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Research Article

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Refined measurement and simulation of vibration characteristics of ballastless track and its effect on wheel-rail broadband dynamic interaction

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Abstract: The wheel-rail dynamic load (WRL) and its vibration energy transfer (VET) are foundational for studying ballastless track dynamics in high-speed railways. In this study, the higher order modal parameters of track beds with different isolating layers were identified experimentally and a vehicle-track coupled dynamic model considering track-bed broadband vibrations (TBVs) was established. The WRL and its VET were investigated, and the contribution law as well as the influence mechanism of TBVs on them were determined. The results showed the WRL and track-bed vibration energy exhibited significant resonances, with more prominent high-frequency resonance peaks in the track-bed vibration energy. TBVs had a significant effect on low-frequency WRLs, and markedly influenced the VET across various frequency bands. Intense low-frequency and weak high-frequency intermodulation effects between the wheel-rail and track beds were observed. The effect of track-bed vibrations can be disregarded when focusing on high-frequency WRLs above 200 Hz. Variations in the isolating layer stiffness have more significant effects on the track-bed vibration energy than the WRL. Rational stiffness of the isolating layer should be selected to avoid mode-coupling resonance from track beds to the wheel-rail subsystem.

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Key words: Ballastless track; Higher order modal testing; Wheel-rail dynamic load; Vibration energy transfer; Intermodulation effect; Rational stiffness

1 Introduction

Clarifying wheel-rail dynamic interactions is key to assessing the safety of train operations and studying the vibration and noise of railway systems. The wheel-rail dynamic load at high speeds is intense and distributed in a broadband vibration zone of from 10 Hz to more than 1000 Hz under the excitation of the wheel polygon and rail corrugation (Zhai et al., 2020). Since the track directly bears and in turn affects the wheel-rail interaction, its reasonable modelling is the premise for accurate simulation of the wheel-rail broadband dynamic response. Most researchers simplify a sleeper or bed-per-span into a mass-spring system that is attached to the rail (Johansson and Nielsen, 2007; Sheng et al., 2016; Mazilu, 2017; Martínez-Casas et al., 2017; Blanco et al., 2017; Torstensson et al., 2019). A wheel-rail dynamic interaction model of a ballasted track has been established, and the broadband distribution characteristics of the wheel-rail vertical force have been well simulated.

Most of China's high-speed railway tracks are ballastless track structures, and complex flexible deformations of track beds need to be characterized. The Euler beam (Gupta and Degrande, 2010; Auersch and Said, 2021), elastic thin plate (Zhai et al., 2009), medium-thick plate theory (Luo et al., 2020; Zhai et al., 2023), finite element (El Kacimi et al., 2013; Xu

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and Zhai, 2019; Ren et al., 2024; Xu et al., 2022; Aggestam and Nielsen, 2020) or modal superposition (Cai et al., 2019; Zhu et al., 2017; Ling et al., 2020; Ma et al., 2023) have been proposed for modelling track beds, and a vehicle-ballastless track dynamic model considering bed or slab damage has been developed (Luo et al., 2019; Yang et al., 2020; Miao et al., 2023). The wheel-rail dynamic loads under the wheel polygon, rail damage, and subgrade settlement have been analyzed based on vehicle-track coupled dynamic models (Chen et al., 2019; Liu et al., 2022; Zhang et al., 2023). Furthermore, the investigation of the transmission law of wheel-rail dynamic loads remains a prominent area of research interest. Sarikavak et al. (2022) and Ren et al. (2019) studied the load transfer law of ballasted track and ballastless track for a high-speed railway. Zhang and Zhao (2020) investigated the dynamic transfer characteristics of vehicle loads for a double-block ballastless track. Zheng et al. (2020) tested the vibration-transfer characteristics of a ballastless track on a long-span bridge. Gou et al. (2023) explored in detail the energy transmission law of ballastless track-bridge systems for high-speed railways. Sainz-Aja et al. (2020) studied the displacement and acceleration of the rail and twin block sleepers based on a detailed 3D slab track model, and clarified their transfer relationship. Hu et al. (2025) studied the deformation transmission law with tunneling beneath the pile-raft foundations. In addition, the environmental vibration caused by high-speed railways and urban rail transit and its transmission laws have been the subject of many studies (Wang et al., 2023; Ni et al., 2024; Xu et al., 2022; Kaynia et al., 2000; Takemiya, 2003; Ghangale et al., 2020). However, the broadband vibration characteristics of track beds have not been accurately considered in previous studies due to the lack of measurement results.

Ballastless track beds are multi-layer heterogeneous structures, composed of a slab, filling layer, isolating layer, and baseplate. The isolating layer is an important structural layer, and has many forms (Fig. 1). It is a 4-mm-thick geotextile in the general section. Instead of the geotextile, a cushion is used on long-span bridges and a vibration-damping layer in environmental protection areas. The stiffness per unit area of various isolating layers varies significantly, which leads to complex broadband-vibration characteristics of track beds. Refined simulation of track-bed broadband vibrations remains inadequate due to the lack of measurement results. The contribution law and influence mechanism of broadband vibrations of track beds on the wheel-rail dynamic load and its vibration-energy transfer have not been fully discussed theoretically.

The first task of this study was to establish a refined broadband-vibration model of a ballastless track and its dynamic model coupled with the vehicle. The second task was to investigate the distribution characteristics of the wheel-rail dynamic load and its vibration energy transfer on the ballastless track. The third task was to study the effects of the broadband vibrations of track beds on the wheel-rail dynamic load and its vibration energy transfer, and to clarify the rational stiffness value of the isolating layer.

Fig. 1 Track isolating layer for different scenarios. (a) a geotextile in the general section; (b) a cushion on a long-span bridge; (c) a vibration-damping layer for environmental protection areas.

2 Measurement and modelling methods

Ballastless track broadband-vibration characteristics were refined, simulated and measured, and a vehicle-ballastless track dynamic interaction model was established.

2.1 Simulation and measurement of ballastless track broadband-vibration characteristics

Taking the CRTS-III ballastless track as the main research object, a three-dimensional (3-D) finite element model of track beds was established to obtain the modal parameters according to the geometric dimensions (length, width and height) and

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mechanical parameters (elastic modulus and density) of each structural layer (slab/ filling layer/ baseplate) (Fig. 2). The isolating layer and elastic foundation were considered to be equivalent to solid elements with identical support stiffness. The conversion relationship between the elastic modulus E_g and support stiffness per unit area k_g is $E_g = k_g d_g$ (where d_g is the structural layer thickness). In the vertical direction, each structural layer within track beds was bound and connected, with the bottom layer fully constrained. In the longitudinal direction, free boundaries were adopted at the slab joints due to separation between the CRTS-III track beds.

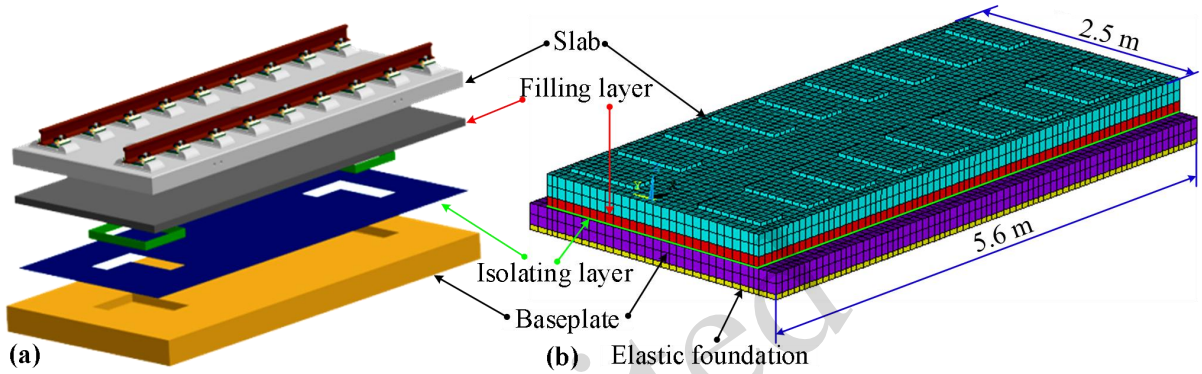

Fig. 2 CRTS-III ballastless track beds. (a) design model (b) refined finite element model.

The finite element mesh size altered the modal parameters, particularly those in the high-frequency modes. Fig. 3 shows that as the finite element mesh size decreases, the total modal order below 2500 Hz increases, and the frequency corresponding to the same high-order bending mode decreases. The modal order and modal frequency stabilize when the mesh size is taken as $c/6$, i.e. 0.105 m. Therefore, the mesh size of the track beds was set to $1/6^{th}$ of the fastener-node spacing. The highest modal frequency was 2500 Hz, which covered the effect of the track bed vibrations on wheel-rail responses in the band of interest. A track bed is a high-frequency modal dense structure, and the total modal order in the range of 2500 Hz exceeds 200. For each track bed, there were 13248 elements and 16505 nodes.

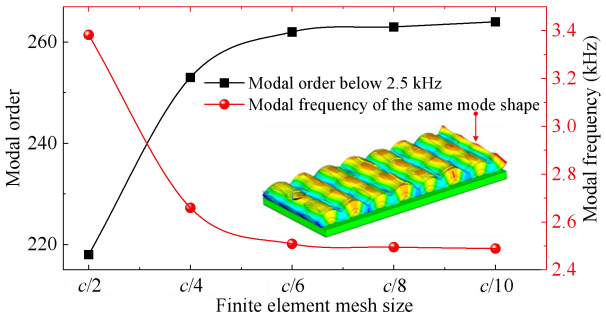

Fig. 3 The effect of finite element mesh size on modal parameters (c is the fastener node spacing).

Accurate analysis of modal parameters is the key to solving vehicle-ballastless track dynamic responses. Since the rail mass was much smaller than the track-bed mass, its effect on the modal parameters of the track bed was negligible. For the CRTS-III ballastless track, a modal identification test for track beds was carried out using multi-input and multi-output methods (MIMO) on actual operating railway lines, (Fig. 4). A force hammer was used to impact the excitation points, and the impulse response was collected by arranging several accelerometers at the response points. To obtain accurate multiple higher-order modes, the following conditions should be met: 1) A dense arrangement of excitation points: 1/2 fastener-node spacing settings, cumulative 19×4=76 excitation points; 2) A multiple asymmetric arrangement of response points: 7 response points throughout the entire track-bed surface. 3) Real-time determination of the coherence function of each response point: if less than 0.95, re-measure to ensure high-quality data.

Higher order modes of track beds are not easily identified by field experiments. For high-speed railway track beds, low-order modes below 150 Hz have been identified experimentally (Zheng et al., 2022), and in this study high-order modes beyond 800 Hz were identified. With the measurement results of the track-bed multimodal parameters as the

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optimization objective, the elastic modulus and density of each structural layer of the track beds were modified, the elastic support role of the isolating layer was fully considered, and its support stiffness value was clarified. A refined broadband-vibration model for track beds was established by integrating measurement results. The key mechanical and geometrical parameters of the track beds are listed in Table 1.

