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A novel resource optimization scheme for multi-cell OFDMA relay network*

Ning DU†[1,2], Fa-sheng LIU[1]

([1]College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China)

([2]Department of Mathematics and Information Engineering, Dongchang College of Liaocheng University, Liaocheng 252000, China)

†E-mail: lczhlydn@126.com

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Abstract: In cellular networks, users communicate with each other through their respective base stations (BSs). Conventionally, users are assumed to be in different cells. BSs serve as decode-and-forward (DF) relay nodes to users. In addition to this type of conventional user, we recognize that there are scenarios users who want to communicate with each other are located in the same cell. This gives rise to the scenario of intra-cell communication. In this case, a BS can behave as a two-way relay to achieve information exchange instead of using conventional DF relay. We consider a multi-cell orthogonal frequency division multiple access (OFDMA) network that comprises these two types of users. We are interested in resource allocation between them. Specifically, we jointly optimize subcarrier assignment, subcarrier pairing, and power allocation to maximize the weighted sum rate. We consider the resource allocation problem at BSs when the end users' power is fixed. We solve the problem approximately through Lagrange dual decomposition. Simulation results show that the proposed schemes outperform other existing schemes.

Key words: Intra-cell communication, Two-way relay, Subcarrier assignment, Subcarrier pairing
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1 Introduction

In less than a decade, the status of smart phones has moved from laboratory breadboard to the world's pervasive consumer electronics products. With a smart phone in hand, a consumer can enjoy many services, such as games and videos. However, a high-speed cellular network becomes a bottleneck in enjoying these services. Orthogonal frequency division multiplexing (OFDM) is a leading technology for high-speed cellular network dividing the frequency-selective wireless channel into a large number of parallel, flat, and narrow-band subcarriers to carry information. Each subcarrier can be assigned to different users and employ different modulation and coding schemes to counter a fading channel. Hence, OFDM has been adopted by various standards as key technology, such as long-term evolution-advanced (LTE-A) and the 802.11 series (Sun and Honig, 2008; Jang *et al.*, 2010; Parkvall *et al.*, 2011).

On the other hand, relay-assisted cooperative communication has attracted tremendous interest, because it can improve the overall system performance in wireless networks, such as coverage extension, power saving, and throughput enhancement (Laneman and Wornell, 2003; Sendonaris *et al.*, 2003;

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ORCID: Ning DU, http://orcid.org/0000-0003-3934-2289

El Gamal and Zahedi, 2005; Yang *et al.*, 2009; Oyman, 2010). Incorporating relay architecture into an OFDM system is a powerful technique for broadband wireless communication. An LTE-A cooperative cellular network was investigated, where multiple decode-and-forward (DF) relay stations (RSs) were deployed to enhance the cell-edge throughput and extend the coverage area. It formulates a sum-rate maximization problem with the total power constraint and a subcarrier assignment (SA) problem. However, subcarrier pairing (SP) was not considered (Zhang *et al.*, 2014). A two-hop cooperative OFDM multi-relay network was investigated, and a joint power allocation (PA) and SP scheme was proposed, which maximizes the transmission rate subject to the total network power constraint (Li *et al.*, 2013). Multiple users and multiple RSs are deployed in a cell to assist uplink and downlink transmission in the LTE system. The joint optimization problem of SP based on relay PA, relay selection, and SA to maximize the sum rate was studied (Zhang *et al.*, 2012). A network with a source node, a DF relay node, and multiple destination nodes was investigated, which maximizes the sum rate of this network under the total power by means of individual power constraint and SA (Hajiaghayi *et al.*, 2012). The subcarrier pair was used in opportunistic OFDM decision feedback relay to minimize the power (Wang *et al.*, 2013). Energy optimization in a multi-input multi-output (MIMO) orthogonal frequency division multiple access (OFDMA) system was also investigated (Xu *et al.*, 2013). A near-optimal resource allocation (RA) algorithm was proposed for uplinking OFDMA systems (Yang *et al.*, 2014). A PA algorithm considering imperfect channel state information was proposed for cooperative communication systems (Devarajan *et al.*, 2013; Mallick *et al.*, 2013). RA policies were optimized to maximize the system throughput for OFDMA amplify-and-forward (AF) and DF relay networks, and the models adopt the AF or DF relay strategy rather than the two-way (TW) relay strategy (Ng and Schober, 2010; 2011). Besides, they just consider SA, which does not take SP into account. However, the fact that many existing works focus on individual uplink or downlink RA in cellular networks or multiple RSs deployed in the cellular networks will increase the complexity and extra investment. We make the base station (BS) serve not only as a DF relay but also as a TW relay to assist cellular network communication, and the uplink and downlink transmission strategy is jointly designed to improve system performance.

