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Stability and agility: biped running over varied and unknown terrain*

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Abstract: We tackle the problem of a biped running over varied and unknown terrain. Running is a necessary skill for a biped moving fast, but it increases the challenge of dynamic balance, especially when a biped is running on varied terrain without terrain information (due to the difficulty and cost of obtaining the terrain information in a timely manner). To address this issue, a new dynamic indicator called the sustainable running criterion is developed. The main idea is to sustain a running motion without falling by maintaining the system states within a running-feasible set, instead of running on a periodic limit cycle gait in the traditional way. To meet the precondition of the criterion, the angular moment about the center of gravity (COG) is restrained close to zero at the end of the stance phase. Then to ensure a small state jump at touchdown on the unknown terrain, the velocity of the swing foot is restrained within a specific range at the end of the flight phase. Finally, the position and velocity of the COG are driven into the running-feasible set. A five-link biped with underactuated point foot is considered in simulations. It is able to run over upward and downward terrain with a height difference of 0.15 m, which shows the effectiveness of our control scheme.

Key words: Underactuated running biped, Dynamic balance, Varied and unknown terrain
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1 Introduction

Human beings perform different types of motions such as walking, running, high jumping, hurdling, and the most complicated movements like those of an acrobat. For bipedal robots, their human-like shape with two legs is suitable for performing various tasks and moving across various environments. Reportedly, however, as of now only a few of them can walk around human-centered settings which include varied terrain (Hirukawa *et al.*, 2007; Shimizu *et al.*, 2007; Li and Chen, 2009). The most impressive biped, Asimo, is capable of walking on a terrain constructed by multiple cells, each with side length 0.025 m (Chestnutt *et al.*, 2005). The improved HRP-2 can cope with an uneven surface with a height difference of less than 0.04 m and an incline of 2.86° (Kaneko *et al.*, 2004; Morisawa *et al.*, 2011). For now the steepest slope a biped can climb is 8.5° in simulations (Seven *et al.*, 2011). Compared with human beings these values are still very small. A biped robot adapting to complex environments has not yet been fully explored.

Possible solutions to the problem of adapting to complex environments are classified into the following three categories: (1) Avoid obstacles or uneven parts of the ground through some successful navigation algorithms (Nishiwaki *et al.*, 2012). This class of strategies requires the ground to have certain conditions, and relies on accurate real-time information on the environment. Due to noisy measurement and slow responses of actuators, it is still challenging

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for a biped robot to measure ground conditions in real time and respond rapidly. (2) Explore the mechanical structure of the feet of a biped to absorb the disturbances from the physical environment (Hashimoto *et al.*, 2006; 2007; Sano *et al.*, 2008; Yamada *et al.*, 2010). As new structures always bring complexity to the biped dynamics, it is difficult to use them appropriately and analyze the stability of the whole biped system. (3) Regenerate new motion trajectories according to the changing ground information based on the zero moment point (ZMP) stability requirement (Hirukawa *et al.*, 2007; Nishiwaki and Kagami, 2007; Huang *et al.*, 2008). However, the ZMP stability indicator is a static criterion, which is not suitable either in fast locomotion or for large changes in the ground (Vukobratović and Borovac, 2004).

In the paper we tackle the problem of a point-foot biped running over varied and unknown terrain. Dynamic balance is difficult for a point-foot running biped without actuators on its ankles, while the challenge is greater when the biped is subject to varied terrain without terrain information. Thus, we take the sustainable running criterion proposed in our previous work (Yi *et al.*, 2015) as a dynamic indicator. It defines a running-feasible set by the center of gravity (COG) state, in which a biped is able to sustain running without falling. A non-period running gait can be realized for a biped robot under disturbances, as long as the COG state is within a running-feasible set. In our previous work, the issue of dynamic balance from a running biped subjected to a sudden push was investigated. In the current work, we focus on the problem of maintaining running gaits (possibly varied) in response to unpredictable and large disturbances from varied terrain. The solution to the problem consists of three components: First, the angular moment about the COG (centroid angular moment) is restrained close to zero at the end of the stance phase. It is taken as the precondition of the sustainable running criterion. Second, the velocity of the swing foot is restrained within a certain range when the biped is touching down on the ground from the flight phase. Thus, the unpredictable disturbances are bounded as well at touchdowns, causing small state jumps. Finally, the position and velocity of the COG are controlled to be within a running-feasible set, so that the sustainable running motion is realized. In simulations, the biped runs up and down the terrain with a height difference of about 0.15 m. To our knowledge, this is the first result on underactuated biped running that shows great adaptability to varied and unknown terrain.

2 Running dynamic model

2.1 A five-link planar biped

The running model considered in this study is a planar point-foot biped (Fig. 1). It has five links and four actuators acting on hips and knees, but without actuators on its ankles. The parameters of the biped are summarized in Table 1.

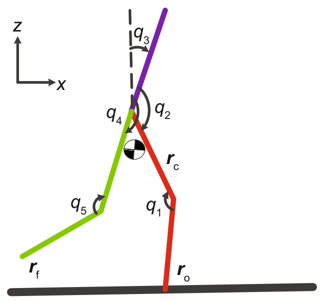

Fig. 1 A five-link planar biped

Table 1 Symbol definition

Symbol	Description	Value
l_b	Length of the body	0.625 m
l_t	Length of the thigh	0.400 m
l_s	Length of the shank	0.400 m
m_b	Mass of the hip	17.0 kg
m_t	Mass of the thigh	6.8 kg
m_s	Mass of the shank	3.2 kg
I_b	Rotational inertia of the hip	1.33 kg·m^2
I_t	Rotational inertia of the thigh	0.47 kg·m^2
I_s	Rotational inertia of the shank	0.20 kg·m^2

Let $\boldsymbol{q}_b = [q_1, q_2, q_3, q_4, q_5]^T$ be the joint angles and $\boldsymbol{\Gamma} = [\tau_1, \tau_2, \tau_3, \tau_4]^T$ the input torques. The absolute position of the COG is specified by the coordinates $\boldsymbol{r}_c = [x_c, z_c]^T$. In addition, we denote the position of the foot that is on the ground by $\boldsymbol{r}_o = [x_o, z_o]^T$ and the position of the other foot by $\boldsymbol{r}_f = [x_f, z_f]^T$. They are expressed as

$$\boldsymbol{r}_c = \boldsymbol{r}_o + \eta_1(\boldsymbol{q}_b), \quad \boldsymbol{r}_f = \boldsymbol{r}_c + \eta_2(\boldsymbol{q}_b), \tag{1}$$

where η_1 and η_2 are the functions of the joint angles describing the shape of the biped.

When the biped stands on the ground, it exerts a downward gross applied force (GAF) to the ground through the contact point. Then an equal upward ground reaction force (GRF) exists on the supporting foot. Denote the GAF by $\boldsymbol{F}_{\mathrm{a}} = [F_{\mathrm{a}x},\ F_{\mathrm{a}z}]^{\mathrm{T}}$, and the GRF by $\boldsymbol{F} = [F_x,\ F_z]^{\mathrm{T}}$. Thus,

$$\boldsymbol{F} = \begin{cases} -\boldsymbol{F}_{\mathrm{a}}, & \text{if } z_{\mathrm{o}} = 0 \text{ and } F_{\mathrm{a}z} < 0, \\ 0, & \text{if } z_{\mathrm{o}} > 0 \ \text{ or } \ F_{\mathrm{a}z} \geq 0. \end{cases} \quad (2)$$

As the GAF is a composite force from the actuators, the GRF can be controlled and expressed by a function of the joint states and torque inputs. Due to the physical constraints on the joint angles, velocities, and the actuators, the GRF has an upper bound denoted by $F^{\max}$. At the same time, it has a unilateral constraint for directing upright and a friction constraint for remaining in the Coulomb cone. Define $\boldsymbol{p} = [\boldsymbol{q}_{\mathrm{b}},\ \boldsymbol{\varGamma},\ \boldsymbol{F}]^{\mathrm{T}}$ and a constraint set C as

$$\begin{aligned} C := \{\boldsymbol{p} | q_i \in [q_i^{\min},\ q_i^{\max}],\ \tau_j \in [\tau_j^{\min},\ \tau_j^{\max}], \\ \|\boldsymbol{F}\| \leq F^{\max},\ |F_x| \leq \mu F_z\}, \\ (i = 1, 2, \cdots, 5,\ j = 1, 2, 3, 4) \end{aligned}$$

where $q_i^{\min}$ and $q_i^{\max}$ represent the minimum and maximum values of q_i respectively, $\tau_j^{\min}$ and $\tau_j^{\max}$ represent the minimum and maximum torque values of the torque input τ_j respectively, and μ is the friction coefficient.