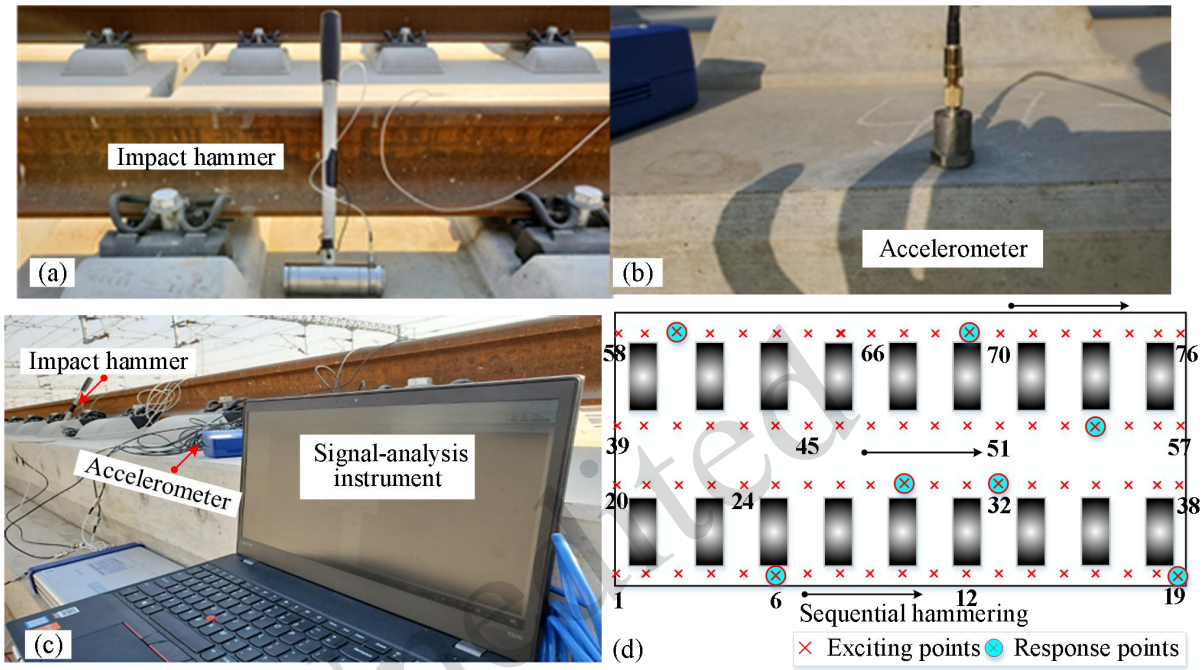

Fig. 4 Modal identification experiment for the CRTS-III ballastless track. (a) Excitation devices; (b) Accelerometer; (c) Measurement scenario; (d) Excitation input-response output point arrangement.

Table 1 Key parameters of the vehicle and track system.

Track		Vehicle	
Parameters	**Values**	**Parameters**	**Values**
Rail mass per unit length (kg)	60.64	Carbody mass (kg)	34934
Rail elastic modulus (GPa)	206	Bogie mass (kg)	3300
Rail Poisson ratio	0.3	Wheelset mass (kg)	1750
Rail 2nd moment of area about vertical direction (m^4)	3.217×10^{-5}	Primary suspension stiffness ($MN\cdot m^{-1}$)	1.1
Fasteners vertical/ lateral stiffness ($MN\cdot m^{-1}$)	30/ 50	Primary suspension damping ($kN\cdot s\cdot m^{-1}$)	10
Fasteners vertical/ lateral damping ($kN\cdot s\cdot m^{-1}$)	30/ 50	Secondary suspension stiffness ($MN\cdot m^{-1}$)	0.24
Fastener node spacing (m)	0.63	Secondary suspension damping ($kN\cdot s\cdot m^{-1}$)	20
Slab/ filling layer/ baseplate density ($kg\cdot m^{-3}$)	2450/ 2350/ 2350	Wheelbase (m)	2.5
Slab/ filling layer/ baseplate modulus (GPa)	42.5/ 40/ 38	Range between adjacent bogies wheels (m)	5
Stiffness per unit area of geotextile/ cushion/ vibration-damping layer ($MPa\cdot m^{-1}$)	900/100/25	Length between bogie centers (m)	17.5
Slab/ filling layer/ baseplate height (m)	0.21/ 0.1/ 0.2	Vehicle speed ($km\cdot h^{-1}$)	300

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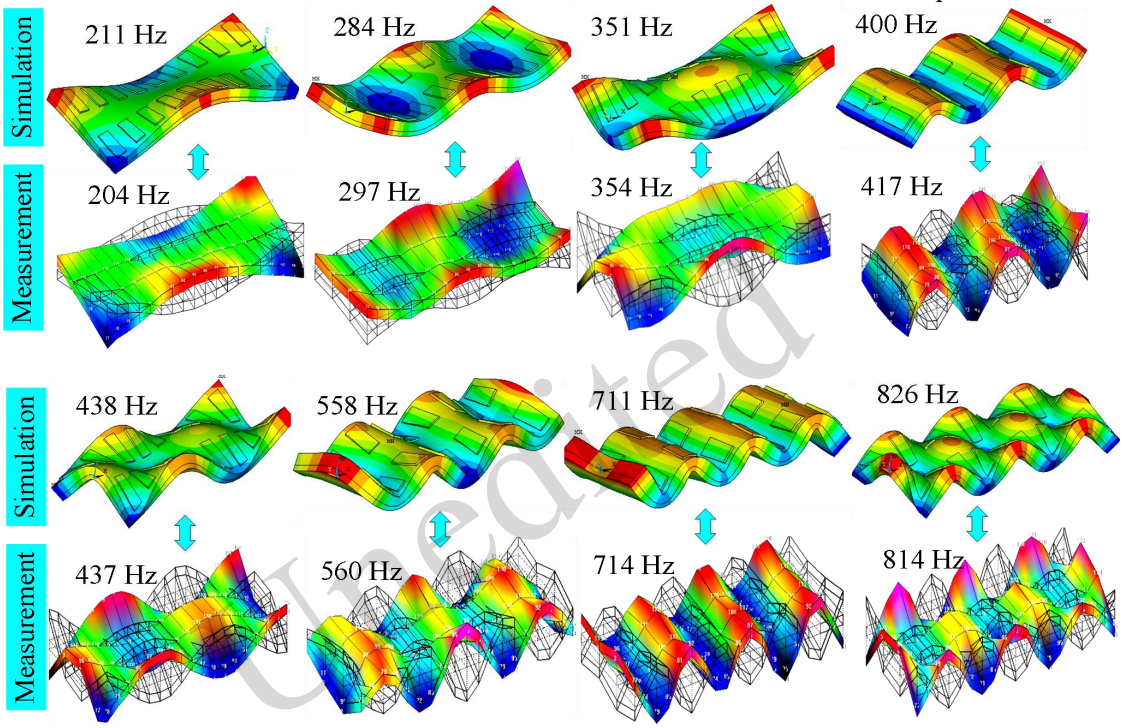
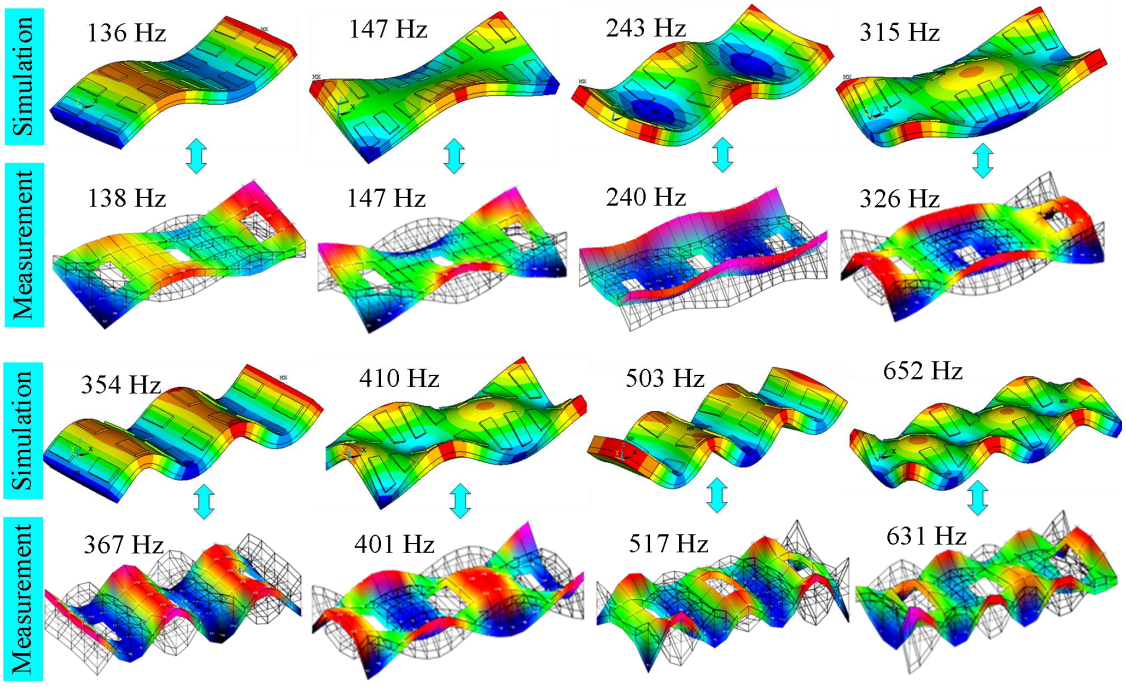
Slab/ filling layer/ baseplate width (m)	2.5/ 2.5/2.9	Wheel-rail adhesion/ friction coefficient	0.04/ 0.17

Results of simulation and measurement comparisons for various track bed modes are shown in Fig. 5. The results verify the reliability of the track-bed vibration model, as the maximum error between the simulated and measured modal frequency was less than 5% for the same modal shape. Comparing Fig. 5a and Fig. 5b, it is evident that the modal frequency of the isolating layer with geotextile was higher than that of the isolating layer with a cushion for the same modal shape.



(a) Isolating layer with geotextile.



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(b) Isolating layer with a cushion.

Fig. 5 Results of simulation and measurement comparison for the track-bed multimode with different isolating layers.

2.2 Vehicle-ballastless track dynamic interaction model considering bed broadband vibrations

A vehicle-ballastless track dynamic interaction model considering bed broadband vibrations was established based on Zhai's vehicle-track coupled dynamics theory (Zhai et al., 2009; Ling et al., 2014) (Fig. 6). Based on the substructure method, the model is composed of a double-vehicle subsystem, rail subsystem, fastener subsystem and track bed broadband-vibration subsystem. A wheel-rail nonlinear contact model is used as the link between the vehicle and rail.

As an infinite-strip periodic structure, the track generates strong vibrations within a finite length. For the track outside this range, the dynamic response is negligible. To improve efficiency within acceptable accuracy limits, a sliding window model (Song et al., 2018) was set up along the moving direction of the train. Only the track within the window was considered for computation. The sliding window can effectively reduce the high computational cost required to simulate the broadband dynamics of track beds.

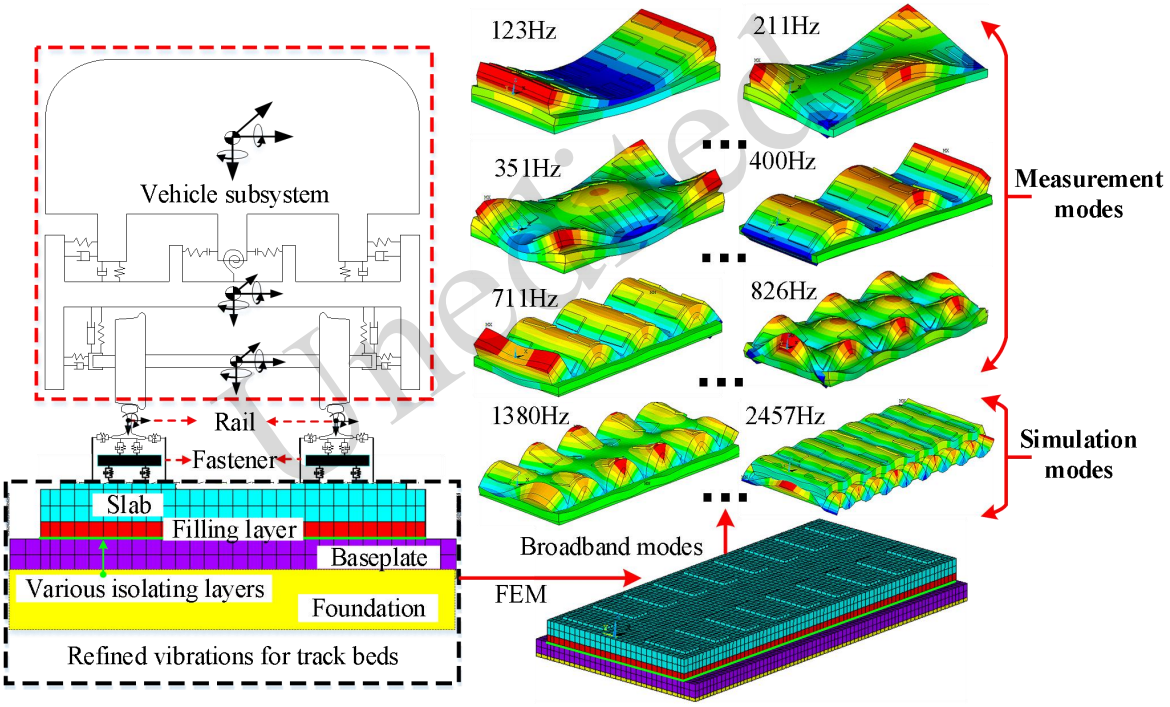

Fig. 6 Vehicle-ballastless track dynamic interaction model with refined consideration of track-bed broadband vibrations.

