluid flow. This vibration mode not only provides a mechanism for energy conversion but also introduces new challenges and opportunities for designing energy harvesters due to its unique nonlinear and multi-degree-of-freedom effects. Vortex-induced vibration energy harvesters typically employ cylindrical shapes as bluff bodies (Jia and Shan, 2020; Wang et al., 2021). Although the lock-in phenomenon of VIV allows for efficient energy conversion at specific wind speeds, this characteristic is only present within a small range of wind speeds. Outside this range, the vibrational mode of the cylinder changes, leading to a rapid decline in energy conversion efficiency. Therefore, to enhance the adaptability and efficiency of energy harvesters, researchers are currently trying to expand the lock-in range through various strategies, for instance by introducing nonlinear mechanisms (Zhang et al., 2017), increasing the number of structural degrees of freedom (Chen et al., 2023), or optimizing the structural design (Zhang et al., 2019).

In real environments, wind speed and direction are often unstable and variable, leading to reduced efficiency and wasted energy; this presents a significant challenge to traditional wind-induced vibration energy harvesters. Multi-directional energy harvesters are advantageous due to their adaptability to varying wind speeds and directions. Wang et al. (2020) summarized the current progress and challenges of converting wind and hydro energy into electrical power, investigating several innovative devices and concepts. A typical method to improve energy harvesting performance is to induce multimodal vibrations by modifying cantilever beams (Hu et al., 2020; Wang et al., 2019). However, this method is limited by narrow wind speed ranges. In contrast, the three-dimensional vortex structure inherent in sphere vortex-induced vibrations exhibits multiple vibrational modes, leading to a wider range of resonance winds and higher response amplitudes; this makes it more effective for capturing vortex-induced vibration energy.

Cylindrical structures typically exhibit vortex-induced vibrations (VIV) with two main resonance modes, depending on the alignment of the vortex shedding frequency with the structure's natural frequency. However, this lock-in phenomenon occurs within a narrow wind speed range, outside of which energy conversion efficiency drops sharply. In contrast, spheres exhibit more complex three-dimensional vortex structures, leading to multiple vibration modes and a broader range of resonant winds. This results in higher response amplitudes and

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a wider operational range for energy harvesting. The advantages of vortex-induced vibrations in spheres include: a broader wind speed range for effective energy conversion, higher response amplitudes due to complex vortex shedding patterns, and the potential for multimodal vibrational energy harvesting in diverse flow conditions. Additionally, the inherent three-dimensional vortex structure of spherical VIVs presents new opportunities for designing energy harvesters that are adaptable to various environmental conditions. For instance, the asymmetrical wake behind a sphere, unlike the symmetrical wake of a cylinder, can lead to more stable and robust energy conversion. In summary, the VIVs of spheres offer significant potential for energy harvesting due to their unique hydrodynamic properties and wider operational capabilities. These advantages make spherical structures a promising avenue for developing the next generation of energy harvesting technologies.

However, the fluid dynamics of sphere VIVs are complex, and researchers are still investigating their underlying mechanisms. Govardhan and Williamson (1997), Williamson and Govardhan (1997), and Rajamuni et al. (2020) each identified four VIV modes in tethered spheres that were immersed in fluid. These are resonance modes characterized by high vibration amplitudes, and frequencies that closely match the natural frequencies. Studies have also focused on the dynamic response of elastically-mounted spheres in uniform flow. For instance, Behara and Sotiropoulos (2016) and Negri et al. (2018) explored multidirectional VIV modes in elastically-tethered spheres. Rajamuni et al. (2019) further compared the multi-degree-of-freedom dynamic characteristics of an elastically-tethered sphere. Specifically, they examined the dynamic response, force coefficients, and wake structures. Also, advances in computing power have made numerical simulation a viable method for studying vortex-induced vibrations. Accordingly, Raza et al. (2020) investigated the VIVs of spheres at moderate Reynolds numbers, and Rajamuni et al. (2018) incorporated transverse rotation into an analysis model for spheres. They found that rotation reduced the oscillation amplitude and shrunk the lock-in region, with the VIV response being related to the Reynolds number.

In this study, we develop a multi-directional energy harvester based on the coupling effect between the vortex-induced vibrations (VIVs) of a sphere and a piezoelectric beam. First, a theoretical model is established and validated through experiments. Second, the impact of the ratio between the natural frequency of the sphere and the natural frequency of the piezoelectric beam on the mechanical response and electrical performance is investigated. Finally, the effect of multi-directional wind on the energy harvesting performance is examined.

2 Modeling and principle

We present an energy harvesting system that utilizes sphere vortex-induced vibrations to drive piezoelectric beams for electricity generation, as depicted in Fig. 1. The proposed device can harvest multi-directional wind energy above the surface of a water body, and supply power to surrounding sensors. The energy harvester is simplified to the computational model depicted in Fig. 2. The sphere is connected via springs to fixed supports above, and to a flexible cantilever beam below. A piezoelectric layer is attached to the flexible cantilever beam. L_p and L_b represent the lengths of the piezoelectric layer and the beam, respectively. The thicknesses of both the piezoelectric layer and the beam are negligible, so shear deformation and rotational inertia can be ignored. The system generates energy through the bending of the beam induced by the sphere's VIVs.