2.2 Dynamic model

The running motion consists of stance and flight phases with instantaneous takeoff and touchdown transitions. Let S_{s} and S_{f} be the sets of valid states and control inputs in the stance phase and flight phase respectively, and let $S_{\mathrm{s}\to\mathrm{f}}$ and $S_{\mathrm{f}\to\mathrm{s}}$ be the sets of valid states at takeoff and touchdown respectively. A visualization of a running step is given in Fig. 2b.

2.2.1 Stance phase

A running biped is in the stance phase when it moves forward with one foot contacting the ground to support the weight of the body. The leg that contacts the ground is called the supporting leg and the other the swing leg (the two legs are distinguished and named in the stance phase, and the names are effective in the whole running step). Let the position of the supporting leg end in stance be the local coordinate origin. As the biped stands still on the ground, there is

$$\boldsymbol{r}_{\mathrm{o}} = [0,\ 0]^{\mathrm{T}}, \quad \dot{\boldsymbol{r}}_{\mathrm{o}} = [0,\ 0]^{\mathrm{T}}. \quad (3)$$

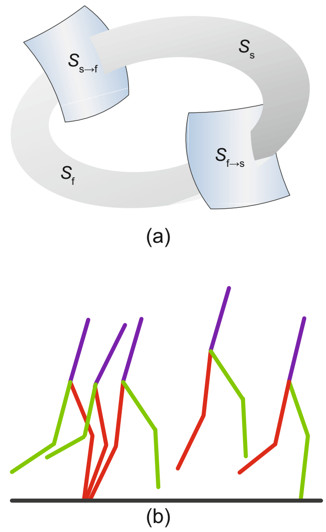

Fig. 2 State sets of a biped running (a) and a visualization of a running step (b)

During biped running the swinging foot should be above the ground and the horizontal velocity should be positive for moving forward. That is, $z_{\mathrm{f}} > 0$ and $\dot{x}_{\mathrm{c}} > 0$. Also, the existence of GRF gives $\|\boldsymbol{F}\| > 0$. So, the state set in stance S_{s} is defined as

$$\begin{aligned} S_{\mathrm{s}} := \{\boldsymbol{p} \in C | \dot{x}_{\mathrm{c}} > 0,\ z_{\mathrm{f}} > 0,\ \|\boldsymbol{F}\| > 0, \\ \boldsymbol{r}_{\mathrm{o}} = [0,\ 0]^{\mathrm{T}},\ \dot{\boldsymbol{r}}_{\mathrm{o}} = [0,\ 0]^{\mathrm{T}}\}. \end{aligned}$$

Denote the vector of the generalized coordinates by $\boldsymbol{q}_{\mathrm{s}} = \boldsymbol{q}_{\mathrm{b}}$. The coordinates of the COG are calculated by substituting Eq. (3) into Eq. (1):

$$\begin{cases} \boldsymbol{r}_{\mathrm{c}} = \eta_1(\boldsymbol{q}_{\mathrm{s}}),\ \dot{\boldsymbol{r}}_{\mathrm{c}} = \nabla\eta_1(\boldsymbol{q}_{\mathrm{s}})\dot{\boldsymbol{q}}_{\mathrm{s}}, \\ \ddot{\boldsymbol{r}}_{\mathrm{c}} = \nabla^2\eta_1(\boldsymbol{q}_{\mathrm{s}})\dot{\boldsymbol{q}}_{\mathrm{s}}^2 + \nabla\eta_1(\boldsymbol{q}_{\mathrm{s}})\ddot{\boldsymbol{q}}_{\mathrm{s}}. \end{cases} \quad (4)$$

Then the GRF is

$$\boldsymbol{F} = M\ddot{\boldsymbol{r}}_{\mathrm{c}} - \boldsymbol{G}, \quad (5)$$

where M is the total mass of the biped and $\boldsymbol{G} = M[0,\ -g]^{\mathrm{T}}$. In addition, the centroid angular moment, which is denoted by $\boldsymbol{H}_{\mathrm{cs}}$ ($\boldsymbol{H}_{\mathrm{cf}}$ in flight), and its derivative, are calculated as

$$\begin{cases} \boldsymbol{H}_{\mathrm{cs}} = \sum_{i=1}^{5} m_i(\boldsymbol{r}_i - \boldsymbol{r}_{\mathrm{c}}) \times (\dot{\boldsymbol{r}}_i - \dot{\boldsymbol{r}}_{\mathrm{c}}), \\ \dot{\boldsymbol{H}}_{\mathrm{cs}} = (\boldsymbol{r}_{\mathrm{o}} - \boldsymbol{r}_{\mathrm{c}}) \times \boldsymbol{F} = -\boldsymbol{r}_{\mathrm{c}} \times \boldsymbol{F}. \end{cases} \quad (6)$$

Here $\boldsymbol{r}_i$ represents the position of the center of mass of the ith link. The Lagrange dynamics is given in the joint space for better understanding the dynamical features. The equation of motion in stance is

$$\boldsymbol{D}_{\mathrm{s}}(\boldsymbol{q}_{\mathrm{s}})\ddot{\boldsymbol{q}}_{\mathrm{s}} + \boldsymbol{C}_{\mathrm{s}}(\boldsymbol{q}_{\mathrm{s}}, \dot{\boldsymbol{q}}_{\mathrm{s}})\dot{\boldsymbol{q}}_{\mathrm{s}} + \boldsymbol{G}_{\mathrm{s}}(\boldsymbol{q}_{\mathrm{s}}) = \boldsymbol{B}_{\mathrm{s}}\boldsymbol{\varGamma}_{\mathrm{s}}, \quad (7)$$

where $\boldsymbol{D}_{\rm s}$ is the inertia matrix, $\boldsymbol{C}_{\rm s}$ is the centrifugal matrix, $\boldsymbol{G}_{\rm s}$ is the gravity term, $\boldsymbol{B}_{\rm s}$ is the effect matrix of the torques, and $\boldsymbol{\Gamma}_{\rm s}$ is the input torque in stance. As $\boldsymbol{\Gamma}_{\rm s}$ is a four-dimensional vector, the biped has one degree of underactuation in stance. Let $\boldsymbol{x}_{\rm s} = [\boldsymbol{q}_{\rm s},\ \dot{\boldsymbol{q}}_{\rm s}]^{\rm T}$ be the system state vector. Then the dynamic equations are written in the standard form as

$$\dot{\boldsymbol{x}}_{\rm s} := f_{\rm s}(\boldsymbol{x}_{\rm s}) + g_{\rm s}(\boldsymbol{x}_{\rm s})\boldsymbol{\Gamma}_{\rm s}, \tag{8}$$

where

$$f_{\rm s} = \begin{bmatrix} \dot{\boldsymbol{q}}_{\rm s} \\ \boldsymbol{D}_{\rm s}^{-1}(-\boldsymbol{C}_{\rm s}\dot{\boldsymbol{q}}_{\rm s} - \boldsymbol{G}_{\rm s}) \end{bmatrix},\ g_{\rm s} = \begin{bmatrix} \boldsymbol{0}_{3\times1} \\ \boldsymbol{D}_{\rm s}^{-1}\boldsymbol{B}_{\rm s} \end{bmatrix}.$$