2.2.1 Dynamic model for track beds

The modal superposition method was used to solve the vibration behavior of track beds. The track-bed modal matrix $\boldsymbol{\Phi}_s$ consists of two parts: the modes $\boldsymbol{\Phi}_s^{\text{mea}}$ below 900 Hz are verified from measurements, while modes $\boldsymbol{\Phi}_s^{\text{sim}}$ above 900 Hz are obtained using the refined finite element model. The track-bed governing equation is as follows:

$$\boldsymbol{M}_{ss}\boldsymbol{a}_s + \boldsymbol{C}_{ss}\boldsymbol{v}_s + \boldsymbol{K}_{ss}\boldsymbol{u}_s = \boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{F}_{sf} \tag{1}$$

$$\boldsymbol{d}_s = \sum_{i=1}^{Ns}\boldsymbol{\phi}_s^i u_s^i = \boldsymbol{\Phi}_s\boldsymbol{u}_s;\quad \boldsymbol{\Phi}_s = \left[\boldsymbol{\Phi}_s^{\text{mea}}\ \boldsymbol{\Phi}_s^{\text{sim}}\right] \tag{2}$$

with

$$\boldsymbol{M}_{ss}^{-1}\boldsymbol{K}_{ss} = diag\left[\omega_1^2\ \omega_2^2\ \ldots\ \omega_s^2\right];\boldsymbol{C}_{ss} = \alpha\boldsymbol{M}_{ss} + \beta\boldsymbol{K}_{ss} \tag{3}$$

where $\boldsymbol{M}_{ss}$, $\boldsymbol{C}_{ss}$ and $\boldsymbol{K}_{ss}$ are the generalized mass, damping and stiffness matrices, respectively, of the track beds; α and β are Rayleigh damping parameters of track beds; $\boldsymbol{a}_s$, $\boldsymbol{v}_s$ and $\boldsymbol{u}_s$ are the generalized acceleration, velocity and displacement vectors; $\boldsymbol{d}_s$ is the actual displacement vector; $\boldsymbol{F}_{sf}$ is

the interaction force vector between the fastener and track beds; ω is the modal frequency.

A Boolean matrix $\boldsymbol{B}_{fs}$ is used to select and arrange the track-bed nodes connected with fasteners, and the dimensions of the generalized load matrix $\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{F}_{sf}$ are unified:

$$\begin{aligned}\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{F}_{sf} &= \boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{K}_{sf}\left(\boldsymbol{u}_f-\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\boldsymbol{u}_s\right)+\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{C}_{sf}\left(\boldsymbol{v}_f-\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\boldsymbol{v}_s\right)\\ &\boldsymbol{M}_{ss}\boldsymbol{a}_s+\left(\boldsymbol{C}_{ss}+\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{C}_{sf}\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\right)\boldsymbol{v}_s+\left(\boldsymbol{K}_{ss}+\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{K}_{sf}\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\right)\boldsymbol{u}_s-\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{C}_{sf}\boldsymbol{v}_f-\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{K}_{sf}\boldsymbol{u}_f=\boldsymbol{0}\end{aligned} \tag{5}$$

2.2.2 Dynamic model for the vehicle-rail system

For the vehicle system, a 3-D multi-body model comprising a two-section vehicle was established. The interactions among vehicles are simulated by linear or nonlinear spring-damper elements based on real constitutive relations. The vehicle dynamic equation can be described as follows:

$$\begin{bmatrix}\boldsymbol{M}_{vv} & \\ & \boldsymbol{M}_{vv}\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_{v1}\\ \boldsymbol{a}_{v2}\end{bmatrix}+\begin{bmatrix}\boldsymbol{C}_{vv} & \boldsymbol{C}_{fr}\\ \boldsymbol{C}_{rf} & \boldsymbol{C}_{vv}\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_{v1}\\ \boldsymbol{v}_{v2}\end{bmatrix}+\begin{bmatrix}\boldsymbol{K}_{vv} & \boldsymbol{K}_{fr}\\ \boldsymbol{K}_{rf} & \boldsymbol{K}_{vv}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}_{v1}\\ \boldsymbol{u}_{v2}\end{bmatrix}=\begin{bmatrix}\boldsymbol{Q}_{wr1}\\ \boldsymbol{Q}_{wr2}\end{bmatrix}+\begin{bmatrix}\boldsymbol{Q}_{inv1}\\ \boldsymbol{Q}_{inv2}\end{bmatrix} \tag{6}$$

where, $\boldsymbol{M}_{vv}$, $\boldsymbol{C}_{vv}$ and $\boldsymbol{K}_{vv}$ are the mass, damping and stiffness matrices, respectively, of a single-segment vehicle; $\boldsymbol{C}_{fr}$ and $\boldsymbol{K}_{fr}$ are the damping and stiffness matrices, respectively, provided for the coupler buffer; $\boldsymbol{a}_v$, $\boldsymbol{v}_v$ and $\boldsymbol{u}_v$ are the vehicle acceleration, velocity and displacement vectors; $\boldsymbol{Q}_{wr1}$ and $\boldsymbol{Q}_{wr2}$ are the wheel-rail nonlinear contact forces including the normal and creep forces; $\boldsymbol{Q}_{inv}$ is the vehicle gravity.

A Timoshenko beam with periodic discrete support is used to simulate the rail vibrations. The rail dynamic equation is solved using the finite element method. The element displacement vector $\boldsymbol{u}_r^{\mathrm{e}}(x,t)$ is expressed as follows:

$$\boldsymbol{u}_r^{\mathrm{e}}(x,t)=\boldsymbol{N}_r^{\mathrm{e}}(x)\left[\left[\boldsymbol{u}_{ri}^{\mathrm{e}}(t)\right]^{\mathrm{T}}\left[\boldsymbol{u}_{rj}^{\mathrm{e}}(t)\right]^{\mathrm{T}}\right]^{\mathrm{T}} \tag{7}$$

with

$$\begin{cases}\boldsymbol{u}_{ri}^{\mathrm{e}}=\left[u_{rxi},u_{ryi},u_{rzi},\theta_{rxi},\theta_{ryi},\theta_{rzi}\right]^{\mathrm{T}}\\ \boldsymbol{u}_{rj}^{\mathrm{e}}=\left[u_{rxj},u_{ryj},u_{rzj},\theta_{rxj},\theta_{ryj},\theta_{rzj}\right]^{\mathrm{T}}\end{cases} \tag{8}$$

In Eqs. (7) & (8), $\boldsymbol{N}_r^{\mathrm{e}}(x)$ is the interpolation function; $\boldsymbol{u}_{ri}^{\mathrm{e}}$ and $\boldsymbol{u}_{rj}^{\mathrm{e}}$ are nodal displacements at end nodes *i* and *j*, respectively.

(4)

where, $\boldsymbol{u}_f$ and $\boldsymbol{v}_f$ are the displacement and velocity vectors of the fastener iron chair; $\boldsymbol{K}_{sf}$ and $\boldsymbol{C}_{sf}$ are stiffness and damping matrices, respectively, provided by the fastener.

By substituting Eq. (4) into Eq. (1), the following equation is obtained:

The dynamic governing equation of the rail is as follows:

$$\boldsymbol{M}_{rr}\boldsymbol{a}_r+\boldsymbol{C}_{rr}\boldsymbol{v}_r+\boldsymbol{K}_{rr}\boldsymbol{u}_r=\boldsymbol{Q}_{rw}+\boldsymbol{Q}_{rf} \tag{9}$$

where $\boldsymbol{M}_{rr}$, $\boldsymbol{C}_{rr}$ and $\boldsymbol{K}_{rr}$ are the mass, damping and stiffness matrices, respectively, of the rail itself; $\boldsymbol{a}_r$, $\boldsymbol{v}_r$ and $\boldsymbol{u}_r$ are the rail acceleration, velocity and displacement vectors; $\boldsymbol{Q}_{rw}$ and $\boldsymbol{Q}_{rf}$ are the wheel-rail load on the rail and the fastener support load, respectively.

A Boolean matrix $\boldsymbol{B}_{fr}$ is used to select and arrange the rail nodes connected with fasteners, and $\boldsymbol{Q}_{rf}$ can be expressed as follows:

$$\boldsymbol{Q}_{rf}=\boldsymbol{K}_{rf}\left(\boldsymbol{B}_{fr}\boldsymbol{u}_r-\boldsymbol{u}_f\right)+\boldsymbol{C}_{rf}\left(\boldsymbol{B}_{fr}\boldsymbol{v}_r-\boldsymbol{v}_f\right) \tag{10}$$

where $\boldsymbol{K}_{rf}$ and $\boldsymbol{C}_{rf}$ are stiffness and damping matrices, respectively, provided for the fasteners.

By substituting Eq. (10) into Eq. (9), the following equation is obtained:

$$\begin{aligned}&\boldsymbol{M}_{rr}\boldsymbol{a}_r+\left(\boldsymbol{C}_{rr}-\boldsymbol{C}_{rf}\boldsymbol{B}_{fr}\right)\boldsymbol{v}_r+\boldsymbol{C}_{rf}\boldsymbol{v}_f+\\ &\left(\boldsymbol{K}_{rr}-\boldsymbol{K}_{rf}\boldsymbol{B}_{fr}\right)\boldsymbol{u}_r+\boldsymbol{K}_{rf}\boldsymbol{u}_f=\boldsymbol{Q}_{rw}\end{aligned} \tag{11}$$

For the fastener system, the dynamic governing equation was adopted from Ma et al. (2023).