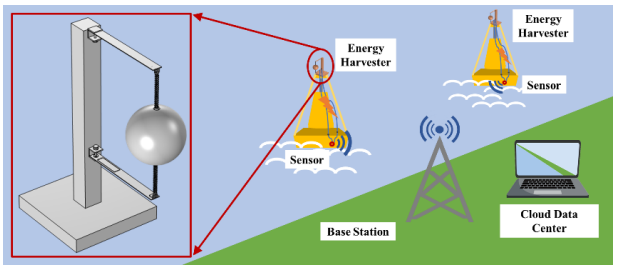

Fig. 1 The proposed energy harvester attached to a buoy on a body of water

The dynamics of vortex-induced vibration (VIV) of an elastically-mounted sphere is simplified to the case of a linear oscillator, with a detailed derivation provided in Section 1 of the Electronic Supplementary Materials. The model of the coupled effect between the piezoelectric beam and sphere is derived as follows:

$$m_s \ddot{w}_s(t) + c\dot{w}_s(t) + (k_1 + k_2) w_s(t) = F_{\mathrm{VIV}}(t) \tag{1}$$

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$$F_{\mathrm{VIV}}(t)=\frac{\pi C_{\mathrm{L}}\rho_{\mathrm{a}}D^2U^2}{16}q(t)-\frac{\pi C_{\mathrm{D}}\rho_{\mathrm{a}}D^2U}{8}\dot{w}_{\mathrm{s}}(t) \quad (2)$$

$$\ddot{q}(t)+\lambda\omega_{\mathrm{s}}\left(q^2(t)-1\right)\dot{q}(t)+\omega_{\mathrm{s}}^2q(t)=\frac{A}{D}\ddot{w}_{\mathrm{s}}(t) \quad (3)$$

$$\ddot{Y}_i+2\varsigma_i\omega_i\dot{Y}_i+\omega_i^2Y_i-\theta_iV=F_{\mathrm{sp}} \quad (4)$$

Here, $w_{\mathrm{s}}(t)$, $\dot{w}_{\mathrm{s}}(t)$, and $\ddot{w}_{\mathrm{s}}(t)$ are the displacement, velocity, and acceleration of the sphere, respectively. The terms m_{s}, c, k_1, and k_2 are the total mass of the sphere and springs, and the structural damping and the stiffness of Spring 1 and Spring 2, respectively; C_{L} denotes the steady mean lift coefficient and C_{D} denotes the steady mean drag coefficient; U denotes the wind speeds; D represents the diameter of the sphere; ρ_{a} denotes the density of the air; $q(t)$ is a fluid–structure coupling term; A represents a constant of the van der Pol wake oscillator model, where A=23; λ is a constant describing vortex street fluctuation, with λ=0.1; both λ and A are determined from experimental results according to Facchinetti (2004); finally, ω_{s} denotes the vortex shedding frequency.

Facchinetti et al. (2004) provides the theoretical foundation for understanding vortex-induced forces and wake oscillator behavior. Eq. (1) and Eq. (2) calculate the vortex-induced force, accounting for the periodicity of the vortex shedding and the fluid pressure distribution on the sphere's surface. Eq. (3) describes the dynamic behavior of the wake oscillator under vortex-induced forces, incorporating the mass, damping characteristics, and torque induced by the vortex force. Assuming the vibrational response of the piezoelectric beam is $w_{\mathrm{b}}(x,t)$, it can be expressed as the product of the beam mode function matrix and the generalized time coordinate matrix, given by $w_{\mathrm{b}}(x,t)=\sum_{i=1}^{n}\Phi_i(x)Y_i(t)$, where $Y_i(t)$ is the mode vector and $\Phi_i(t)$ is the vibration mode amplitude of the piezoelectric beam. The calculation method using the Galerkin method follows the approach detailed by Dai et al. (2014).

The vortex-induced force is transmitted to the piezoelectric beam through Spring 2. According to Hooke's Law, the spring force is given by:

$$F_{\mathrm{sp}}=k_2(l_2^{'}+w_{\mathrm{s}}(t)-w_{\mathrm{b}}(L,t)) \quad (5)$$

where $w_{\mathrm{b}}(L,t)$ denotes the end displacement of the piezoelectric beam; $l_1^{'}$ and $l_2^{'}$ represent the initial elongation of Spring 1 and Spring 2, respectively; L_0, l_1, and l_2 represent the total height of the sphere-spring system, the length of Spring 1, and the length of Spring 2, respectively, all of which can be measured directly.

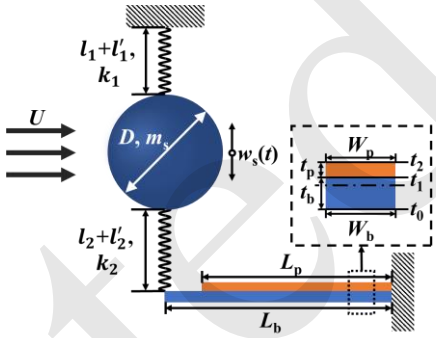

Fig. 2 Simplified computational model of the proposed device

According to Ohm's law, the voltage $V(t)$ across a resistor is directly proportional to the current passing through it. By also considering the definition of current, the following equation can be obtained:

$$C_{\mathrm{p}}\dot{V}(t)+\frac{V(t)}{R}+\sum_{i=1}^{n}\theta_i\dot{Y}_i=0 \quad (6)$$

where C_{p}, R, and θ_i denote the capacitance, resistance load, and electromechanical coupling coefficient of the piezoelectric beam, respectively.