2.2.2 Flight phase

In flight all parts of the biped are in the air. The set $S_{\rm f}$ is defined as

$$S_{\rm f} := \{\boldsymbol{p} \in C \mid z_{\rm f} > 0,\ z_{\rm o} > 0,\ \|\boldsymbol{F}\| = 0\}.$$

As all parts of the biped move with respect to the world frame, the position of the biped COG is introduced in the vector of generalized coordinates in flight, $\boldsymbol{q}_{\rm f} = [\boldsymbol{q}_{\rm b}, \boldsymbol{r}_{\rm c}]^{\rm T}$. The equation of motion is then given as

$$\boldsymbol{D}_{\rm f}\begin{bmatrix} \ddot{\boldsymbol{q}}_{\rm b} \\ \ddot{\boldsymbol{r}}_{\rm c} \end{bmatrix} + \boldsymbol{C}_{\rm f}\begin{bmatrix} \dot{\boldsymbol{q}}_{\rm b} \\ \dot{\boldsymbol{r}}_{\rm c} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0}_{5\times1} \\ \boldsymbol{G} \end{bmatrix} = \boldsymbol{B}_{\rm f}\begin{bmatrix} \boldsymbol{\Gamma}_{\rm f} \\ \boldsymbol{0}_{2\times1} \end{bmatrix}, \tag{9}$$

where

$$\boldsymbol{D}_{\rm f} = \begin{bmatrix} \boldsymbol{D}_{\rm s} & \boldsymbol{0}_{5\times2} \\ \boldsymbol{0}_{2\times5} & M\boldsymbol{I}_{2\times2} \end{bmatrix},\ \boldsymbol{C}_{\rm f} = \begin{bmatrix} \boldsymbol{C}'_{\rm f} \\ \boldsymbol{0}_{2\times7} \end{bmatrix},\ \boldsymbol{B}_{\rm f} = \begin{bmatrix} \boldsymbol{B}_{\rm s} \\ \boldsymbol{0}_{2\times4} \end{bmatrix}$$

are the inertia matrix, centrifugal matrix, and effect matrix of the torques in flight, respectively. The dynamic equation shows that the biped has three degrees of underactuation in flight. Since the biped is only subject to the unchangeable gravity in flight, the variables of the COG are not controllable. Thus, the horizontal velocity of the biped remains constant, and the vertical velocity of the biped decreases with the gravitational acceleration. As no moment with respect to the COG exists, the centroid angular moment becomes zero. Therefore, we have

$$\ddot{x}_{\rm c} = 0,\quad \ddot{z}_{\rm c} = -g,\quad \dot{\boldsymbol{H}}_{\rm cf} = \boldsymbol{0}. \tag{10}$$

From Eq. (1), the coordinates of the swing foot in flight are calculated as

$$\begin{cases} \boldsymbol{r}_{\rm f} = \boldsymbol{r}_{\rm c} + \eta_2(\boldsymbol{q}_{\rm b}),\ \dot{\boldsymbol{r}}_{\rm f} = \dot{\boldsymbol{r}}_{\rm c} + \nabla\eta_2(\boldsymbol{q}_{\rm b})\dot{\boldsymbol{q}}_{\rm b}, \\ \ddot{\boldsymbol{r}}_{\rm f} = \ddot{\boldsymbol{r}}_{\rm c} + \nabla^2\eta_2(\boldsymbol{q}_{\rm b})\dot{\boldsymbol{q}}_{\rm b}^2 + \nabla\eta_2(\boldsymbol{q}_{\rm b})\ddot{\boldsymbol{q}}_{\rm b}. \end{cases} \tag{11}$$

Finally, let $\boldsymbol{x}_{\rm f} = [\boldsymbol{q}_{\rm f},\ \dot{\boldsymbol{q}}_{\rm f}]^{\rm T}$ be the system state vector in flight and the dynamic equations are written in the standard form as

$$\dot{\boldsymbol{x}}_{\rm f} := f_{\rm f}(\boldsymbol{x}_{\rm f}) + g_{\rm f}(\boldsymbol{x}_{\rm f})\boldsymbol{\Gamma}_{\rm f}, \tag{12}$$

where

$$f_{\rm f} = \begin{bmatrix} \dot{\boldsymbol{q}}_{\rm f} \\ \boldsymbol{D}_{\rm f}^{-1}(-\boldsymbol{C}_{\rm f}\dot{\boldsymbol{q}}_{\rm f} - \boldsymbol{G}_{\rm f}) \end{bmatrix},\ g_{\rm f} = \begin{bmatrix} \boldsymbol{0}_{4\times1} \\ \boldsymbol{D}_{\rm f}^{-1}\boldsymbol{B}_{\rm f} \end{bmatrix}.$$

2.2.3 Takeoff

The transition from the stance phase to the flight phase is defined by an instantaneous takeoff. It occurs when the biped moves upwards and has its supporting foot about to leave the ground. The transition set $S_{\rm s\to f}$ is defined as

$$\begin{aligned} S_{\rm s\to f} :=\{&\boldsymbol{p} \in C \mid \dot{z}_{\rm c} > 0,\ \dot{z}_{\rm o} > 0,\ z_{\rm o} = 0, \\ &z_{\rm f} > 0, \|\boldsymbol{F}\| = 0\}. \end{aligned}$$

Denote the state vectors just before and after a takeoff by $[\boldsymbol{q}_{\rm s}^-,\ \dot{\boldsymbol{q}}_{\rm s}^-]$ and $[\boldsymbol{q}_{\rm f}^+,\ \dot{\boldsymbol{q}}_{\rm f}^+]$, respectively. The generalized coordinates and their velocities do not change at the takeoff transition. Thus, the instantaneous map is written as

$$\begin{bmatrix} \boldsymbol{q}_{\rm f}^+ \\ \dot{\boldsymbol{q}}_{\rm f}^+ \end{bmatrix} = \boldsymbol{\Delta}_{\rm s}^{\rm f} := \begin{bmatrix} \boldsymbol{q}_{\rm s}^- \\ \eta_1(\boldsymbol{q}_{\rm s}^-) \\ \dot{\boldsymbol{q}}_{\rm s}^- \\ \nabla\eta_1(\boldsymbol{q}_{\rm s}^-)\dot{\boldsymbol{q}}_{\rm s}^- \end{bmatrix}. \tag{13}$$

2.2.4 Touchdown

The transition from the flight phase to the stance phase is defined by an instantaneous touchdown. It occurs when the swing foot contacts the ground with a downward speed, since the leg becomes the supporting leg and exerts a GAF downwards for the next running step. The set of the touchdown $S_{\rm f\to s}$ is defined as

$$S_{\rm f\to s} := \{\boldsymbol{p} \in C \mid \dot{z}_{\rm f} < 0,\ z_{\rm f} = 0,\ z_{\rm o} > 0, \|\boldsymbol{F}\| = 0\}.$$

The generalized coordinates change continuously but the joint velocities jump at touchdown. Denote the state vectors just before and after the touchdown by $[\boldsymbol{q}_{\rm f}^-,\ \dot{\boldsymbol{q}}_{\rm f}^-]$ and $[\boldsymbol{q}_{\rm s}^+,\ \dot{\boldsymbol{q}}_{\rm s}^+]$, respectively, and we have

$$\boldsymbol{q}_{\rm s}^+ = \boldsymbol{N}\boldsymbol{q}_{\rm b}^-,\ \boldsymbol{N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{14}$$

where $\boldsymbol{q}_\mathrm{b}^-$ is the sub-vector of $\boldsymbol{q}_\mathrm{f}^-$ corresponding to the joint angles. To derive the velocity map, we introduce

$$\boldsymbol{q}_\mathrm{w}^+ = \begin{bmatrix} \boldsymbol{q}_\mathrm{b}^+ \\ \boldsymbol{r}_\mathrm{c}^+ \end{bmatrix}, \ \boldsymbol{J} = \frac{\partial \boldsymbol{r}_\mathrm{f}}{\partial \boldsymbol{q}_\mathrm{f}},$$

where $\boldsymbol{J}$ is the Jacobian matrix which gives

$$\boldsymbol{J}\dot{\boldsymbol{q}}_\mathrm{f}^- = \dot{\boldsymbol{r}}_\mathrm{f}^-. \tag{15}$$