For the wheel-rail contact model, the wheel-rail contact is the core bond between the vehicle and track systems. The normal and tangential contact are solved by the Kik-Piotrowski non-Hertz algorithm (Piotrowski and Kik, 2008) and the Fastsim algorithm (Kalker, 1982), respectively. The excitation sources of the vehicle-track system are the surface roughness of the rail or wheel and the random irregularity of the track. The wheel-rail excitation comprises the measured irregularity of China's high-speed railway lines and the wheel-rail surface roughness (the short-wavelength irregularity). The wheel-rail surface roughness adopts the Sato union roughness spectrum

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(Sato, 1977).

$$Q_{\mathrm{nc}}=\frac{\pi E\delta}{2\left(1-\mu^2\right)}\frac{\int_{y_r}^{y_l}\int_{-x_l}^{x_l}\sqrt{x_l^2(y)-x^2}\,dxdy}{\int_{y_r}^{y_l}\int_{-x_l}^{x_l}\frac{\sqrt{x_l^2(y)-x^2}}{\sqrt{x^2+y^2}}dxdy} \tag{12}$$

$$p_{\mathrm{n}}\left(x,y\right)=\frac{N\sqrt{2R_{\mathrm{w}}\varepsilon\delta}}{\int_{y_r}^{y_l}\int_{-x_l}^{x_l}\sqrt{x_l^2(y)-x^2}\,dxdy}\frac{\sqrt{x_l^2(y)-x^2}}{x_l(0)} \tag{13}$$

$$\delta=\frac{z_{\mathrm{c}}^{\mathrm{wheel}}-z_{\mathrm{c}}^{\mathrm{rail}}-z_0}{\cos\left(\theta+\varphi\right)} \tag{14}$$

In Eqs. (12)-(14), Q_{nc} is the normal contact force; E is the elastic modulus and μ is the Poisson's ratio; δ is the virtual penetration; ε is the correction factor; R_{w} is the wheel rolling radius; x_l, y_l and y_r denote the edges of the interpenetration area; $z_{\mathrm{c}}^{\mathrm{wheel}}$ and $z_{\mathrm{c}}^{\mathrm{rail}}$ are the vertical displacement of the wheel and rail, respectively, at the contact patch; z_0 is the wheel-rail surface roughness; θ is the contact angle and φ is the relative roll angle of the wheel-rail tread.

The governing equations of the vehicle-ballastless track system are obtained by combining the dynamic equations of the vehicle, rail, fastener, and track beds.

$$\begin{bmatrix}\boldsymbol{M}_{tt}&&&\\&\boldsymbol{M}_{rr}&&\\&&\boldsymbol{M}_{ff}&\\&&&\boldsymbol{M}_{ss}\end{bmatrix}\begin{bmatrix}\boldsymbol{a}_t\\\boldsymbol{a}_r\\\boldsymbol{a}_f\\\boldsymbol{a}_s\end{bmatrix}+\begin{bmatrix}\boldsymbol{C}_{tt}&&&\\&\boldsymbol{C}_{rr}-\boldsymbol{C}_{rf}\boldsymbol{B}_{fr}&\boldsymbol{C}_{rf}&\\&\boldsymbol{C}_{fr}&\boldsymbol{C}_{ff}&-\boldsymbol{C}_{fs}\boldsymbol{\Phi}_s\\&-\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{C}_{sf}&\boldsymbol{C}_{ss}+\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{C}_{sf}\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\end{bmatrix}\begin{bmatrix}\boldsymbol{v}_t\\\boldsymbol{v}_r\\\boldsymbol{v}_f\\\boldsymbol{v}_s\end{bmatrix}+$$
$$\begin{bmatrix}\boldsymbol{K}_{tt}&&&\\&\boldsymbol{K}_{rr}-\boldsymbol{K}_{rf}\boldsymbol{B}_{fr}&\boldsymbol{K}_{rf}&\\&\boldsymbol{K}_{fr}&\boldsymbol{K}_{ff}&-\boldsymbol{K}_{fs}\boldsymbol{\Phi}_s\\&-\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{K}_{sf}&\boldsymbol{K}_{ss}+\boldsymbol{\Phi}_s^{\mathrm{T}}\boldsymbol{K}_{sf}\boldsymbol{B}_{fs}\boldsymbol{\Phi}_s\end{bmatrix}\begin{bmatrix}\boldsymbol{u}_t\\\boldsymbol{u}_r\\\boldsymbol{u}_f\\\boldsymbol{u}_s\end{bmatrix}=\begin{bmatrix}\boldsymbol{Q}_{wr}+\boldsymbol{Q}_{int}\\\boldsymbol{Q}_{rw}\\\boldsymbol{0}\\\boldsymbol{0}\end{bmatrix} \tag{15}$$

where $\boldsymbol{M}_{tt}$, $\boldsymbol{C}_{tt}$ and $\boldsymbol{K}_{tt}$ are the mass, damping and stiffness matrices of vehicles, respectively. The other variables are defined as in the other equations.

2.3 Track dynamic response measurement and model validation

The model's reliability across different frequency bands was validated through on-site dynamic response measurements of the track. The rail and track-bed dynamic responses for general ballastless tracks were tested on-site. The mid-span rail acceleration and fastener position track-bed acceleration were obtained through simulation. The proposed vehicle-ballastless track coupled model was validated by comparing simulated and measured data (Figs. 7- 9). Note that low-pass filtering below 1500 Hz was performed on the measured and simulated data.

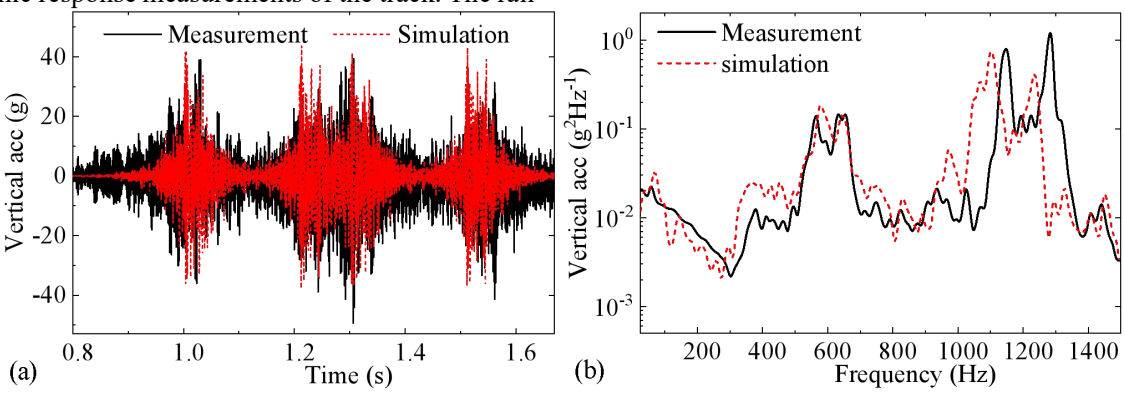

Fig. 7 Simulation and measurement of rail acceleration. (a) Time domain; (b) Frequency domain.

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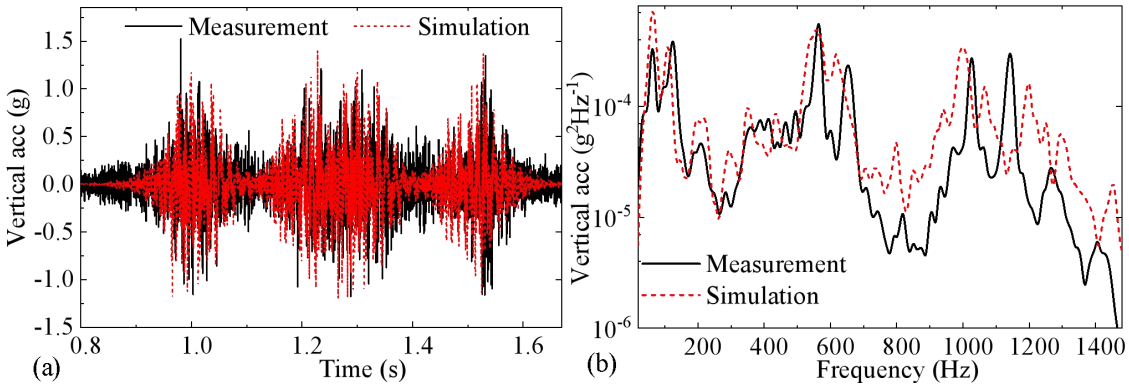

Fig. 8 Simulation and measurement of track bed acceleration. (a) Time domain; (b) Frequency domain.

As shown in Fig. 7 and Fig. 8, the simulated and measured results of rail and track-bed acceleration were consistent in terms of both the time-domain and frequency-domain distributions. From Fig. 7(b), two resonance peaks appear at 550-650 Hz and 1050-1250 Hz for the rail acceleration. Combined with the results of analysis of the wheel-rail dynamic load in Section 3.2, these resonances are caused by the rail 3rd order bending and pinned-pinned vibration modes between bogie wheels, respectively. In Fig. 8(b), the track bed acceleration shows significant resonances at 30-100, 550-650 and 900-1100 Hz. The low-frequency resonance at 30-100 Hz is generated by the P2 resonance mode and track bed low-order modes. The P2 resonance is the equal-amplitude, in-phase vibration mode of the wheel relative to the rail.

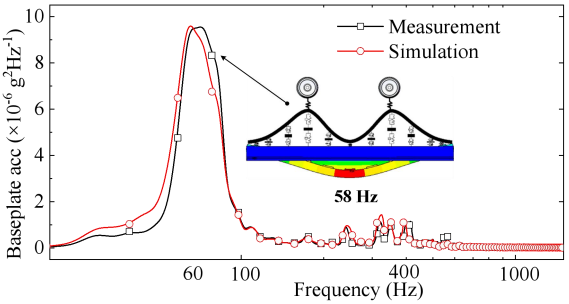

Fig. 9 Simulation and measurement of baseplate acceleration for the vibration-damping track.

To verify the accuracy of the track-bed model with a vibration-damping isolating layer, Fig. 9 shows the simulation and measurement results of the baseplate acceleration for the vibration-damping track. The vehicle operating speed on this track test section is 250 km·h^{-1}. The simulation and measurement results of the baseplate acceleration are in good agreement. In addition, setting a vibration-damping layer causes the baseplate acceleration to resonate near 60 Hz, which is known from Fig. 21 to be caused by the coupled resonance of the wheel-rail P2 resonance mode and the track-bed 1st order bending mode.

In general, the ballastless track model is doubly validated in time and frequency within 1500 Hz based on the rectification of higher order modal measurements and the verification of broadband dynamic response measurements.

3 Simulation results

In this section we first introduce the indicators for evaluating wheel-rail broadband dynamic interactions. Then, taking the CRTS-III ballastless track as an example, the wheel-rail dynamic load and its vibration energy transfer characteristics on track beds are simulated.

3.1 Indicators for evaluating dynamic interaction between wheel-rail and track beds

The dynamic interaction between the wheel-rail and track beds considers the wheel-rail dynamic load and its vibration transfer to the track beds. The vibration transfer between the wheel-rail and track beds is a diffusion process from point (wheel-rail contact point) to surface (track-bed surface), as shown in Fig. 10. To evaluate the vibration energy transfer on the ballastless track, the power flow method (Hussein and Hunt, 2006; Li et al., 2016; Clot et al., 2014) has been widely adopted. Therefore, wheel-rail force and power indicators were used to measure the dynamic interaction between the wheel-rail and track beds.

The impact power E^{t}_{impact} input to the track is

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considered, which can be expressed as the product of the wheel-rail dynamic contact force and the contact-point velocity caused by the track dynamic deformation. Considering that the vehicle is operating on a straight track, the wheel-rail vertical contact force is much greater than the lateral force. The impact power can be expressed as follows:

$$E_{\text{impact}}^{\text{t}} = F_z \frac{du_z^{\text{t}}}{dt} \tag{16}$$

where F_z is the wheel-rail vertical contact force and u_z^{t} is the vertical deflection of the contact point caused by the track dynamic deflection.

The energy transferred to the track beds $E_{\text{impact}}^{\text{s}}$, i.e. the track bed vibration energy, can be expressed as the sum of the product of the fastener force acting on the track beds and the corresponding track bed node-velocity within the wheel load impact range. Owing to the large amplitude variation in the entire frequency range, the vibration level E_{VL}^{s} is used for convenience.

$$E_{\text{impact}}^{\text{s}} = \sum_{i=1}^{Nf} F_i^{\text{s}} \frac{du_{zi}^{\text{s}}}{dt} \tag{17}$$

$$E_{\text{VL}}^{\text{s}} = 10\lg\left(\frac{E_{\text{impact}}^{\text{s}}}{E_0}\right) \tag{18}$$

where F_i^{s} is the fastener force acting on track beds; u_{zi}^{s} is the track-bed node deflection; Nf is the total number of load-bearing fasteners under the wheel load; E_0 is a reference value, taken as 1×10^{-12} W.