3 Validation of the theoretical model

3.1 Determination of the vortex-induced force

Fig. 3 illustrates a schematic of vortex-induced vibration of a bluff body sphere, with Fig. 3(a) depicting the variation of lift/drag force coefficients C_{L} and C_{D} with reduced velocity U_{r}, as reported by Sareen et al. (2018). At the onset of the lock-in region, C_{L} initially increases and then continuously decreases. During this reduction, small amplitude oscillations occur. Beyond U_{r} =10, C_{L} almost remains constant; C_{D} also increases sharply when the lock-in region begins. The accuracy of the theoretical model is validated through wind tunnel experiments, using

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the setup detailed in Section 2 of the Electronic Supplementary Materials.

In Fig. 3(b) we showcase the variation of the sphere's vibrational amplitude with wind speed. When U is below 4.47 m/s, a gradual increase in amplitude ensues, reaching a peak at U=4.47 m/s. In the subsequent range of 4.47 m/s to 8.06 m/s, the amplitude gradually decreases. A noteworthy shift occurs in the range U>8.06 m/s, which is characterized by a slight rebound. A comparative analysis of the experimental and numerical results shows consistent maximum and minimum amplitudes under the same velocity conditions. However, a slight discrepancy emerges when the wind speed exceeds 8.06 m/s. Numerical simulations suggest an increase in amplitude, indicative of a transition from vibration mode I to II. In contrast, the experimental results initially show an increase in amplitude, followed by a slight decrease. This variation in amplitude is attributed to the fluid flow characteristics observed during the experiment, where the three-dimensional nature of the wake generates a significant vibration component. Consequently, the restraining springs apply both downstream and cross-flow constraints, causing a deviation in the sphere's vibration mode from the anticipated cross-flow mode.

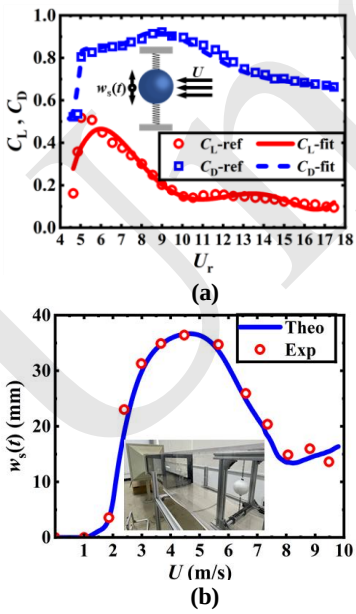

Fig. 2 (a) Schematic model of an elastically-mounted sphere and variations of lift/drag force coefficient C_L and C_D with reduced velocity U_r. (b) The experimental setup and the theoretical and experimental displacements of the sphere under different wind speeds.

Within the self-locking region, the sphere primarily exhibits vertical vibrations due to the periodic force provided by the spring. These vibrations are the most effective for energy harvesting as they directly cause deformation of the piezoelectric beam. When the wind speed deviates from the self-locking region, the sphere's horizontal vibration amplitude increases while its vertical vibration amplitude decreases. Although the sphere moves laterally under these conditions, its contribution to the piezoelectric beam's deformation is minimal, limiting its impact on the power generation efficiency. At higher wind speeds, the sphere stabilizes at the equilibrium position due to the combined effects of the wind and spring forces, further reducing the vibration amplitude. Consequently, both the deformation of the piezoelectric beam and its power generation capacity are significantly reduced. This study focuses on optimizing vertical vibrations within the self-locking region, as this mechanism provides the most effective energy conversion for vortex-induced vibrations. Although the sphere also exhibits horizontal vibrations under other wind speed conditions, these contribute less to energy harvesting and are therefore not considered as a primary factor.

3.2 Experimental validation

To validate the feasibility of the proposed device and theoretical model, a series of wind tunnel experiments are conducted. The sample and experimental setup are shown in Fig. 4(a) and (b), while detailed geometric and physical characteristics of the sample are listed in Table 1. Details of the experimental setup and the fabrication of the proposed device are provided in Section 2 of the Electronic Supplementary Materials.

Looking at Fig. 4(c), the curve depicting the variation of generated voltage with wind speed (as predicted using the Galerkin discretization method) reveals the significant influence of mode number on prediction accuracy. With only one or two modes, the predicted voltage values do not converge, resulting in an underestimation of both voltage and power output. This indicates that models with one or two modes cannot fully capture the complexity of the power generation. Increasing the number of modes to three improves prediction accuracy, but it still does not

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achieve the desired level of convergence. However, including four or five modes results in consistent and nearly identical predicted voltage and power values, indicating better convergence. This finding suggests that at least four modes are needed in the Galerkin discretization to ensure accuracy of the voltage prediction. While increasing the number of modes can enhance the prediction accuracy, it is also essential to balance this accuracy with computational cost. Therefore, selecting the appropriate number of modes is crucial for efficient and accurate predictions, and as such should be determined based on the specific application scenario and precision requirements.

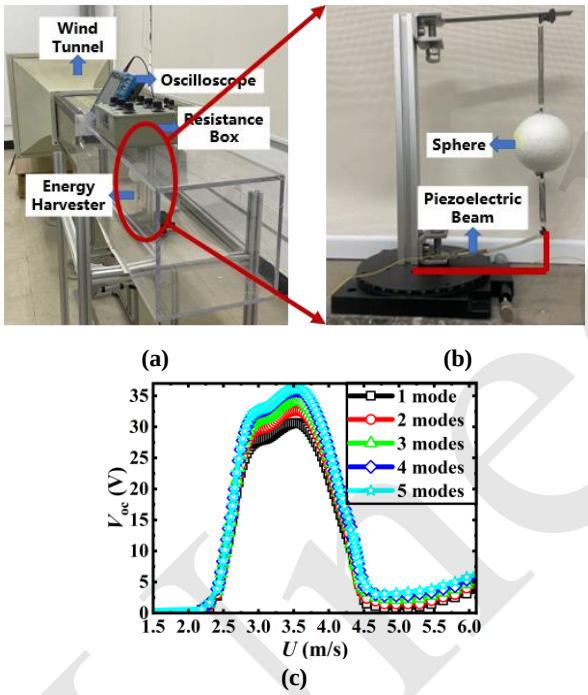

Fig. 3. (a) Experimental setup of the proposed device and (b) a photograph of the sample. (c) The peak open-circuit voltage of the device varies with wind speed under different modes.