Denote $\boldsymbol{F}_\mathrm{I}$ as the impulsive GRF at touchdown. The touchdown model (Fujimoto, 2004) is given as

$$\begin{cases} \boldsymbol{D}_\mathrm{f}(\boldsymbol{q}_\mathrm{w}^+ - \boldsymbol{q}_\mathrm{f}^-) = \boldsymbol{J}^\mathrm{T}\boldsymbol{F}_\mathrm{I}, \\ \dot{\boldsymbol{r}}_\mathrm{f}^+ = \boldsymbol{J}\dot{\boldsymbol{q}}_\mathrm{w}^+ = \boldsymbol{0}_{2\times 1}. \end{cases} \tag{16}$$

Combining Eq. (15), the equations are solved as

$$\begin{cases} \boldsymbol{F}_\mathrm{I} = -(\boldsymbol{J}\boldsymbol{D}_\mathrm{f}^{-1}\boldsymbol{J}^\mathrm{T})^{-1}\dot{\boldsymbol{r}}_\mathrm{f}^-, \\ \dot{\boldsymbol{q}}_\mathrm{w}^+ - \dot{\boldsymbol{q}}_\mathrm{f}^- = -\boldsymbol{D}_\mathrm{f}^{-1}\boldsymbol{J}^\mathrm{T}(\boldsymbol{J}\boldsymbol{D}_\mathrm{f}^{-1}\boldsymbol{J}^\mathrm{T})^{-1}\dot{\boldsymbol{r}}_\mathrm{f}^-, \end{cases} \tag{17}$$

which gives

$$\dot{\boldsymbol{q}}_\mathrm{s}^+ = \boldsymbol{N}\dot{\boldsymbol{q}}_\mathrm{b}^+, \tag{18}$$

where $\dot{\boldsymbol{q}}_\mathrm{b}^+$ is a sub-vector of $\dot{\boldsymbol{q}}_\mathrm{w}^+$. Then the instantaneous map $\boldsymbol{\Delta}_\mathrm{f}^\mathrm{s}$ at touchdown can be calculated from Eqs. (14) and (18). We have

$$\begin{bmatrix} \boldsymbol{q}_\mathrm{s}^+ \\ \dot{\boldsymbol{q}}_\mathrm{s}^+ \end{bmatrix} = \boldsymbol{\Delta}_\mathrm{f}^\mathrm{s}. \tag{19}$$

Remark 1 Eq. (17) reveals that if $\dot{\boldsymbol{r}}_\mathrm{f}^-$ is small enough, the amount of velocity change $\dot{\boldsymbol{q}}_\mathrm{w}^+ - \dot{\boldsymbol{q}}_\mathrm{f}^-$ is also small, so is $\boldsymbol{F}_\mathrm{I}$. Thus, small $\dot{\boldsymbol{r}}_\mathrm{f}^-$ is desired at touchdown.

In summary, the complete hybrid system model for the biped running motion is given as follows:

$$\begin{cases} \dot{\boldsymbol{x}}_\mathrm{s} = f_\mathrm{s}(\boldsymbol{x}_\mathrm{s}) + g_\mathrm{s}(\boldsymbol{x}_\mathrm{s})\boldsymbol{\Gamma}_\mathrm{s}, & \boldsymbol{x}_\mathrm{s} \in S_\mathrm{s}, \\ \boldsymbol{x}_\mathrm{f}^+ = \boldsymbol{\Delta}_\mathrm{s}^\mathrm{f}(\boldsymbol{x}_\mathrm{s}^-), & \boldsymbol{x}_\mathrm{s}^- \in S_{\mathrm{s}\to\mathrm{f}}, \\ \dot{\boldsymbol{x}}_\mathrm{f} = f_\mathrm{f}(\boldsymbol{x}_\mathrm{f}) + g_\mathrm{f}(\boldsymbol{x}_\mathrm{f})\boldsymbol{\Gamma}_\mathrm{f}, & \boldsymbol{x}_\mathrm{f} \in S_\mathrm{f}, \\ \boldsymbol{x}_\mathrm{s}^+ = \boldsymbol{\Delta}_\mathrm{f}^\mathrm{s}(\boldsymbol{x}_\mathrm{f}^-), & \boldsymbol{x}_\mathrm{f}^- \in S_{\mathrm{f}\to\mathrm{s}}. \end{cases} \tag{20}$$

3 Reference gait generation

3.1 Running-feasible conditions

In a complex and varied ground environment, a biped needs to change its running gait at every step for better adaption to the environment. The problem is rather difficult when the ground information is unknown and the biped has degrees of underactuation. We adopt the sustainable running criterion, which was originally developed in our previous work (Yi *et al.*, 2015), for the purpose of sustaining running in a rough ground environment.

Definition 1 (Running-feasible state) A state of a biped is said to be 'running-feasible' if there exists a control input such that, starting from this state, the biped results in a trajectory consisting of an infinite number of running steps.

Definition 2 (Sustainable running) A biped is said to be in 'sustainable running' if every system state is running-feasible.

Thus, a running-feasible initial state is a necessary condition for successful running. Given a running-feasible state, a biped is able to generate a running step gait. Given that the initial state of the next step is determined by the end state of the current step, we consider a 'running-feasible step' if it goes though the stance, takeoff, flight, and touchdown phases and the state after touchdown is running-feasible. In our previous work, an estimation for a set of running-feasible states is given by the following lemma:

Lemma 1 (Running-feasible set (Yi *et al.*, 2015)) Assume the centroid angular moment is ignored and a running biped is modeled by an inverted pendulum model. The COG state of the biped $[x_\mathrm{c},\ z_\mathrm{c},\ \dot{x}_\mathrm{c},\ \dot{z}_\mathrm{c}]^\mathrm{T}$ is a running-feasible state if it belongs to the set

$$\Omega(\boldsymbol{x}) = \{\boldsymbol{x}_\mathrm{s} \in S_\mathrm{s} | x_\mathrm{c} < 0,\ \dot{z}_\mathrm{c} < 0,\ \dot{x}_\mathrm{c}^\mathrm{min} \le \dot{x}_\mathrm{c} \le \dot{x}_\mathrm{c}^\mathrm{max}\},$$

where

$$\dot{x}_\mathrm{c}^\mathrm{max} = \frac{F^\mathrm{max}}{Ml_\mathrm{o}}\frac{x_\mathrm{c}z_\mathrm{c}}{\dot{z}_\mathrm{c}} - \frac{x_\mathrm{c}g}{\dot{z}_\mathrm{c}},$$
$$\dot{x}_\mathrm{c}^\mathrm{min} = \frac{x_\mathrm{c}\dot{z}_\mathrm{c} - x_\mathrm{c}\sqrt{\dot{z}_\mathrm{c} + 4gz_\mathrm{c}}}{2z_\mathrm{c}},$$

and $l_\mathrm{o} = \|\boldsymbol{r}_\mathrm{c} - \boldsymbol{r}_\mathrm{o}\|$ represents the distance between the COG and the standing foot in the inverted pendulum model.

To tackle the problem of a biped running over varied terrain, the placement of the swing foot before touchdown should be adjusted to ensure that the state after touchdown is within the running-feasible set. Therefore, the running-feasible set is rewritten as

$$\Omega^1(\boldsymbol{x}) = \{\boldsymbol{p} \in C | x_\mathrm{c} < 0,\ \dot{z}_\mathrm{c} < 0,\ x_\mathrm{c}^\mathrm{min} \le x_\mathrm{c} \le x_\mathrm{c}^\mathrm{max}\},$$

where

$$\begin{cases} x_{\mathrm{c}}^{\max} = \dfrac{Ml_{\mathrm{o}}\dot{x}_{\mathrm{c}}\dot{z}_{\mathrm{c}}}{F^{\max}z_{\mathrm{c}} - Mgl_{\mathrm{o}}}, \\ x_{\mathrm{c}}^{\min} = \dfrac{2\dot{x}_{\mathrm{c}}z_{\mathrm{c}}}{\dot{z}_{\mathrm{c}} - \sqrt{\dot{z}_{\mathrm{c}} + 4gz_{\mathrm{c}}}}. \end{cases} \tag{21}$$

Note that x_{c} here is the horizontal position of the COG in the local frame with its origin at the position of the supporting foot. As shown in Fig. 3, the gray area represents the estimation of the running-feasible set at the beginning of every step. The biped has to adjust its posture to place the swing foot in this safe area.