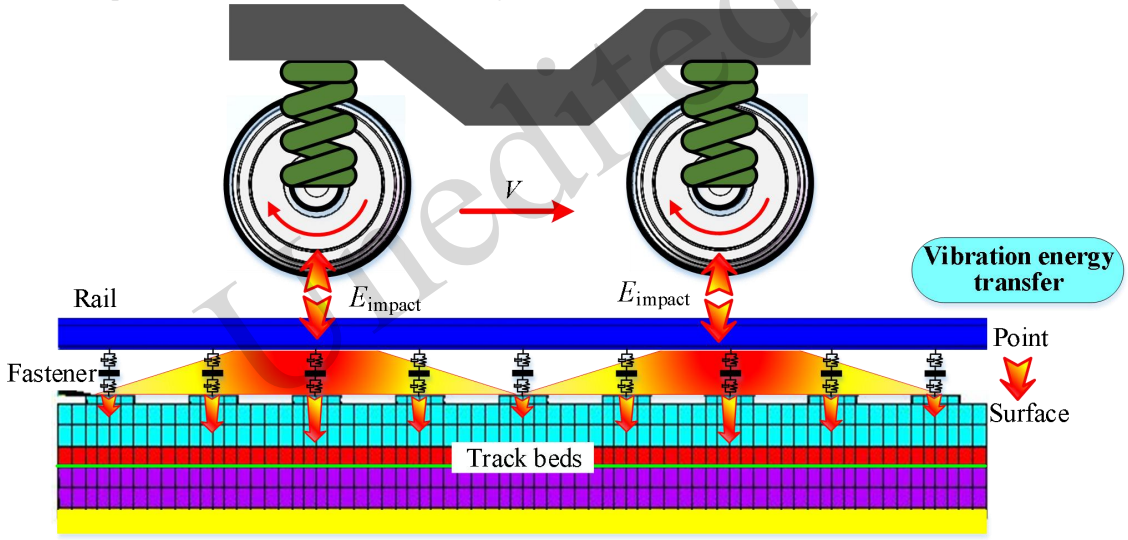


Fig. 10 Schematic diagram of the wheel-rail dynamic load and its vibration energy transfer on the track.

E_{TL} is used to evaluate the degree of energy transmission loss from the wheel-rail to the track beds.

$$E_{\text{TL}} = 10\lg\left(\frac{E_{\text{impact}}^{\text{t}}}{E_0}\right) - 10\lg\left(\frac{E_{\text{impact}}^{\text{s}}}{E_0}\right) = 10\lg\left(\frac{E_{\text{impact}}^{\text{t}}}{E_{\text{impact}}^{\text{s}}}\right) \tag{19}$$

3.2 Wheel-rail broadband dynamic load characteristics

The wheel-rail dynamic load within the 1500 Hz broadband was studied (Fig. 11).

As shown in Fig. 11, there are significant resonances for the wheel-rail dynamic load, in which the impact power resonates around 45, 600 and 1000 Hz, and the vertical force resonates around 45, 360, 600 and 1000 Hz. Furthermore, the 45 Hz resonance belongs to the low frequency range, and its resonance peak is the most significant in the entire frequency range. The other three resonances belong to the high frequency range. In addition, unlike for the vertical force, there is no 2nd resonance for the impact power.

We believe that the P2 resonance and the rail 2nd and 3rd order bending and pinned-pinned vibration modes between the bogie wheels result in wheel–rail coupled resonances around 45, 360, 600 and 1000 Hz, respectively (Ma et al., 2021). The absence of the 2nd resonance for impact power is determined by the rail 2nd order bending modal shape. For this mode, the wheels are located exactly at the wave node of the rail

bending mode, so the rail dynamic deflection and velocity at the wheel-rail contact point will be minimal. As a result, when this mode is excited, an impact power resonating near 360 Hz will not be generated.

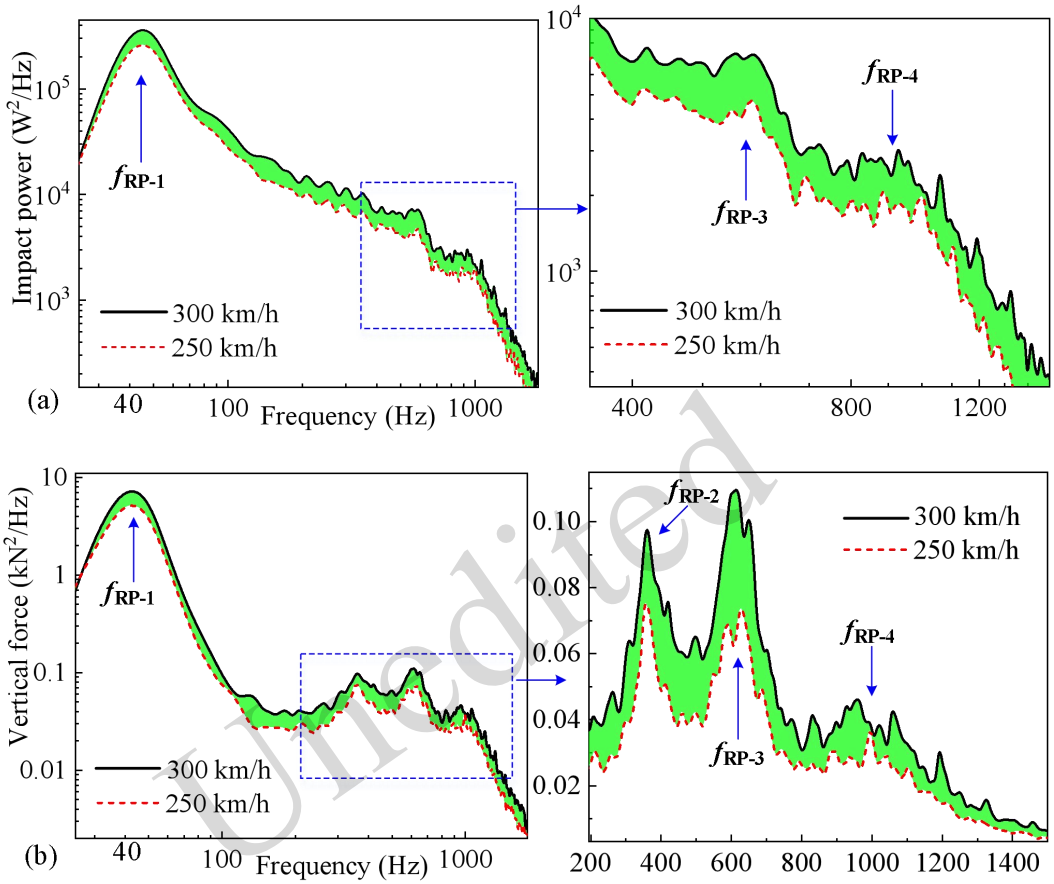
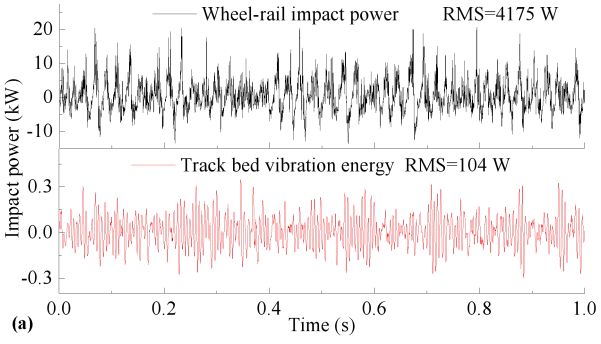


Fig. 11 Wheel-rail dynamic load characteristics. (a) Impact power; (b) Vertical force. f_{RP-i} denotes the i^{th} resonance band.

3.3 Vibration energy transfer characteristics for a ballastless track

The impact power transmitted to track beds, i.e. the track bed vibration energy, is solved, and the vibration energy transfer characteristics from the wheel-rail to track beds are determined, as shown in Fig. 12.



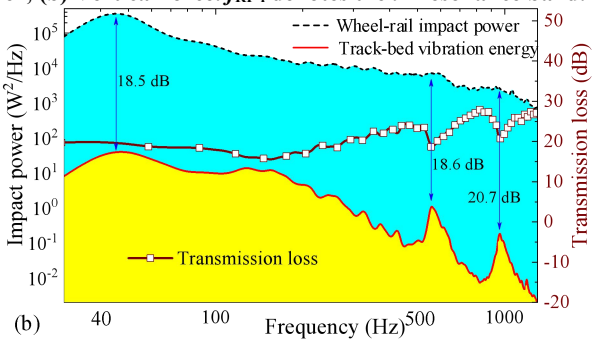


Fig. 12 Vibration energy transfer characteristics from the wheel-rail system to the track beds. (a) Time domain; (b) Frequency domain. RMS is the root mean square value.

Fig. 12(a) shows that the impact power from the wheel-rail system to the track beds decreases from 4175 W to 104 W, and decays to 1/40. The wheel-rail impact energy is 40 times greater than the track-bed vibration energy. The track-bed vibration energy is greatly weakened due to the significant

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vibration-isolating effects of track fasteners.

Fig. 12(b) shows that the impact power transmitted to the track beds resonates significantly at 40-80 Hz, 530-630 Hz, and 850-1000 Hz. Moreover, the energy transmission loss from the wheel-rail system to the track beds is 18.5, 18.6 and 20.7 dB at 45, 580 and 950 Hz, respectively. Overall, the energy transmission loss tends to increase with increasing frequency, indicating that the high-frequency energy is more difficult to transfer from the wheel-rail system to the track beds compared to low-frequency energy. Specifically, when the wheel-rail impact undergoes high-frequency resonances, the track-bed vibration energy exhibits more prominent resonance peaks, while the energy transmission losses exhibit valleys. The wheel-rail impact energy at resonant frequency bands is more prone to transfer to the track beds than that at adjacent non-resonant frequency bands.

4 Discussion

The contribution law of track bed broadband vibrations to the wheel-rail dynamic load and its vibration energy transfer was studied, and the influence mechanism of track-bed broadband vibrations on wheel-rail dynamics was revealed from the perspective of the intermodulation effect. Finally, the rational stiffness of the isolating layer is discussed from the viewpoint of avoiding mode-coupling resonance between subsystems. Unless otherwise specified, the track bed structure in this section is the CRTS-III ballastless track bed.

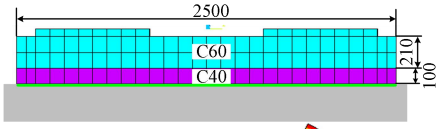
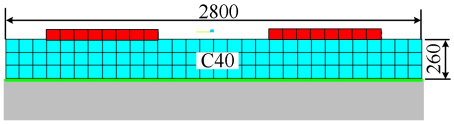
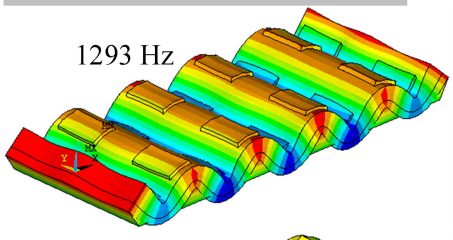
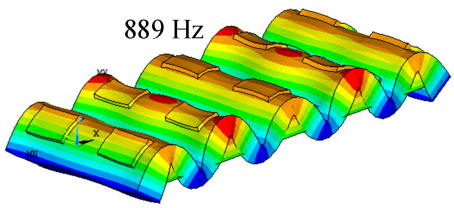
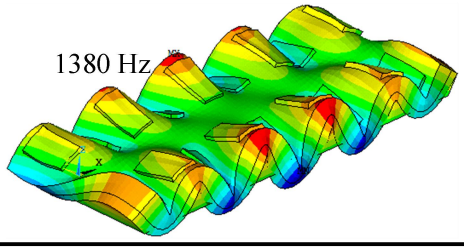
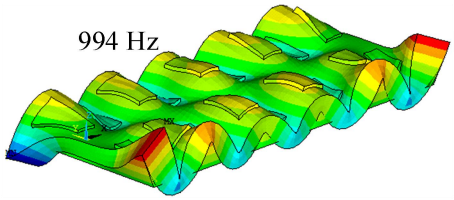
4.1 Influence law of track-bed vibration on wheel-rail dynamic interaction

The variation of the track-bed type and the material of the isolating layer (geotextile/ cushion/ vibration-damping layer) significantly changes the broadband vibration characteristics of track beds. This section focuses on the effects of the track-bed broadband vibrations on the wheel-rail dynamic load and its vibration energy transfer.