The vibration amplitudes of the spherical bluff body and the beam end are shown for various wind speeds in Fig. 5(a) and (b). We can observe different phenomena here as compared to Fig. 3(b). The lock-in region for the model in Fig. 5(a) is 0.64 m/s~6.04 m/s, while in Fig. 3 it is 1.63 m/s~8.06 m/s, demonstrating that the designed structure effectively converts fluid flow energy, even at low speeds. Notably, in the plots of Fig. 5, at U=3.2 m/s the amplitudes exceed the maximum amplitude of 38 mm observed in the model in Fig. 3(b). This sudden increase is due to the support design and the elasticity of the underlying cantilever beam. Looking closer at Fig. 5(b), it appears that the structure reaches resonance at the wind speed of U=3.2 m/s, significantly increasing the amplitude and enhancing the wind energy conversion efficiency. This design advantage allows the model to achieve high energy output even at relatively low wind speeds. Moreover, adjusting the stiffness of the piezoelectric beam enables tuning of the structure's natural frequency, thereby enhancing performance at different wind speeds.

Table 1 Geometric and physical properties of the device components

Components	Properties	Values
Sphere	Mass, m_s (g)	2.02
	Diameter, D (mm)	56.8
Spring	Stiffness of Spring 1, k_1 (N/m)	11.9
	Stiffness of Spring 2, k_2 (N/m)	11.9
Beam	Length, L_b (mm)	120
	Width, W_b (mm)	10
	Thickness, t_b (mm)	0.6
	Mass density, ρ_b (kg/m3)	2700
	Young's modulus, E_b (GPa)	70
MFC	Length, L_p (mm)	28
	Width, W_p (mm)	7
	Thickness, t_p (mm)	0.3
	Mass density, ρ_p (kg/m3)	7700
	Young's modulus, E_p (GPa)	30.336
	Strain coefficient, d_{31} (pm/V21)	-190
	Permittivity at constant strain, ε_{33} (nF/m)	13.28
	Capacitance, C_p (nF)	15.1

The open-circuit voltage of the designed device is shown in Fig. 5(c). The electrical response of the structure is consistent with the vibration characteristics depicted in Fig. 5(a) and (b). As we can see, the lock-in region is approximately 2.63 m/s~5.30 m/s. As the vibration amplitude increases, the charge generated by the piezoelectric effect also increases, resulting in a higher output voltage. Outside this region, the output voltage rapidly decreases. At the critical wind speed of U=2.63 m/s, the open-circuit voltage peaks at 36.8 V (experimentally) and 36.2 V (theoretically). The close match between the experi-

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mental and theoretical results further demonstrates the accuracy of the proposed model. The open-circuit voltage sharply declines to its lowest point at U=5.30 m/s. When the wind speed exceeds 5.30 m/s, the sphere oscillates laterally, causing the cantilever beam to bend due to spring expansion and contraction; this leads to a rebound in the open-circuit voltage.

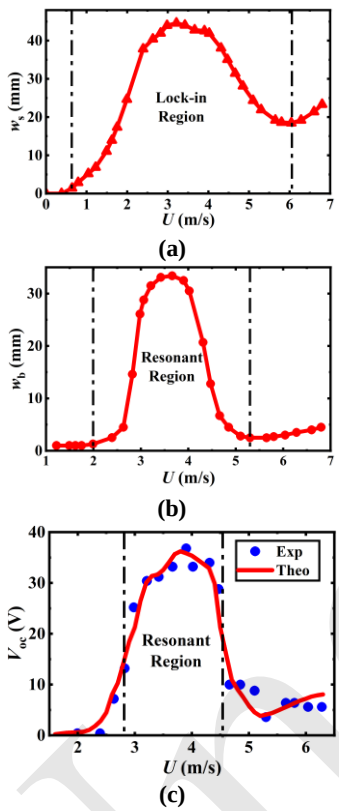

Fig. 5 (a) The vibration amplitude of the sphere in the proposed device varies with wind speed. (b) The end displacement of the piezoelectric beam changes with wind speed. (c) The peak open-circuit voltage of the proposed device varies with wind speed.

To investigate the optimal load resistance R, the variations of root mean square voltage V_{rms} and average output power P_{avg} with load resistance are shown in ESM-Fig. 1. It is known that structural resonance significantly impacts energy harvesting efficiency (Wu et al., 2021). Thus, three wind speeds were analyzed: the wind speed corresponding to the maximum amplitude, the initial wind speed of the lock-in region, and the wind speed at the end of the lock-in region. The variations of RMS voltage and average power with load resistance at the three representative wind speeds are shown in ESM-Fig. 1(a) and (b). As shown in ESM-Fig. 1(a), the V_{rms} increases with increasing load resistance. This is due to the power generation characteristics of piezoelectric elements: without an external load, charges generated by the piezoelectric effect accumulate, increasing the electric field strength until balance is reached. Accordingly, the ability to generate additional charge is reduced. Introducing an external load resistor provides a discharge path, enabling the piezoelectric material to continue generating charge.