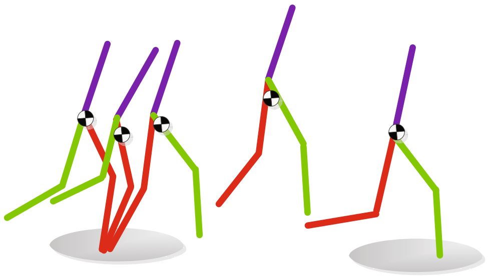

Fig. 3 A running biped with a pre-known safe area

3.2 Reference gait generation

Sustainable running allows a biped to change its running gaits in an external varied environment. Given the initial state of a running step within a running-feasible set, we propose the following specifications for the reference gait generation at the beginning of every running step:

1. Make the centroid angular moment close to zero in the stance phase. By Eq. (10), the centroid angular moment will remain constant in the flight phase. So, the assumption in Lemma 1 is satisfied.

2. Make the foot placement before touchdown within the running-feasible set in Eq. (21). As long as the state jump at touchdown is limited, a running-feasible gait is obtained.

3. Make the velocity of the touching foot near to zero when the biped is about to touchdown. As commented in Remark 1, if the velocity of the touching foot is small enough, the amount of velocity change before and after touchdown is also small, which limits the state jump at touchdown.

4. Maintain the upper body upwards to avoid damage to the on-board hardware inside it.

To meet the above specifications 1–4, we consider controlling a gait of four variables $\boldsymbol{\psi}_{\mathrm{s}} = [\boldsymbol{H}_{\mathrm{cs}}, q_3, q_5, z_{\mathrm{c}}]^{\mathrm{T}}$ in stance and $\boldsymbol{\psi}_{\mathrm{f}} = [q_2, q_3, x_{\mathrm{f}}, z_{\mathrm{f}}]^{\mathrm{T}}$ in flight. In a running step, denote the initial and end time instants of a stance phase by t_{so} and t_{se} respectively, and denote the initial and end time instants of a flight phase by t_{fo} and t_{fe} respectively. Here t_{se} is also taken as the beginning time of the flight phase. Then the reference gait represented by the four variables is given as

$$\boldsymbol{\psi}_{\mathrm{s}}^{\mathrm{ref}}(t) = \begin{bmatrix} \boldsymbol{H}_{\mathrm{cs}}^{\mathrm{ref}}(t) \\ q_3^{\mathrm{ref}}(t) \\ q_5^{\mathrm{ref}}(t) \\ z_{\mathrm{c}}^{\mathrm{ref}}(t) \end{bmatrix}, \ \boldsymbol{\psi}_{\mathrm{f}}^{\mathrm{ref}}(t) = \begin{bmatrix} q_2^{\mathrm{ref}}(t) \\ q_3^{\mathrm{ref}}(t) \\ x_{\mathrm{f}}^{\mathrm{ref}}(t) \\ z_{\mathrm{f}}^{\mathrm{ref}}(t) \end{bmatrix}.$$

The fifth-order polynomial is used to generate the reference trajectory in the stance phase. That is,

$$\boldsymbol{\psi}_{\mathrm{s}}^{\mathrm{ref}}(t) = \boldsymbol{a}_{n0} + \boldsymbol{a}_{n1}t + \boldsymbol{a}_{n2}t^2 + \boldsymbol{a}_{n3}t^3 + \boldsymbol{a}_{n4}t^4 + \boldsymbol{a}_{n5}t^5, \tag{22}$$

where the parameters $\boldsymbol{a}_{nj}$ ($n = 1, 2, 3, 4, j = 0, 1, \ldots, 5$) are determined by the initial state $\boldsymbol{\psi}_{\mathrm{s}}(t_{\mathrm{so}})$ of the current step, the desired ending state $\boldsymbol{\psi}_{\mathrm{s}}(t_{\mathrm{se}})$, and their first and second derivatives $\dot{\boldsymbol{\psi}}_{\mathrm{s}}(t_{\mathrm{so}})$, $\ddot{\boldsymbol{\psi}}_{\mathrm{s}}(t_{\mathrm{so}})$, $\dot{\boldsymbol{\psi}}_{\mathrm{s}}(t_{\mathrm{se}})$, $\ddot{\boldsymbol{\psi}}_{\mathrm{s}}(t_{\mathrm{se}})$. In this study, the initial state of the reference trajectory is set to be the true state of the biped at the moment of starting a new running step such that the tracking control starts with zero tracking error.

However, when the information of the varied terrain environment is not given, the touchdown time t_{fe} becomes unpredictable, which means that the ending state of the flight phase $\boldsymbol{\psi}_{\mathrm{f}}(t_{\mathrm{fe}})$ cannot be designed. For this reason the fifth-order polynomial method is unsuitable for realizing specifications 1 and 3 in the flight phase. To solve this problem, we decompose the flight phase into the rising sub-phase and falling sub-phase. Namely, in the rising sub-phase the biped has positive or zero vertical velocity, and in the falling sub-phase the biped has negative vertical velocity. Then specifications 2 and 3 are replaced by the following alternatives:

a. Make both legs of the biped draw upwards in the rising sub-phase and then put downwards in the falling sub-phase. So, the biped has a touchdown with a negative vertical velocity in the falling sub-phase; i.e., $\dot{z}_{\mathrm{c}} < 0$ is satisfied for the running-feasible set in Eq. (21).

b. Make the velocity of the swinging foot within a small range in the whole falling sub-phase. The occurrence of a touchdown requires the swinging foot to move downward with a negative vertical velocity. When the swinging foot moves faster toward the ground, the time duration of the falling sub-phase becomes smaller, which makes the vertical velocity of the COG larger. Based on Eq. (21) a larger running-feasible set will also be obtained. However, a fast swinging foot will cause an undesirable and large state jump at touchdown. Considering both factors, the velocity of the swinging leg should be a compromise design between zero and far below zero during the whole falling sub-phase.

c. Make the horizontal position of the swinging foot within the running-feasible set in the whole falling sub-phase. Thus, whenever the biped touches the terrain, specification 2 is satisfied.

Denote the divided reference gait in the flight phase by $\boldsymbol{\psi}_{\rm f}=[\boldsymbol{\psi}_{\rm f1},\ \boldsymbol{\psi}_{\rm f2}]^{\rm T}$ and the ending time instant of the rising sub-phase by $t_{\rm fm}$. The fifth-order polynomial is used to generate the reference trajectory of the rising sub-phase to meet the above alternative specifications. That is,

$$\boldsymbol{\psi}_{\rm f1}^{\rm ref}(t)=\boldsymbol{b}_{n0}+\boldsymbol{b}_{n1}t+\boldsymbol{b}_{n2}t^2+\boldsymbol{b}_{n3}t^3+\boldsymbol{b}_{n4}t^4+\boldsymbol{b}_{n5}t^5, \quad (23)$$

where the parameters $\boldsymbol{b}_{nj}$ ($n=1,\ 2,\ 3,\ 4,\ j=0,\ 1,\ \ldots,\ 5$) are determined by the ending state of the stance phase $\boldsymbol{\psi}_{\rm f1}(t_{\rm se}),\ \dot{\boldsymbol{\psi}}_{\rm f1}(t_{\rm se}),\ \ddot{\boldsymbol{\psi}}_{\rm f1}(t_{\rm se})$ and the design of the ending state of the rising sub-phase $\boldsymbol{\psi}_{\rm f1}(t_{\rm fm}),\ \dot{\boldsymbol{\psi}}_{\rm f1}(t_{\rm fm}),\ \ddot{\boldsymbol{\psi}}_{\rm f1}(t_{\rm fm})$.