4.1.1 Track-bed type

The CRTS-III track slab and double-block track bed are ballastless track structures commonly used for high-speed railways in China. Table 2 lists the typical higher order modal characteristics of these two track structures. The modal parameters of the two track structures differ: the modal frequency of the CRTS-III track slab is significantly higher than that of the double-block track bed under the same modal shape. The wheel-rail dynamic load and its energy transfer under different track-bed structures were simulated by introducing the modal parameters of these two track beds into the vehicle-track dynamic interaction model (Fig. 13 and Fig. 14).

Table 2 Comparison of modal parameters between the CRTS-III and double-block track beds.

Type	CRTS-III track slab	Double-block track bed
Structural parameters		
9th order co-direction bending mode		
9th order reverse bending mode		

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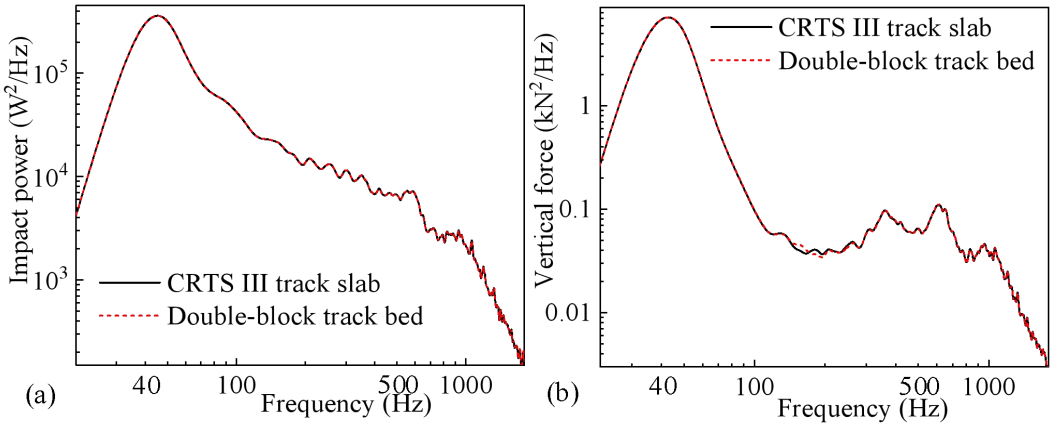

Fig. 13 Influence of the track-bed type on the wheel-rail dynamic load. (a) Impact power; (b) Vertical force.

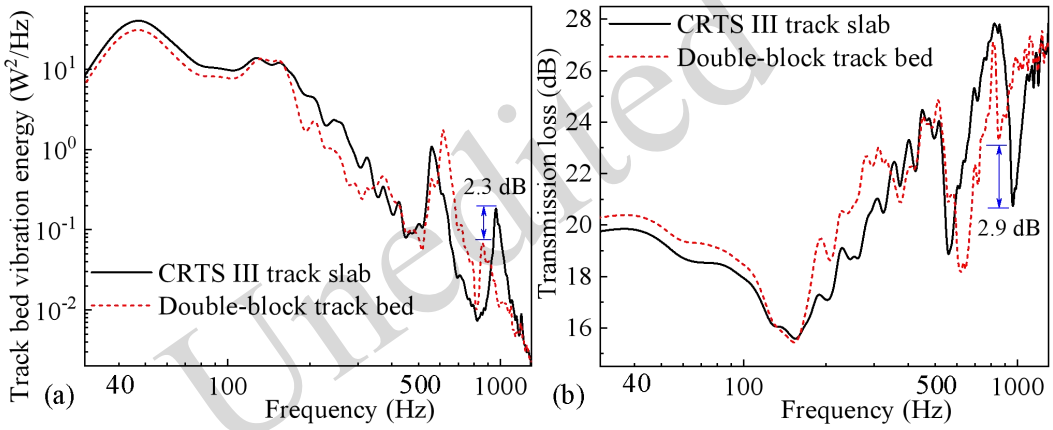

Fig. 14 Influence of the track-bed type on wheel-rail vibration energy transfer. (a) Track bed vibration energy; (b) Energy transmission loss.

Fig. 13 shows that the difference in the wheel-rail dynamic load between CRTS-III and double-block track beds is not significant, even though the modal differences between these two track structures are significant. The double-block track bed has less vibration energy and higher energy transmission losses below 400 Hz than the CRTS-III track slab (Fig. 14). We infer that the vibration energy below 400 Hz is not easily transmitted from the wheel-rail system to the double-block track bed.

In addition, when the two track structures undergo high-frequency resonance, the vibration energy of the double-block track bed is enhanced at 580 Hz and significantly weakened at 950 Hz. When the track resonates at 950 Hz, the rail exhibits pinned-pinned resonance, while the double-block track bed exhibits the 9th order bending resonance (Fig. 15). The rail and the track bed form a coupled resonance with a phase difference of π/2, which is responsible for the significant weakening of the vibration energy. In general, the change in broadband-vibration characteristics of the ballastless track bed has little influence on the wheel-rail dynamic load, but significantly contributes to its vibration energy transfer.

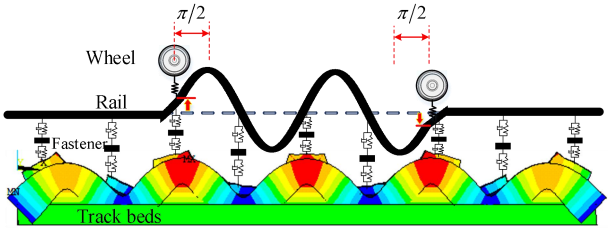

Fig. 15 Coupled resonance of rail pinned-pinned vibration and track-bed 9th order bending mode.

4.1.2 Isolating cushion on a long-span bridge

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To coordinate the deformation and ensure the smoothness of a ballastless track on a long-span bridge, an isolating cushion is laid underneath the track bed. The stiffness of the isolating cushion is 100 MPa·m^{-1}. Laying isolating cushions changes the broadband vibration characteristics of the track bed itself. The influence of laying isolating cushions on the wheel-rail dynamic load and its vibration energy transfer is shown in Fig. 16 and Fig. 17.

Fig. 16 shows that laying isolating cushions weakens the wheel-rail P2 resonance energy at 40-50 Hz but has little effect on the high-frequency wheel-rail interaction above 200 Hz. Fig. 17 shows that laying isolating cushions influences the energy transfer from the wheel-rail system to the track beds across the entire frequency band, and the contribution is more significant below 200 Hz. Specifically, laying isolating cushions aggravates the track-bed vibrations by about 10 dB, reduces the energy transmission loss significantly, and makes the high-frequency energy distribution of the track beds more uniform. We conclude that laying isolating cushions slightly attenuates the wheel-rail impacts and significantly aggravates the track-bed vibrations.

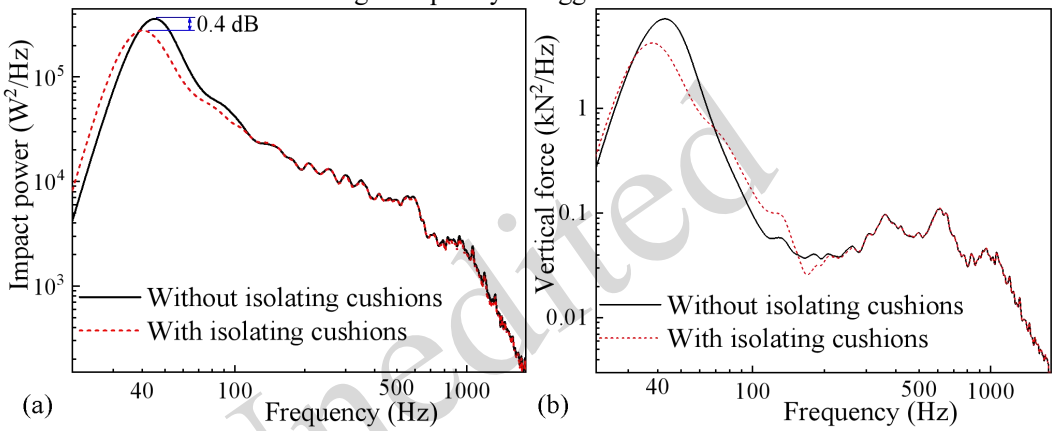

Fig. 16 Influence of laying isolating cushions on the wheel-rail dynamic load. (a) Impact power; (b) Vertical force.

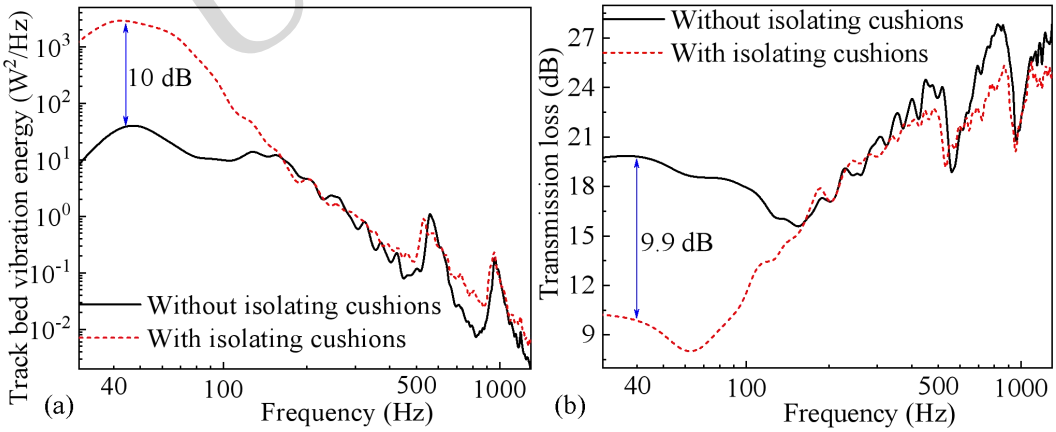

Fig. 17 Influence of laying isolating cushions on the wheel-rail vibration energy transfer. (a) Track bed vibration energy; (b) Energy transmission loss.

4.1.3 Vibration-damping layer for environmental protection areas

A vibration-damping ballastless track is often installed in environmental protection areas, i.e., the vibration-damping layer is laid under the track beds to form floating track structures. The stiffness of the vibration-damping layer is 25 MPa·m^{-1}. The effect of the vibration-damping track on the wheel-rail dynamic load and its energy transfer are shown in Fig. 18 and Fig. 19, respectively.

Fig. 18 shows that the vibration-damping layer significantly reduces the wheel-rail impacts below 200 Hz, whereas it has limited effect on the high-frequency wheel-rail interactions. The laying of a vibration-damping layer significantly weakens the

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P2 resonance energy. Moreover, the vibration-damping layer makes the wheel-rail impacts resonate near 60 Hz.