2 System model

We consider a wireless OFDMA network consisting of G cells. Let S denote the index set of the cells, i.e., $S = \{1, 2, \ldots, G\}$. A simple example that we consider is illustrated in Fig. 1. In this figure, we divide users into two groups in each cell, including conventional users and intra-cell users. A conventional user refers to a user who wants to exchange information with a user in another cell. For example, in Fig. 1, user 1 in cell 1 wants to exchange information with its peer user 3 in cell 2. Let Ψ_k and K_k denote the set of conventional users in cell k and its cardinality. An intra-cell user refers to a user who wants to exchange information with another user in the same cell. For example, in Fig. 1, users 2 and 3 are considered a pair of intra-cell users in cell 1. We define Γ_k as the set of intra-cell user pairs in cell k, and M_k as its cardinality. For example, in Fig. 1, $\Gamma_1 = \{(2, 3)\}$ for cell 1 and $\Gamma_2 = \{(1, 2)\}$ for cell 2. We consider that the available bandwidth in each cell is divided into N equally spaced subcarriers. We focus on RA including SA and SP together with PA in the TW relay system under a common assumption of perfect knowledge of channel state information (Shim *et al.*, 2010). The result can be taken as a performance criterion for practical implementation.

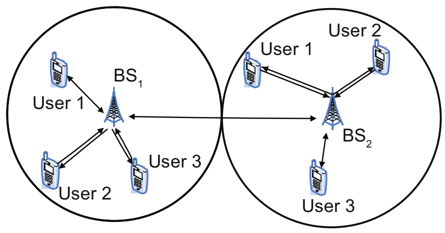

Fig. 1 System model

We illustrate the communication protocol of the system as follows: the transmission for exchanging information between either a pair of conventional users or a pair of intra-cell users is divided into two time slots. In time slot 1, conventional user 1 in cell 1 and user 3 in cell 2 transmit their signals to BS_1 and BS_2 in uplink subcarriers i and i', respectively. Then, in time slot 2, BS_1 and BS_2

transmit the exchanged information to their served conventional user in downlink subcarriers j and j', respectively. In this protocol, we assume that the transmission between BS_1 and BS_2 is usually linked with a high-speed fiber network and that the information exchange time between them is negligible. At the same time, in cells 1 and 2, intra-cell communication pairs want to exchange information with each other. In this case, the BSs can also behave as relay nodes to assist information exchange among the intra-cell communication pairs. However, to achieve higher spectral efficiency, a TW relay is used to assist such a communication. For example, in time slot 1, users 2 and 3 in cell 1 transmit their information to BS_1 using the same subcarrier through the uplink. Next, in time slot 2, BS_1 decodes and transmits the received signal and forwards it using possibly another downlink subcarrier. Suppose that in cell k, subcarriers i and j are assigned to user α for its uplink and downlink transmission, respectively. $h_{k,\alpha,i}$ is used to denote the uplink channel response gain, and $g_{k,\alpha,i}$ is used to denote the downlink channel response gain. Here, user α can be either a conventional cellular user or a user of an intra-cell communication pair. We also denote $p_{k,\alpha,i}$ as the transmit power of user α at subcarrier i for uplink transmission, and $q_{k,\alpha,i}$ and $q_{k,\alpha,\beta,j}$ as the transmit power of a BS at subcarrier j for the conventional user and the intra-cell pair, respectively.

Furthermore, we denote $\bar{q}_k$ as the total power constraint at BS_k. The allocations of subcarriers i and j to the user for its uplink and downlink are called SA and SP, respectively. Let us introduce an indicator function $\rho_{k,\alpha}^{i,j}$ for conventional user α and $\rho_{k,(\alpha,\beta)}^{ij}$ for intra-cell communication pair (α,β) in cell k. For cell k, if the uplink subcarrier is paired with downlink subcarrier j and this paired subcarrier is allocated to user α or a pair of users (α,β), then $\rho_{k,\alpha}^{i,j}=1$ or $\rho_{k,(\alpha,\beta)}^{i,j}=1$; otherwise, $\rho_{k,\alpha}^{i,j}=0$ or $\rho_{k,(\alpha,\beta)}^{i,j}=0$. Furthermore, SP is a one-to-one mapping between uplink and downlink subcarriers. Since each subcarrier pair can be assigned to one user or one pair only, the indicator function needs to satisfy the following constraints:

$$\sum_{i=1}^{N}\left(\sum_{\alpha\in\Psi_k}\rho_{k,\alpha}^{ij}+\sum_{(\alpha,\beta)\in\Gamma_k}\rho_{k,(\alpha,\beta)}^{ij}\right)=1 \quad \forall j,\ \forall k\in\{1,2,\ldots,G\}, \tag{1}$$

$$\sum_{j=1}^{N}\left(\sum_{\alpha\in\Psi_k}\rho_{k,\alpha}^{ij}+\sum_{(\alpha,\beta)\in\Gamma_k}\rho_{k,(\alpha,\beta)}^{ij}\right)=1 \quad \forall i,\ \forall k\in\{1,2,\ldots,G\}. \tag{2}$$

Our objective is to maximize the weighted sum rate of the whole cellular network of G cells through PA, SA, and SP. To achieve this, we can divide this problem into two parts.

For conventional users in the whole network, the weighted sum rate can be expressed as

$$\sum_{k=1}^{G}\sum_{\alpha\in\Psi_k}\sum_{l=k+1}^{G}\sum_{\beta\in\Psi_l}w_{k,l,\alpha,\beta}I(k,l,\alpha,\beta)R_{k,l,\alpha,\beta}, \tag{3}$$

where $w_{k,l,\alpha,\beta}$ is the relative weight assigned to user α in cell k and user β in cell l for the information exchange rate. $I(k,l,\alpha,\beta)$ represents the indicator function. $I(k,l,\alpha,\beta)=1$ when user α in cell k and user β in cell l are conventional users who want to exchange information with each other in different cells; otherwise, $I(k,l,\alpha,\beta)=0$.

$R_{k,l,\alpha,\beta}$ (bit per channel use) means the exchanged information rate for user α in cell k and user β in cell l, which can be expressed as

$$R_{k,l,\alpha,\beta}=\frac{r_{\alpha\to\beta}^{kl}+r_{\beta\to\alpha}^{lk}}{2}. \tag{4}$$

The denominator 2 accounts for the fact that two time slots are needed for the whole information exchange process, and

$$\begin{aligned}r_{\alpha\to\beta}^{kl}=\min\Bigg\{&\sum_{i=1}^{N}\rho_{k,\alpha}^{ij}\cdot C\left(\frac{p_{k,\alpha,i}|h_{k,\alpha,i}|^2}{N_0}\right),\\&\sum_{j'=1}^{N}\rho_{l,\beta}^{i'j'}\cdot C\left(\frac{q_{l,\beta,j'}|g_{l,\beta,j'}|^2}{N_0}\right)\Bigg\},\end{aligned} \tag{5}$$

$$\begin{aligned}r_{\beta\to\alpha}^{lk}=\min\Bigg\{&\sum_{i'=1}^{N}\rho_{l,\beta}^{i'j'}\cdot C\left(\frac{p_{l,\beta,i'}|h_{l,\beta,i'}|^2}{N_0}\right),\\&\sum_{j=1}^{N}\rho_{k,\alpha}^{ij}\cdot C\left(\frac{q_{k,\alpha,j}|g_{k,\alpha,j}|^2}{N_0}\right)\Bigg\},\end{aligned} \tag{6}$$

where $C(x)=\log(1+x)$, and N_0 refers to the variance of complex-valued white Gaussian noise.

For intra-cell communication pairs in the whole network, the weighted sum rate can be expressed as

$$\sum_{k=1}^{G}\sum_{(\alpha,\beta)\in\Gamma_k}w_{k,k,\alpha,\beta}R_{k,k,\alpha,\beta}, \tag{7}$$

where $w_{k,k,\alpha,\beta}$ is the relative weight for intra-cell communication pair (α,β), and $R_{k,k,\alpha,\beta}$ (bit per channel use) means the information exchange rate for intra-cell users α and β in cell k:

$$R_{k,k,\alpha,\beta}=\sum_{i,j}\rho_{k,(\alpha,\beta)}^{ij}R_{k,k,\alpha,\beta}^{ij}, \tag{8}$$