Now we design the reference trajectory during the falling sub-phase. Let the joint angles q_2 and q_3 remain constant during this sub-phase,

$$q_2^{\rm ref}(t)=q_2(t_{\rm fm}),\ q_3^{\rm ref}(t)=q_3(t_{\rm fm}). \quad (24)$$

Note that after touchdown the swinging foot becomes the supporting foot on the ground for the next step, and its horizontal position becomes the origin of the local coordinates of the biped. Thus, before touchdown the horizontal distance between the COG and the swinging foot represents the initial COG horizontal position of the next step, which is designed as a value in the middle of the running-feasible set Ω^1. As the horizontal COG position in flight is calculated as

$$x_{\rm c}(t)=x_{\rm c}(t_{\rm fo})+\dot{x}_{\rm c}(t_{\rm fo})(t-t_{\rm fo}),$$

the horizontal swinging foot in the falling sub-phase is designed as

$$x_{\rm f}^{\rm ref}(t)=x_{\rm c}(t_{\rm fo})+\dot{x}_{\rm c}(t_{\rm fo})(t-t_{\rm fo})-\frac{x_{\rm c}^{\min}+x_{\rm c}^{\max}}{2}. \quad (25)$$

Then the alternative specification c is ensured. For the vertical velocity of the swinging foot, we set

$$\dot{z}_{\rm f}^{\rm ref}(t)=-2,$$

which gives the reference gait

$$z_{\rm f}^{\rm ref}(t)=-2(t-t_{\rm fm}). \quad (26)$$

By Eqs. (24)–(26), the falling sub-phase reference gait $\boldsymbol{\psi}_{\rm f2}^{\rm ref}$ is determined as long as the rising gait states $x_{\rm f}(t_{\rm fo})$ and $x_{\rm f}(t_{\rm fm})$ are given. Now a reference running-feasible step gait is generated. In the next section, we will design a control law to track the reference gait with zero initial tracking error.

4 Tracking control design

In this section we develop a running control law for a five-link planar point-foot biped based on the results in the preceding sections. We develop a tracking control law for the biped to track the designed reference trajectory $\boldsymbol{\psi}_{\rm s}^{\rm ref}$ in the stance phase and $\boldsymbol{\psi}_{\rm f}^{\rm ref}$ in the flight phase.

Define an output $\boldsymbol{y}_{\rm s}=[\boldsymbol{y}_{\rm s1},\ \boldsymbol{y}_{\rm s2}]^{\rm T}$ in the stance phase as

$$\begin{cases}\boldsymbol{y}_{\rm s1}=h_{\rm s1}(\boldsymbol{q}_{\rm s},\dot{\boldsymbol{q}}_{\rm s}):=\boldsymbol{H}_{\rm cs}-\boldsymbol{H}_{\rm cs}^{\rm ref},\\ \boldsymbol{y}_{\rm s2}=h_{\rm s2}(\boldsymbol{q}_{\rm s}):=\begin{bmatrix}q_3\\ q_5\\ z_{\rm c}\end{bmatrix}-\begin{bmatrix}q_3^{\rm ref}\\ q_5^{\rm ref}\\ z_{\rm c}^{\rm ref}\end{bmatrix}.\end{cases}$$

Calculating the Lie derivatives of $f_{\rm s},\ f_{\rm f},\ g_{\rm s}$, and $g_{\rm f}$, we obtain

$$\begin{cases}\dfrac{{\rm d}\boldsymbol{y}_{\rm s1}}{{\rm d}t}=L_{f_{\rm s}}h_{\rm s1}(\boldsymbol{q}_{\rm s},\dot{\boldsymbol{q}}_{\rm s})+L_{g_{\rm s}}h_{\rm s1}(\boldsymbol{q}_{\rm s},\dot{\boldsymbol{q}}_{\rm s})\boldsymbol{\Gamma}_{\rm s},\\ \dfrac{{\rm d}\boldsymbol{y}_{\rm s2}}{{\rm d}t}=L_{f_{\rm s}}h_{\rm s2}(\boldsymbol{q}_{\rm s}),\\ \dfrac{{\rm d}^2\boldsymbol{y}_{\rm s2}}{{\rm d}t^2}=L_{f_{\rm s}}^2h_{\rm s2}(\boldsymbol{q}_{\rm s})+L_{g_{\rm s}}L_{f_{\rm s}}h_{\rm s2}(\boldsymbol{q}_{\rm s})\boldsymbol{\Gamma}_{\rm s}.\end{cases}$$

For $\boldsymbol{y}_{\rm s1}=\boldsymbol{y}_{\rm s2}\equiv\boldsymbol{0}$, the system state in the stance phase evolves on the set

$$Z_{\rm s}:=\{\boldsymbol{x}_{\rm s}\in S_{\rm s}|h_{\rm s1}=\boldsymbol{0},\ h_{\rm s2}=\boldsymbol{0},\ L_{f_{\rm s}}h_{\rm s2}=\boldsymbol{0}\}. \quad (27)$$

Here we write $h_{\rm s1}=h_{\rm s1}(\boldsymbol{q}_{\rm s},\dot{\boldsymbol{q}}_{\rm s}),\ h_{\rm s2}=h_{\rm s2}(\boldsymbol{q}_{\rm s})$ for brevity. The feedback control $\boldsymbol{\Gamma}_{\rm s}^*$ is then chosen to be continuous and to render $Z_{\rm s}$ invariant under the closed-loop dynamics. That is,

$$\begin{cases} L_{f_{\rm s}}h_{\rm s1}+L_{g_{\rm s}}h_{\rm s1}\boldsymbol{\Gamma}_{\rm s}^*=\mathbf{0},\\ L_{f_{\rm s}}^2h_{\rm s2}+L_{g_{\rm s}}L_{f_{\rm s}}h_{\rm s2}\boldsymbol{\Gamma}_{\rm s}^*=\mathbf{0}.\end{cases} \tag{28}$$

When $h_{\rm s1}\neq\mathbf{0}$, $h_{\rm s2}\neq\mathbf{0}$, $L_{g_{\rm s}}h_{\rm s1}\neq\mathbf{0}$, and $L_{g_{\rm s}}L_{f_{\rm s}}h_{\rm s2}\neq\mathbf{0}$ are ensured, the control law $\boldsymbol{\Gamma}_{\rm s}^*$ is written as

$$\boldsymbol{\Gamma}_{\rm s}^*=-\begin{bmatrix} L_{g_{\rm s}}h_{\rm s1}\\ L_{g_{\rm s}}L_{f_{\rm s}}h_{\rm s2}\end{bmatrix}^{-1}\begin{bmatrix} L_{f_{\rm s}}h_{\rm s1}\\ L_{f_{\rm s}}^2h_{\rm s2}\end{bmatrix}. \tag{29}$$

Define an output $\boldsymbol{y}_{\rm f}=[\boldsymbol{y}_{\rm f1},\ \boldsymbol{y}_{\rm f2}]^{\rm T}$ for the rising and falling sub-phases as

$$\begin{cases} \boldsymbol{y}_{\rm f1}=h_{\rm f1}(\boldsymbol{q}_{\rm f}):=\boldsymbol{\psi}_{\rm f}-\boldsymbol{\psi}_{\rm f1}^{\rm ref},\\ \boldsymbol{y}_{\rm f2}=h_{\rm f2}(\boldsymbol{q}_{\rm f}):=\boldsymbol{\psi}_{\rm f}-\boldsymbol{\psi}_{\rm f2}^{\rm ref}.\end{cases}$$