However, the average output power P_{avg} does not increase monotonically with load resistance R. As the load resistance R increases from 0.05 MΩ to 1.0 MΩ, and then to 10 MΩ, the average output power P_{avg} initially increases and then decreases. This occurs because in energy harvesting systems, the energy transfer efficiency is maximized when the load resistance matches the system's dynamic impedance. If the load resistance is too low, significant energy is lost within the system; if it is too high, the system cannot efficiently transfer energy to the load. The variations in P_{avg} suggest the existence of an optimal load resistance that maximizes the average output power P_{avg}. ESM-Fig. 1 shows that the average power P_{avg} reaches a peak at a load resistance of R=1.0 MΩ. At this optimal load resistance, the energy conversion efficiency is highest and maximum power output is provided.

4 Output performance results

4.1 Influence of natural frequencies on output performance

To optimize the mechanical and electrical performance, the natural frequencies of both the sphere system and the piezoelectric beam must be tuned. This relationship can be described by a dimensionless frequency (η), defined as the ratio of the natural frequency of the sphere system (f_s) to that of the piezoelectric beam (f_b), expressed as $\eta=f_s/f_b$. To elucidate the relationship, we vary the diameters of the sphere (D) from 50 mm to 80 mm, while we vary the stiffnesses of the springs (k_1, k_2) from 6 N/m to 16 N/m, respectively. The resulting calculations show the relationships between sphere diameter, spring stiffness, and natural frequency, as shown in ESM-Fig. 2. ESM-Fig. 2(a) shows that the natural frequency increases with increasing spring stiffness. This plot also

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indicates that larger sphere diameters strongly influence the natural frequency. ESM-Fig. 2(b) illustrates the relationship between the piezoelectric beam length (L_b) and the first natural frequency (f_b) for different beam widths (w_b). Specifically, it shows that the natural frequency decreases with increasing beam length, indicating a reduced stiffness of the piezoelectric beam as length increases. Similarly, increasing the beam width raises its natural frequency.

ESM-Fig. 2(c) shows the influence curve of dimensionless frequency η and the vortex-induced vibration amplitude w_s of the sphere, for a wind speed of U=3.90 m/s. The natural frequency of the sphere-spring system depends on its diameter (D) and the stiffness of the tension springs. Larger sphere diameters and lower spring stiffness tend to reduce the structure's natural frequency. The natural frequency of the piezoelectric beam is influenced by its dimensions (length and width). Increasing the beam length typically lowers its natural frequency, whereas increasing the width raises it. At η=0, the piezoelectric beam is treated as a rigid support with infinite stiffness, resulting in a low vibration amplitude for the sphere. This indicates that with rigid support, the sphere's response to wind excitation is constrained.

As η increases, the sphere's amplitude first increases slowly and then decreases rapidly. This trend suggests an optimal dimensionless frequency at which the sphere's VIV response is maximized. At η=1.34, the sphere's amplitude peaks at 43.8 mm. This is because the matching of the natural frequencies of the sphere system and the piezoelectric beam leads to resonance, which maximizes the energy transfer efficiency. It also indicates that the coupling effect between the structure's natural frequency and the sphere's VIV characteristics is optimized at this dimensionless frequency. When η exceeds 1.5, the sphere's amplitude decreases rapidly. This is due to the equivalent stiffness of the piezoelectric beam being too low to maintain the sphere's vibration mode, resulting in reduced energy transfer efficiency.

In ESM-Fig. 2(d) we present the relationship between the dimensionless frequency η and the average output power P_{avg}, at a wind speed of U=3.90 m/s. As one can observe, the average output power P_{avg} reaches a peak at η=1.34. This suggests that when the natural frequency of the sphere system is 1.34 times that of the piezoelectric beam, the energy transfer efficiency of the system is highest, reflecting optimal resonance. The lock-in region for the spherical bluff body helps maintain the resonant mode. Thus, by setting a fixed vibration frequency, the system can operate at optimal performance. ESM-Fig. 2 indicates that resonance is essential to our proposed piezoelectric energy harvesting system. By tuning the natural frequencies of both the sphere system and the piezoelectric beam, maximum energy harvesting efficiency can be achieved.

ESM-Fig. 3 shows the FFT (Fast Fourier Transform) of the output power signal at a wind speed of U=3.90 m/s for various dimensionless frequencies. This plot illustrates the effect of frequency ratio on electrical performance by presenting the FFT of the output power signal, which indicates the resonant frequency of the piezoelectric beam. In ESM-Fig. 3 (a), when the wind speed is 3.90 m/s and the dimensionless frequency is 0.569, the average output power of the energy harvester reaches a peak at 11.11 Hz. This indicates that at this frequency ratio, the harvester’s vibration frequency aligns with the vortex shedding frequency caused by the wind, leading to efficient energy conversion. At other frequencies, the output power fluctuates less, suggesting that these frequencies do not synchronize with the fluid-induced vibration, resulting in reduced energy conversion efficiency. Looking at ESM-Fig. 3 (b), with a wind speed of 3.90 m/s and a dimensionless frequency of 1.341, the average output power of the energy harvester reaches a peak at 14.22 Hz. At this frequency ratio, the harvester synchronizes with the vortex shedding frequency at a higher range, achieving efficient energy conversion.