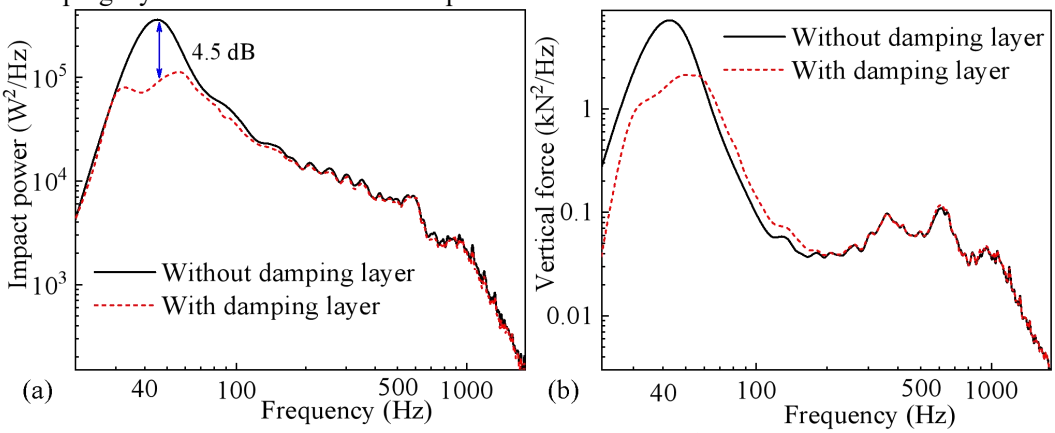

Fig. 18 Influence of the vibration-damping layer on the wheel-rail dynamic load. (a) Impact power; (b) Vertical force.

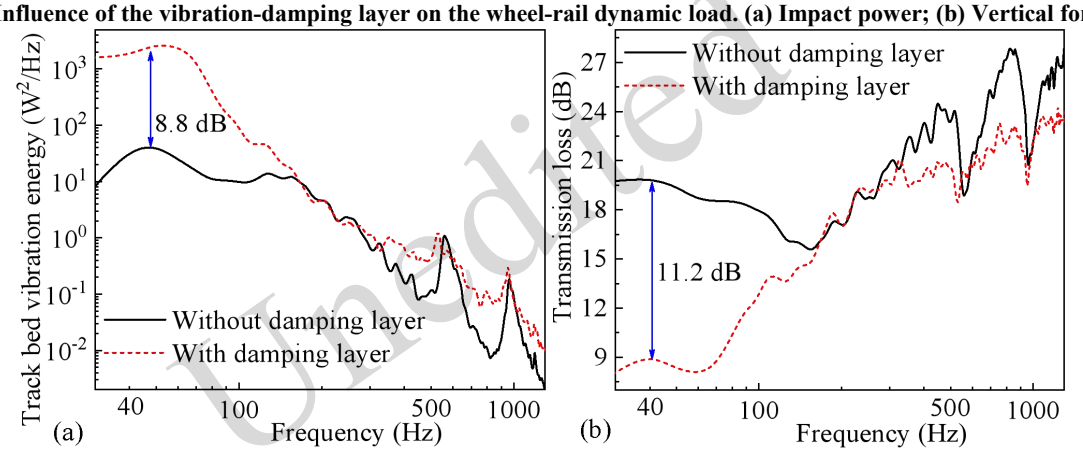

Fig. 19 Influence of the vibration-damping layer on wheel-rail vibration energy transfer. (a) Track bed vibration energy; (b) Energy transmission loss.

Fig. 19 shows that the vibration-damping layer influences energy transfer from the wheel-rail system to the track beds across the whole frequency band, and the contribution is more significant below 200 Hz. Specifically, the introduction of the vibration-damping layer increases track-bed vibrations by nearly 8.8 dB and reduces energy transmission loss by nearly 11.2 dB, and makes the high-frequency energy distribution of the track beds more uniform. We conclude that laying the vibration-damping layer significantly attenuates wheel-rail impacts and exacerbates track-bed vibration.

4.2 Influence mechanism of track-bed vibrations on wheel-rail dynamic interaction

From the wheel-rail dynamic load characteristics under various isolating layer stiffnesses, we conclude that the track-bed broadband vibrations significantly contribute to low-frequency but not high-frequency wheel-rail dynamic load. This section discusses the causes of this phenomenon.

4.2.1 Intermodulation effects between the track beds and wheel-rail coupled subsystem

To analyze the influence mechanism of the track bed vibrations on wheel-rail dynamic load, a modal analysis model for coupling the track beds and wheel-rail coupled subsystem was established by taking the vibration-damping ballastless track as an example (Fig. 20). The interaction between the wheel-rail coupled subsystem and track beds is realized by the fasteners. The intermodulation effects between the track beds and wheel-rail coupled subsystem are shown in Fig. 21 and Fig. 22.

P2 resonance is a key mode affecting the wheel-rail dynamic load. In Fig. 21, compared with the P2 resonance mode (45 Hz) without the influence

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of track beds, the P2 resonance mode with the influence of track beds is resolved into two vibration modes: in-phase coupling (25 Hz) and inverse coupling (58 Hz) with the track-bed bending vibration. The P2 resonance modal frequency and shape change significantly when considering track bed vibrations, indicating that the wheel-rail coupled vibration modes in the low-frequency range are susceptible to track beds.

In Fig. 22, for a certain track bed mode, the modal frequency with effects of the wheel-rail system compared with that without those effects differs significantly at low frequencies. The difference between the two cases decreases to a negligible level as the modal frequency increases to 200 Hz. This indicates that the track bed modes at high frequencies are less affected by the wheel-rail coupled subsystem.

In summary, the modes of the track bed and wheel-rail coupled subsystem are mutually affected in the low-frequency range, but no mutual effects are observed at high frequencies. Hence, we conclude that the track bed modes and the wheel-rail coupled vibration modes exert intense low-frequency and weak high-frequency intermodulation effects.

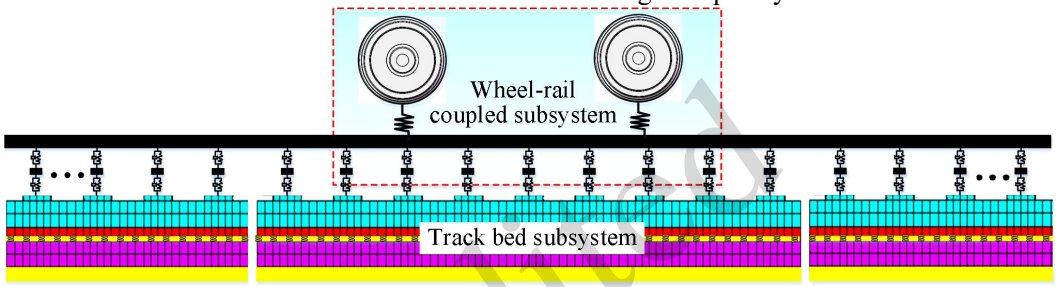

Fig. 20 Mode-coupling model between the track beds and the wheel-rail coupled subsystem.

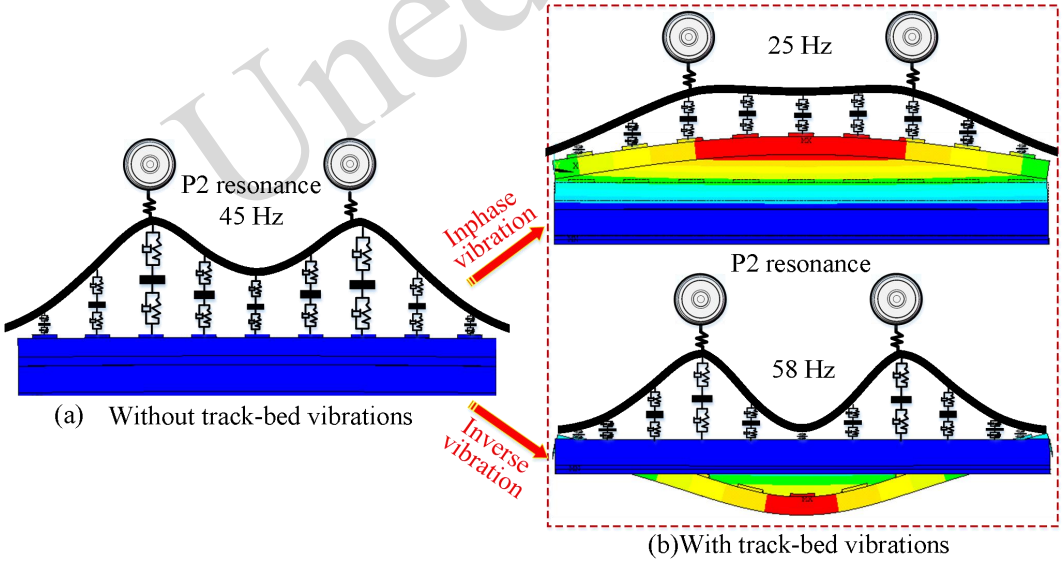

Fig. 21 The wheel-rail P2 resonance mode: (a) Without effects of track beds or (b) With effects of track beds.

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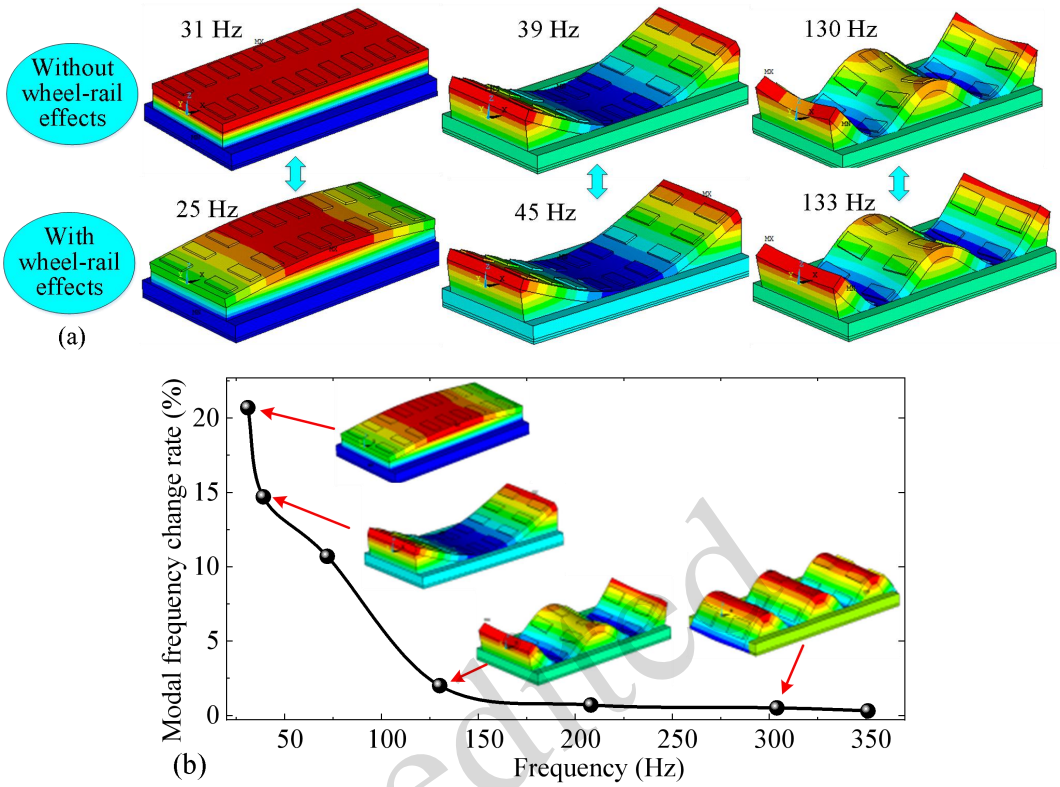
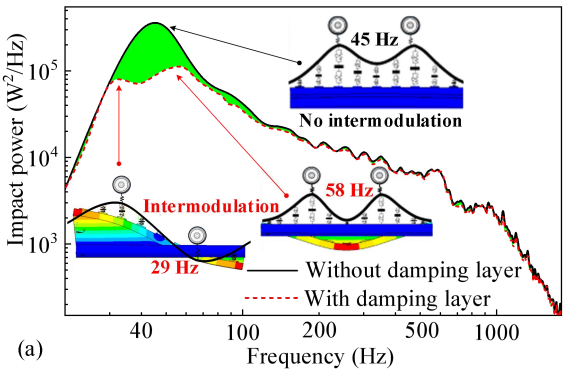


Fig. 22 The track bed modes with effects of the wheel-rail system. (a) Differences in track bed modal shapes with and without wheel-rail effects; (b) Difference of track bed modal frequencies with and without wheel-rail effects.