Also, denote the control input in the rising and falling sub-phases by $\boldsymbol{\Gamma}_{\rm f}=[\boldsymbol{\Gamma}_{\rm f1},\ \boldsymbol{\Gamma}_{\rm f2}]^{\rm T}$. Thus, we have

$$\begin{cases} \dfrac{{\rm d}\boldsymbol{y}_{{\rm f}i}}{{\rm d}t}=L_{f_{\rm f}}h_{{\rm f}i},\\ \dfrac{{\rm d}^2\boldsymbol{y}_{{\rm f}i}}{{\rm d}t^2}=L_{f_{\rm f}}^2h_{{\rm f}i}+L_{g_{\rm f}}L_{f_{\rm f}}h_{{\rm f}i}\boldsymbol{\Gamma}_{{\rm f}i},\end{cases} \tag{30}$$

where $i=1,2$. For $\boldsymbol{y}_{\rm f1}=\boldsymbol{y}_{\rm f2}\equiv\mathbf{0}$, the system state evolves on the set

$$Z_{\rm f}:=\{\boldsymbol{x}_{\rm f}\in S_{\rm f}|h_{{\rm f}i}=\mathbf{0},\ L_{f_{\rm s}}h_{{\rm f}i}=\mathbf{0},\ i=1,2\}. \tag{31}$$

When $h_{\rm f1}\neq\mathbf{0}$, $h_{\rm f2}\neq\mathbf{0}$, $L_{g_{\rm f}}L_{f_{\rm f}}h_{\rm f1}\neq\mathbf{0}$, and $L_{g_{\rm f}}L_{f_{\rm f}}h_{\rm f2}\neq\mathbf{0}$ are ensured, consider the following control to render $Z_{\rm f}$ invariant under the closed-loop dynamics:

$$\boldsymbol{\Gamma}_{{\rm f}i}^*=-(L_{g_{\rm f}}L_{f_{\rm f}}h_{{\rm f}i})^{-1}L_{f_{\rm f}}^2h_{{\rm f}i},\quad i=1,2. \tag{32}$$

Evaluating system (20) on the zero output sets (27) and (31) yields the zero dynamics

$$\begin{cases} \dot{\boldsymbol{x}}_{\rm s}=f_{\rm s}(\boldsymbol{x}_{\rm s})+g_{\rm s}(\boldsymbol{x}_{\rm s})\boldsymbol{\Gamma}_{\rm s}^*, & \boldsymbol{x}_{\rm s}\in Z_{\rm s},\\ \boldsymbol{x}_{\rm f}^+=\boldsymbol{\Delta}_{\rm s}^{\rm f}(\boldsymbol{x}_{\rm s}^-), & \boldsymbol{x}_{\rm s}^-\in S_{\rm s\to f},\\ \dot{\boldsymbol{x}}_{\rm f}=f_{\rm f}(\boldsymbol{x}_{\rm f})+g_{\rm f}(\boldsymbol{x}_{\rm f})\boldsymbol{\Gamma}_{\rm f}^*, & \boldsymbol{x}_{\rm f}\in Z_{\rm f},\\ \boldsymbol{x}_{\rm s}^+=\boldsymbol{\Delta}_{\rm f}^{\rm s}(\boldsymbol{x}_{\rm f}^-), & \boldsymbol{x}_{\rm f}^-\in S_{\rm f\to s}.\end{cases} \tag{33}$$

Note that once $\boldsymbol{\psi}_{\rm s}^{\rm ref}(t)$ in the stance phase and $\boldsymbol{\psi}_{\rm f}^{\rm ref}(t)$ in the flight phase are given, the reference gait consisting of all the states is encoded. Thus, the choice of two reference signals $\boldsymbol{\psi}_{\rm s}^{\rm ref}(t)$ and $\boldsymbol{\psi}_{\rm f}^{\rm ref}(t)$ should also ensure that all the other states are bounded for zero dynamics (33). Finally, to converge to the zero output set, we add $\boldsymbol{\nu}_{\rm s}$ and $\boldsymbol{\nu}_{\rm f}$ in the feedback control law, i.e.,

$$\boldsymbol{\Gamma}_{\rm s}=\boldsymbol{\Gamma}_{\rm s}^*+\boldsymbol{\nu}_{\rm s},\ \boldsymbol{\Gamma}_{\rm f}=\boldsymbol{\Gamma}_{\rm f}^*+\boldsymbol{\nu}_{\rm f},$$

which results in the following closed-loop error tracking system ($i=1,2$):

$$\begin{cases} \ddot{\boldsymbol{y}}_{{\rm s}i}+\boldsymbol{K}_{\rm d}^{\rm s}\dot{\boldsymbol{y}}_{{\rm s}i}+\boldsymbol{K}_{\rm p}^{\rm s}\boldsymbol{y}_{{\rm s}i}=\mathbf{0},\\ \ddot{\boldsymbol{y}}_{{\rm f}i}+\boldsymbol{K}_{\rm d}^{\rm f}\dot{\boldsymbol{y}}_{{\rm f}i}+\boldsymbol{K}_{\rm p}^{\rm f}\boldsymbol{y}_{{\rm f}i}=\mathbf{0}.\end{cases} \tag{34}$$

By choosing negative $\boldsymbol{K}_{\rm d}^{\rm s}$, $\boldsymbol{K}_{\rm d}^{\rm f}$, $\boldsymbol{K}_{\rm p}^{\rm s}$, $\boldsymbol{K}_{\rm p}^{\rm f}$, the solution of Eq. (34) converges exponentially quickly to zero. In conclusion, the tracking control law is

$$\begin{cases} \boldsymbol{\Gamma}_{\rm s}=\boldsymbol{\Gamma}_{\rm s}^*+\begin{bmatrix} L_{g_{\rm s}}h_{\rm s1}\\ L_{g_{\rm s}}L_{f_{\rm s}}h_{\rm s2}\end{bmatrix}^{-1}\\ \qquad\cdot\left(\boldsymbol{K}_{\rm p}^{\rm s}\begin{bmatrix} h_{\rm s1}\\ h_{\rm s2}\end{bmatrix}+\boldsymbol{K}_{\rm d}^{\rm s}\begin{bmatrix} L_{f_{\rm s}}h_{\rm s1}\\ L_{f_{\rm s}}h_{\rm s2}\end{bmatrix}\right),\\ \boldsymbol{\Gamma}_{{\rm f}i}=\boldsymbol{\Gamma}_{{\rm f}i}^*+(L_{g_{\rm f}}L_{f_{\rm f}}h_{{\rm f}i})^{-1}(\boldsymbol{K}_{\rm p}^{\rm f}h_{{\rm f}i}+\boldsymbol{K}_{\rm d}^{\rm f}L_{f_{\rm f}}h_{{\rm f}i}),\ i=1,2.\end{cases}$$

5 Simulation results

Assume $F^{\max}=2Mg$ and $\mu=0.4$ for the five-link running biped. Given the initial state of the stance phase $\boldsymbol{x}(t_{\rm so})=[2.6469,\ 2.4861,\ 0,\ 3.2575\ 2.7057,\ -13.7305,\ -2.4342,\ -1,\ 0.9667,\ -2]^{\rm T}$, the initial condition of the stance reference gait is calculated for $\boldsymbol{\psi}_{\rm s}^{\rm ref}(t_{\rm so})$, $\dot{\boldsymbol{\psi}}_{\rm s}^{\rm ref}(t_{\rm so})$ and designed for $\ddot{\boldsymbol{\psi}}_{\rm s}^{\rm ref}(t_{\rm so})$ as

$$\begin{bmatrix} \boldsymbol{\psi}_{\rm s}^{\rm ref}(t_{\rm so})^{\rm T}\\ \dot{\boldsymbol{\psi}}_{\rm s}^{\rm ref}(t_{\rm so})^{\rm T}\\ \ddot{\boldsymbol{\psi}}_{\rm s}^{\rm ref}(t_{\rm so})^{\rm T}\end{bmatrix}=\begin{bmatrix} 0.1 & 0.05 & -0.27 & 0.70\\ 3.03 & -0.60 & 2.82 & -0.03\\ 0 & 0 & -100 & g\end{bmatrix}.$$