Comparing ESM-Fig. 3 (a) and (b), it is evident that the frequency ratio affects the resonant frequency of the device. According to Govardhan and Williamson (1997), vortex-induced vibrations of a sphere exhibit a self-locking effect. Specifically, within a certain frequency range, the vibration amplitude of the sphere significantly increases, leading to resonance. However, when the excitation frequency deviates from this range, the vibration amplitude of the sphere sharply decreases, resulting in almost no vibration. Near the resonant frequency, energy transfer efficiency is maximized, enabling effective vibra-

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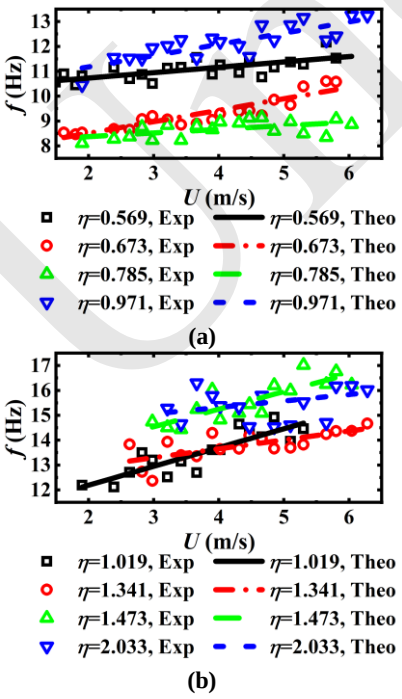
tional energy transfer to the cantilever beam and yielding significant electrical output. Conversely, at non-resonant frequencies, reduced vibration amplitude decreases the energy transfer efficiency, significantly reducing the cantilever beam's deformation and the power generation capability. This explains why only one significant resonance peak is observed in the FFT spectrum.

To summarize the effect of frequency ratio on the resonant frequency, Fig. 6(a) and (b) show the change in vibration frequency with wind speed as the dimensionless frequency varies. Inspecting Fig. 6(a) and (b), we can see that at a constant dimensionless frequency η, the relationship between vibration frequency and wind speed is approximately linear. Additionally, the smaller the dimensionless frequency η, the lower the wind speed required to initiate vibration. Fig. 6(c) and (d) depict the change in the average output power P_{avg} with wind speed under different dimensionless frequencies. Clearly, as the dimensionless frequency η increases, the lock-in region shifts rightward, causing the peak output power to shift accordingly. This occurs because a larger dimensionless frequency η corresponds to a higher natural frequency of the structure, which requires a higher flow rate to achieve the same excitation frequency.



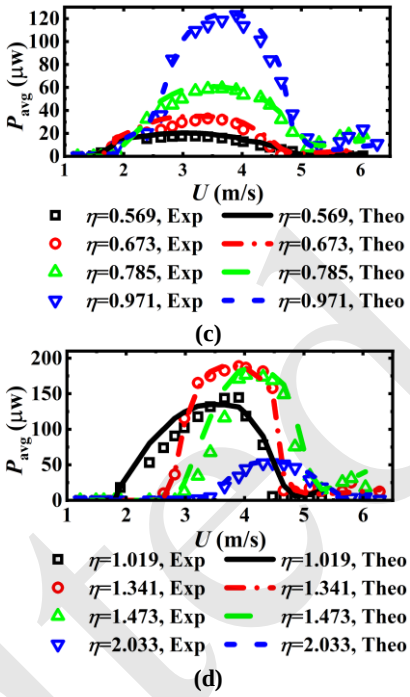


Fig. 4. (a) - (b) Vibrational frequency (f) varies with the wind speed of various dimensionless frequencies (η). (c) - (d) Average output power (P_{avg}) varies with wind speed of various dimensionless frequencies (η).

Comparing Fig. 6(c) and (d) reveals that when η ranges from 0.971 to 1.473, the output power is relatively high, indicating optimal energy harvesting efficiency within this range. The coupling vibrations between the piezoelectric beam and the sphere system influence each other. This interaction optimizes vibration patterns and energy transfer at specific dimensionless frequencies, improving output performance. At η=1.341, the output power reaches peaks of 189.72 μW (theoretically) and 188.75 μW (experimentally), indicating that this dimensionless frequency provides optimal resonance conditions and maximizes the coupling effect between the piezoelectric beam and the sphere system. This comparison between the experimental and theoretical values helps validate the theoretical model.

Fig. 7 shows the variation of the aero-electro-mechanical efficiency ψ_{aem} of the proposed energy harvester as a function of resistance R, for different dimensionless frequencies η. The formula for aero-electro-mechanical efficiency ψ_{aem} is provided in Section 3 of the Electronic Supplementary Materials. As the resistance R increases from

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0.05 MΩ to 10 MΩ, ψ_{aem} initially increases and then decreases. This occurs because at low resistance values, the output voltage is relatively low, reducing the energy harvester's efficiency. However, as resistance increases to an optimal value, the output voltage reaches its maximum. The peak aero-electro-mechanical efficiency is found at R=1 MΩ, with a peak value of 1.05%. When η is less than or equal to 1.341, increasing the value of η results in a higher ψ_{aem}. However, when η exceeds 1.341, further increases in η lead to a decrease in ψ_{aem}. This is because an excessively high η value causes the energy harvester's vibration mode to misalign with the fluid flow characteristics, reducing the energy conversion efficiency.