4.2.2 Intermodulation effects between the track beds and wheel-rail coupled subsystem

As an example of a vehicle and vibration-damping track, Fig. 23 illustrates the influence of intermodulation effects on the wheel-rail dynamic load. The coupling resonance of the track-bed bending mode with the wheel-rail P2 resonance mode alters the wheel-rail P2 resonance response without mode coupling, causing the wheel-rail dynamic load to exhibit a new resonance at 58 Hz.



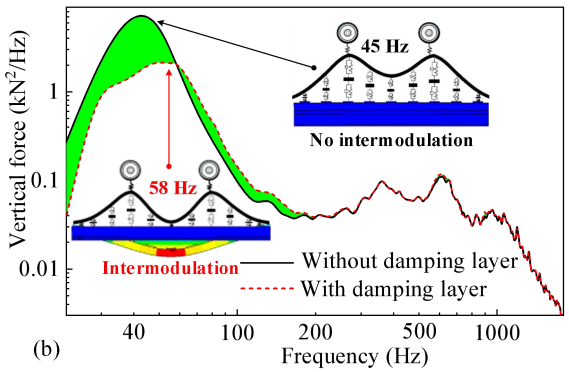


Fig. 23 Influence of intermodulation effects on wheel-rail dynamic load. (a) Impact power; (b) Vertical force.

More generally, the frequency range where the track beds and wheel-rail coupled subsystem exert intense intermodulation effects (Fig. 21 and Fig. 22) is the same as that where the track bed vibrations significantly affect the wheel-rail dynamic load (Fig. 23). We infer that, whether the track beds and wheel-rail coupled subsystem produce the intense intermodulation effects is the prerequisite to whether the track bed vibrations change the wheel-rail dynamic load. If the intermodulation effects are

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intense, the track bed vibrations significantly alter the wheel-rail dynamic load, but if they are weak, the track bed vibrations exhibit a moderate effect on the wheel-rail dynamic load.

Regarding the ballastless tracks in China, although the broadband vibration characteristics of track beds are affected by the track-bed type and the stiffness of isolating layer, intense intermodulation effects of the track beds and wheel-rail coupled subsystem occur mainly below 200 Hz. Consequently, the track bed vibrations make significant contributions to the low-frequency wheel-rail dynamic load below 200 Hz, but exert only a minor effect on the high-frequency wheel-rail dynamic load.

4.3 Rational stiffness of isolating layer based on avoiding mode-coupling resonance

The isolating layer stiffness is an important parameter that affects the vibration characteristics of the track beds. To derive the rational stiffness value of the isolating layer, we analyzed the contribution law of the variation of the isolating layer stiffness on the wheel-rail dynamic load and its vibration energy transfer (Fig. 24).

Fig. 24 shows that with the decrease in isolating layer stiffness, the track-bed vibration energy increases slowly and then rapidly, reaching the peak value at 60 MPa·m^{-1}. Initially, the wheel-rail impact energy tends to show a gradual reduction but later it decreases rapidly. When the isolating layer stiffness is reduced from 1000 MPa·m^{-1} to 60 MPa·m^{-1}, the wheel-rail impact energy is weakened by only 18%, while the track-bed vibration energy is dramatically increased by 9.4 times. This suggests that variations in the isolating layer stiffness have more significant effects on the track-bed vibration energy than on the wheel-rail impact energy.

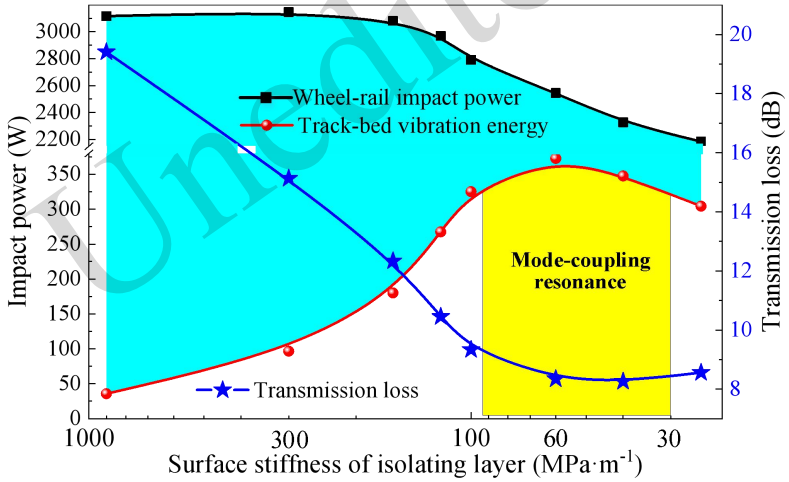

Fig. 24 Contribution of isolating layer stiffness to wheel-rail dynamic load and its vibration energy transfer.

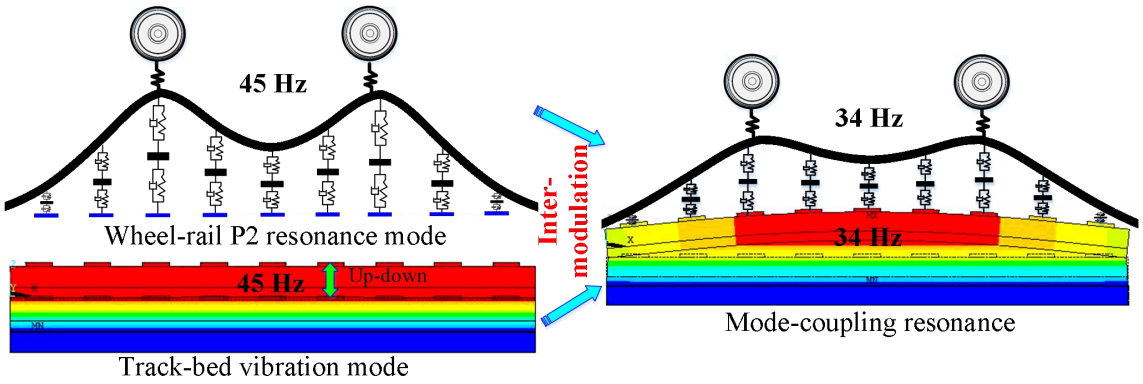

Fig. 25 Mode-coupling resonance between the track beds and wheel-rail coupled subsystem.

We infer that when the isolating layer stiffness is lower than 90 MPa·m^{-1}, the wheel-rail system and

track beds produce intense intermodulation effects. Fig. 25 shows that when the isolating layer stiffness is 60 MPa·m^{-1}, the vibration frequency of the track beds is 45 Hz, which is equal to the wheel-rail P2 resonance modal frequency, and the intermodulation effects of the two are the most intense. At this time, the mode-coupling resonance from the track beds to the wheel-rail coupled subsystem is formed and the track-bed vibration energy is severe, which may accelerate structural damage to the track. This suggests that the isolating layer stiffness should be selected to avoid the mode-coupling resonance between wheel-rail system and track beds. For the CRTS-III ballastless track, an isolating layer stiffness value of between 30 and 90 MPa·m^{-1} should be avoided.

5 Conclusions

In this study, we experimentally identified the higher-order modal parameters of track beds and established a vehicle-ballastless track coupled dynamic model that more accurately considers track-bed broadband vibrations. We simulated the wheel-rail dynamic load and its vibration-energy transfer characteristics, investigated the influence of track-bed broadband vibrations on the wheel-rail dynamic load and its vibration energy transfer, and drew the following main conclusions:

(1) There are three significant resonances for the wheel-rail impact and track-bed vibration energy, with the wheel-rail impact energy resonating around 45, 600 and 1000 Hz, and the track-bed vibration energy resonating at 40-80, 530-630 and 850-1000 Hz. The high-frequency resonance peaks of track-bed vibration energy are more prominent than those of the wheel-rail impact energy.

(2) The major contribution by track-bed broadband vibrations is to a low-frequency wheel-rail load, and markedly influences its vibration energy transfer across various frequency bands. Installing an isolating cushion and vibration-damping layer significantly enhances the low-frequency vibration energy of the track beds and makes its high-frequency energy distribution more uniform.

(3) The intense intermodulation effects of the track beds and wheel-rail coupled subsystem are the prerequisite for track bed vibrations changing the wheel-rail dynamic load. Intense low-frequency and weak high-frequency intermodulation effects between the track beds and wheel-rail coupled subsystem were found. The influence of track-bed vibrations can be disregarded when focusing only on high-frequency wheel-rail dynamic loads above 200 Hz.

(4) The more intense the intermodulation effects and mode-coupling resonance of the wheel-rail system and track beds, the more severe the track-bed vibration energy. The isolating layer stiffness should be selected to avoid the mode-coupling resonance between the wheel-rail system and track beds. For the CRTS-III ballastless track, an isolating layer stiffness value of between 30 and 90 MPa·m^{-1} should be avoided.

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Author contributions

Chaozhi MA designed the research. Chaozhi MA, Pu WANG and Bolun AN processed the corresponding data. Chaozhi MA wrote the first draft of the manuscript. Liang GAO and Peng ZHOU helped to organize the manuscript. Pu WANG and Mahantesh M NADAKATTI revised and edited the final version.

Conflict of interest

Chaozhi MA, Liang GAO, Pu WANG, Bolun AN, Peng ZHOU, and Mahantesh M NADAKATTI declare that they have no conflict of interest.

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中文概要

题　目：无砟轨道振动特性精细化辨识及其对轮轨宽频动

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态相互作用的影响机理

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目　的：明确轮轨动载及其在无砟轨道中传递规律对提升高铁轨道动力学性能具有重要意义。本文旨在探讨无砟道床振动特性对轮轨动载及其振动能量传递的贡献规律，阐明无砟道床参振对轮轨宽频动态相互作用的影响机理。

创新点：1. 首次通过现场试验辨识了铺设不同隔离层（土工布、缓冲垫层等）下无砟道床的高阶模态参数；2. 构建了精细考虑无砟轨道宽频振动特性的车辆-轨道动态相互作用分析模型。3. 从模态互调的角度阐明了无砟道床参振对轮轨宽频相互作用的贡献规律和影响机理。

方　法：1. 通过布设密集的激励点和多个非对称排列的响应点，结合每个响应点相干函数的实时评估及优化，辨识出无砟道床的高阶模态参数（图 4 和图 5）；2. 融合试验辨识模态和理论仿真模态，运用模态叠加法精细构建了车辆-无砟轨道相互作用模型（图 6）；3. 基于无砟道床和轮轨子系统各自模态分析以及耦合下模态互调分析，阐明无砟道床参振对轮轨宽频相互作用的影响机理（图 21、图 22 和图 23）。

结　论：1. 无砟道床参振仅对轮轨低频动载有显著贡献，对无砟轨道全频段振动能量传递规律均有影响；2.无砟道床和轮轨子系统间分别在低频和高频存在强烈和微弱的模态互调效应，当聚焦 200Hz 以上轮轨高频动态相互作用分析时可忽略无砟道床参振的影响；3. 隔离层合理刚度的选取应避免无砟道床与轮轨子系统间的模态耦合共振，以减弱无砟轨道的振动能量。

关键词：无砟轨道；高阶模态辨识；轮轨动载；振动能量传递；互调效应；合理刚度

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