Consider the fifth-order polynomial reference gaits in the stance phase and the rising sub-phase. As the ending state at the stance phase results in the beginning state of the rising sub-phase in flight, only the values of $\boldsymbol{\psi}_{\rm s}^{\rm ref}(t_{\rm se})$, $\boldsymbol{\psi}_{\rm f}^{\rm ref}(t_{\rm fm})$, and their first and second derivatives need to be determined. We set

$$\begin{bmatrix} \boldsymbol{\psi}_{\rm s}(t_{\rm se})^{\rm T}\\ \dot{\boldsymbol{\psi}}_{\rm s}(t_{\rm se})^{\rm T}\\ \ddot{\boldsymbol{\psi}}_{\rm s}(t_{\rm se})^{\rm T}\end{bmatrix}=\begin{bmatrix} 0 & q_3(t_{\rm se}) & 2 & z_{\rm c}(t_{\rm so})\\ 0 & 0 & -5 & 0.6\\ 0 & 0 & 100 & g\end{bmatrix},$$

$$\begin{bmatrix} \boldsymbol{\psi}_{\rm f}(t_{\rm fm})^{\rm T}\\ \dot{\boldsymbol{\psi}}_{\rm f}(t_{\rm fm})^{\rm T}\\ \ddot{\boldsymbol{\psi}}_{\rm f}(t_{\rm fm})^{\rm T}\end{bmatrix}=\begin{bmatrix} 2 & 0.1 & x_{\rm f}(t_{\rm fm}) & 0.2\\ -1 & -1 & \dot{x}_{\rm c}(t_{\rm fo}) & 0\\ \varsigma_1 & \varsigma_2 & \varsigma_3 & \varsigma_4\end{bmatrix},$$

with

$$q_3(t_{\mathrm{se}}) = q_3(t_{\mathrm{so}}) + 0.2,$$
$$x_{\mathrm{f}}(t_{\mathrm{fm}}) = x_{\mathrm{c}}(t_{\mathrm{fo}}) - \dot{x}_{\mathrm{c}}(t_{\mathrm{fo}})\frac{\dot{z}_{\mathrm{c}}(t_{\mathrm{fo}})}{g},$$
$$\varsigma_k = \frac{\dot{\psi}_k(t_{\mathrm{fm}}) - \dot{\psi}_k(t_{\mathrm{f0}})}{t_{\mathrm{fe}} - t_{\mathrm{fo}}},\ k = 1, 2, 3, 4.$$

By Eqs. (24)–(26), when the ending state of the rising sub-phase is determined, the reference gait of the falling sub-phase is also generated.

First, we let the biped run over the terrain with one upward stair and one downward stair of a height difference of 0.15 m. As in Figs. 4 and 5 the biped handles this challenge appropriately.

Then we set the biped running over continuously varied terrain (Fig. 6). The trajectories of the COG are shown in Fig. 7. It shows the height of the COG with respect to the varied ground level, z_{c}, transiting between near 0.7 m and 0.8 m and that the velocity of the COG changes instantaneously at discrete times due to touchdowns. When the robot recognizes these changes in the varied terrain, it regenerates a new reference trajectory and a new tracking control law to maintain the running motion. Thus, the COG is always able to return back to a proper position and velocity without divergence. The non-periodic joint trajectories are shown in Fig. 8. It shows that the biped uses its body posture to remain upright and maintains a steady human-like posture for running.

6 Conclusions

In the paper we proposed a strategy for an underactuated biped running on unknown and varied terrain. By controlling the height of the COG upwards and the centroid angular moment close to zero, the biped incorporates the environmental change into its velocity. Based on a sustainable running creation, we set the proper foot placement before touchdown to make the biped sustain a running motion in the presence of terrain variation. The simulation results demonstrate that the biped is able

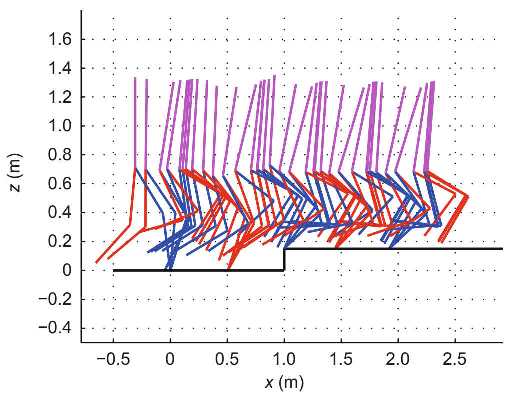

Fig. 4 Running up a 0.15-m stair

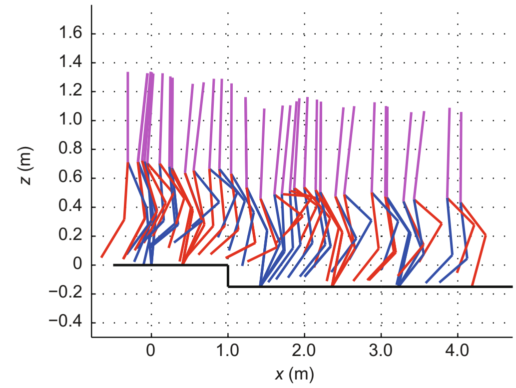

Fig. 5 Running down a 0.15-m stair

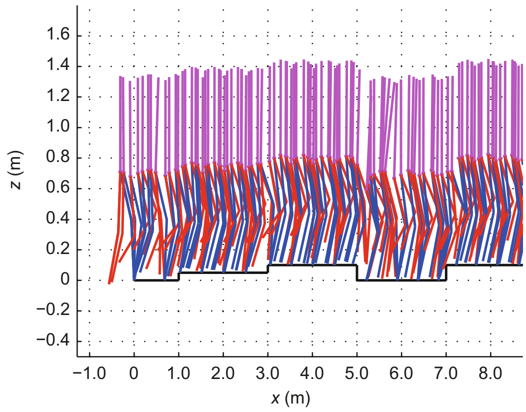

Fig. 6 Running over varied terrain

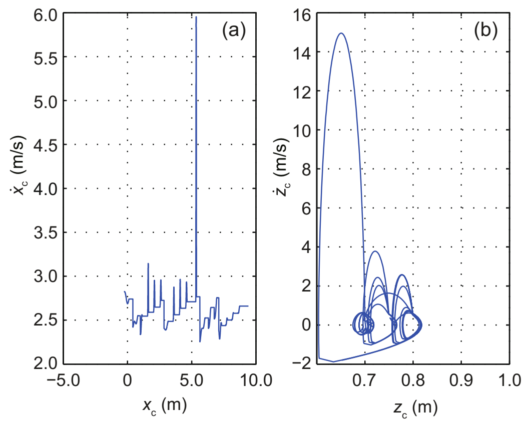

Fig. 7 The dynamics of the COG in the horizontal (a) and vertical (b) directions

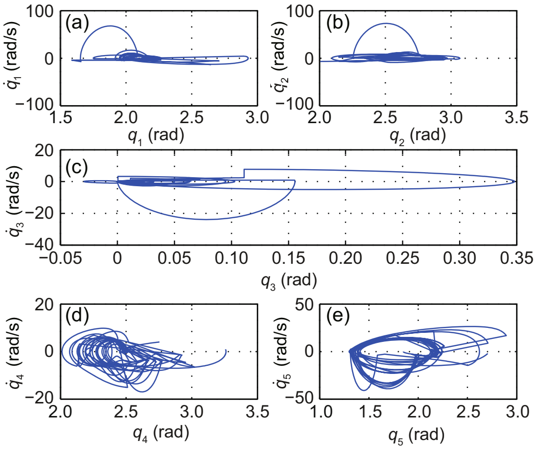

Fig. 8 The non-periodic trajectories of q_1 with respect to $\dot{q}_1$ (a), q_2 with respect to $\dot{q}_2$ (b), q_3 with respect to $\dot{q}_3$ (c), q_4 with respect to $\dot{q}_4$ (d), and q_5 with respect to $\dot{q}_5$ (e)

to react swiftly, resembling the reaction patterns of human beings.

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