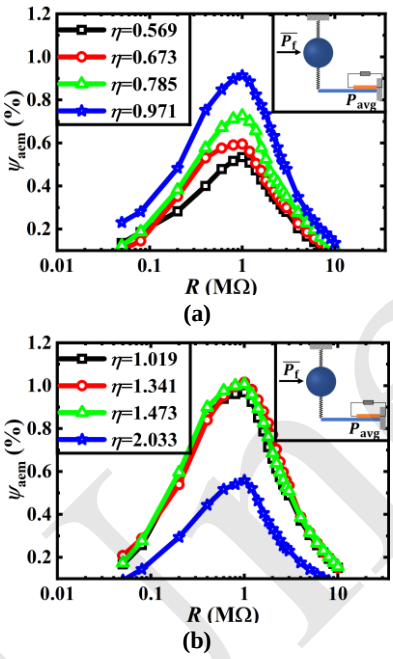

Fig. 5. The aero-electro-mechanical efficiency (Ψ_{aem}) varies with resistance and dimensionless frequencies (a) η<1.000 and (b) η>1.000.

We next show variation curves of the electro-mechanical efficiency ψ_{em} of the designed energy harvester with respect to the resistance R at different dimensionless frequencies η, as presented in Fig. 8. The expression for the electro-mechanical efficiency ψ_{em} is given in Section 3 of the Electronic Supplementary Materials. Notably, the trends for ψ_{em} and ψ_{aem} are similar, both peaking at R=1 MΩ. This is because this resistance value aligns with the dynamic characteristics of the structure, thereby optimizing the energy conversion efficiency. However, ψ_{em} exhibits significantly higher values, with a peak reaching 28.4%. This indicates that the process of converting mechanical energy from the piezoelectric beam into electrical energy is more efficient than the process of converting fluid energy into electrical energy. ψ_{em} is a measure of the efficiency of converting piezoelectric vibrations into electrical energy, and it tends to have a higher value since it only considers the electromechanical conversion step. On the other hand, ψ_{aem} describes the entire energy conversion process, including fluid-driven sphere vibration, sphere-driven piezoelectric beam vibration, and ultimately the conversion of piezoelectric beam vibrations into electrical energy. Consequently, ψ_{aem} might be influenced by energy losses in the earlier stages of the energy conversion process.

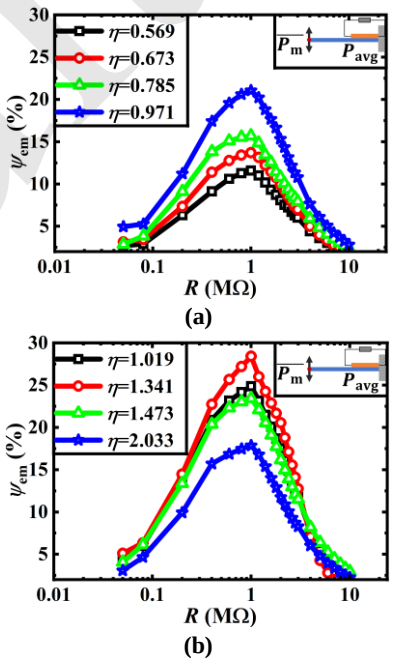

Fig. 6. The electro-mechanical efficiency (Ψ_{em}) varies with resistance and dimensionless frequencies (a) η<1.000 and (b) η>1.000.

4.2 Influence of wind direction on output performance

To investigate the output performance in multiple directions, the inflow angle θ of the device is adjusted by an indexing dial in an experiment. With

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the base of the sample mounted on an indexing plate, the direction of the device with respect to the flow can be controlled by an indexing dial. When the inflow direction aligns with the line connecting the sphere and the support column, the angle θ is considered as 0°. Changing the angle θ indicates a change in the inflow direction.

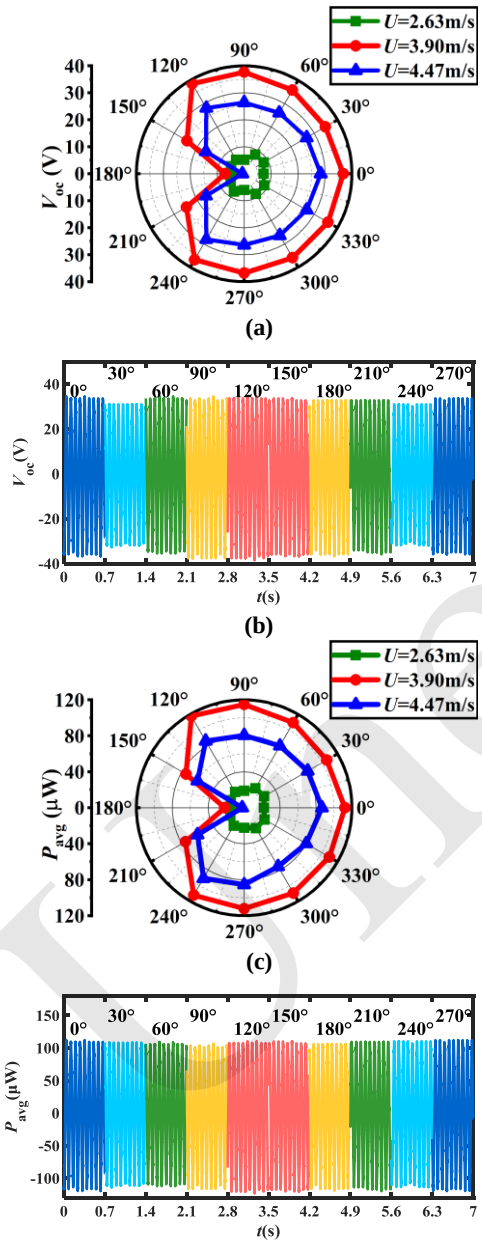

Fig. 7. (a) The output open-circuit voltage varies with flow-facing angle θ at three representative wind speeds. (b) Time history curves of peak open-circuit voltage of various angles for U=3.90 m/s. (c) The average output power varies with the flow-facing angle θ at three representative wind speeds. (d) Time history curves of the average output power of various angles for U=3.90 m/s.

The open-circuit voltage V_{oc} and average output power P_{avg} are recorded at various angles. Fig. 9(a) and (c) show how the open-circuit voltage V_{oc} and average output power P_{avg} vary with wind direction at three typical wind speeds. The results were recorded at 12 evenly-spaced angles increasing in a 30° interval. The figures reveal that both V_{oc} and P_{avg} exhibit 180° symmetry, indicating that the energy harvester has strong adaptability to different wind directions. Only in the range of 150°$\leqslant\theta\leqslant$210° is there a significant drop in output performance. When the inflow angle is between 150°~210°, the sphere is positioned behind the support column. The support column obstructs the airflow toward the sphere, causing a change in the airflow pattern around the sphere. This alteration affects the relationship between the lift and drag forces, reducing the vortex force acting on the sphere. As a result, the vibration mode of the sphere changes from vertical to horizontal. When the sphere's vibration mode becomes horizontal, the piezoelectric beam cannot bend, and thus the piezoelectric effect cannot occur, leading to a decrease in energy conversion efficiency. Fig. 9(b) and (d) show the time history curves of the open-circuit voltage V_{oc} and the average output power P_{avg} for different wind directions, at a wind speed of U=3.90 m/s. When 0°$\leqslant\theta\leqslant$120° and 240°$\leqslant\theta\leqslant$360°, the open-circuit voltage V_{oc} and average output power P_{avg} fluctuate by no more than 5.5%, indicating that the energy harvesting performance is relatively stable across different wind directions. For example, the open-circuit voltage is V_{oc}=36.8 V at θ=0° and V_{oc}=34.9 V at θ=300°, a difference of only 5.1%.

5 Conclusions

A novel piezoelectric energy harvesting device was proposed, which utilizes vortex-induced vibrations of a sphere coupled with a piezoelectric beam to capture wind energy from varying directions. First, a coupled dynamic theoretical model describing the vibration behaviors and the electrical performance of

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the energy harvester was developed. The impact of the frequency ratio between the elastic-supported sphere and the piezoelectric beam on the electrical power and lock-in region was investigated, revealing the coupled effect between the sphere and beam oscillations. Moreover, experimental results showed that this new piezoelectric device can harvest wind energy from multiple incoming flow directions.

The fluid forces acting on the sphere to produce vortex-induced vibration behavior were determined through a wake oscillator model. Then, a coupled dynamic model was constructed to analyze vibration responses and interactions between the sphere and the piezoelectric beam. These models were validated by experimental results. Our theoretical and experimental results indicated that the frequency ratio between the sphere and the piezoelectric beam has a strong impact on the electrical performance of the energy harvester. When the frequency ratio was 1.34, the average output power reached a peak of 0.19 mW at a wind speed of 3.9 m/s. Additionally, the output voltage of the energy harvester was consistent across varying wind speeds and flow directions. This was especially true in the lock-in region, where the peak output voltage varied by less than 5.5%.

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Author contributions

Shun Weng contributed to the methodology, software development, and validation. Liying Wu contributed to the methodology, software development, validation, processed the corresponding data and organized the manuscript. Lanbin Zhang contributed to the methodology, experiment designing and software development. Ke Gao, Junshu Zhang, and Zhiyue Zhang assisted in organizing the manuscript. Huliang Dai design and coordinated the manuscript.

Conflict of interest

Shun WENG, Liying WU, and Huliang DAI declare that they have no conflict of interest.

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Electronic supplementary materials

Section S1–S3, Figs. S1–S3

中文概要

题　目：基于压电梁与弹性支撑球体耦合效应的多方向风能采集

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目　的： 无线传感器网络在结构健康监测中关键，但受限于电池寿命，因此迫切需要环境能量收集技术以实现长期稳定监测。本文旨在探索和开发一种新型的压电能量收集器，利用压电梁和弹性支撑球之间的耦合效应，实现多方向风能的有效捕获和转换。

创新点： 1. 提出了压电梁和弹性支撑球之间的耦合效应来捕获风能的新方法；2. 基于欧拉-拉格朗日方程开发了非线性耦合动力学理论模型；3. 实验验证了耦合系统的动态模型，并研究了频率比对输出性能的影响；4. 研究了来流方向对电气性能的影响，展示了在复杂环境中的潜在应用价值。

方　法： 1. 利用压电梁和弹性支撑球的耦合效应设计能量收集器；2. 建立基于欧拉-拉格朗日方程的非线性耦合动力学理论模型；3. 通过实验验证理论模型，并进行 Galerkin 离散化分析；4.在风洞中调整收集器与来流之间的角度，测试不同来流方向下的电气性能。

结　论： 1. 提出弹性支撑与压电梁耦合的球体涡激振动能量采集设计，动力学理论模型得到了实验验证；2. 该俘能器可以有效的俘获多个来流方向的风能，相差不超过 5%；3. 理论和实验结果表明，球体固有频率与压电梁固有频率之比为 1.34 时有最大输出功率，3.9m/s 风速时平均输出功率约为 0.19mW。

关键词： 风能采集；涡激振动；压电效应；自锁区间

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