$$\begin{aligned}R_{k,k,\alpha,\beta}^{ij}=&\frac{1}{2}C\left(\frac{p_{k,\alpha,i}|h_{k,\alpha,i}|^2q_{k,\alpha,\beta,j}|g_{k,\beta,j}|^2}{(q_{k,\alpha,\beta,j}|g_{k,\beta,j}|^2+u_{k,(\alpha,\beta)}^{ij})N_0}\right)\\&+\frac{1}{2}C\left(\frac{p_{k,\beta,i}|h_{k,\beta,i}|^2q_{k,\alpha,\beta,j}|g_{k,\alpha,j}|^2}{(q_{k,\alpha,\beta,j}|g_{k,\alpha,j}|^2+u_{k,(\alpha,\beta)}^{ij})N_0}\right),\end{aligned} \tag{9}$$

$$u_{k,(\alpha,\beta)}^{ij}=1+p_{k,\alpha,i}|h_{k,\alpha,i}|^2+p_{k,\beta,i}|h_{k,\beta,i}|^2. \tag{10}$$

Then our problem can be formulated as

$$\begin{aligned}\max\sum_{k=1}^{G}\Bigg(&\sum_{\alpha\in\Psi_k}\sum_{l=k+1}^{G}\sum_{\beta\in\Psi_l}w_{k,l,\alpha,\beta}I(k,l,\alpha,\beta)R_{k,l,\alpha,\beta}\\&+\sum_{(\alpha,\beta)\in\Gamma_k}w_{k,k,\alpha,\beta}R_{k,k,\alpha,\beta}\Bigg)\end{aligned} \tag{11}$$

$$\text{s.t. Eqs. (1) and (2)}, \rho\in\{0,1\}, \tag{12}$$

$$\sum_{j=1}^{N}\left(\sum_{\alpha\in\Psi_k}q_{k,\alpha,j}+\sum_{(\alpha,\beta)\in\Gamma_k}q_{k,\alpha,\beta,j}\right)\leq\overline{q}_k \quad \forall i \text{ and } \forall k\in\{1,2,\ldots,G\}, \tag{13}$$

$$\sum_{i=1}^{N}p_{k,\alpha,i}\leq\overline{p}_{k,\alpha},\ 0\leq p_{k,\alpha,i}\leq\overline{p}_i, \quad \forall\alpha,\ \forall i,\ \text{and } \forall k\in\{1,2,\ldots,G\}, \tag{14}$$

where $\rho=\{\rho_{k,\alpha}^{ij},\rho_{k,(\alpha,\beta)}^{ij}\}$ is the set of SP and assignment. This is a non-convex problem due to the fact that SA and the pairing indicator are restricted to be 0 or 1. In fact, the problem is a mixed-integer program. In addition, it needs a central controller to acquire the channel response gain for all the users in the whole network, which is impractical and intractable. So, the transformation and simplification of this problem are stated in the next section.

3 Problem transformation and simplification

To make this problem tractable, we consider the following approximation of the original problem such that it can be decomposed into G parallel problems. According to the inequality $\min\{a,b\}\leq(a+b)/2$, Eqs. (5) and (6) can be expressed as follows:

$$\begin{aligned}r_{\alpha\to\beta}^{kl}\leq\frac{1}{2}\Bigg(&\sum_{i=1}^{N}\rho_{k,\alpha}^{ij}\cdot C\left(\frac{p_{k,\alpha,i}|h_{k,\alpha,i}|^2}{N_0}\right)\\&+\sum_{j'=1}^{N}\rho_{l,\beta}^{i'j'}\cdot C\left(\frac{q_{l,\beta,j'}|g_{l,\beta,j'}|^2}{N_0}\right)\Bigg),\end{aligned} \tag{15}$$

$$\begin{aligned}r_{\beta\to\alpha}^{lk}\leq\frac{1}{2}\Bigg(&\sum_{i'=1}^{N}\rho_{l,\beta}^{i'j'}\cdot C\left(\frac{p_{l,\beta,i'}|h_{l,\beta,i'}|^2}{N_0}\right)\\&+\sum_{j=1}^{N}\rho_{k,\alpha}^{ij}\cdot C\left(\frac{q_{k,\alpha,j}|g_{k,\alpha,j}|^2}{N_0}\right)\Bigg).\end{aligned} \tag{16}$$

With the above inequalities, we can obtain

$$R_{k,l,\alpha,\beta}=\sum_{ij}\rho_{k,\alpha}^{ij}R_{k,\alpha}^{ij}+\sum_{i'j'}\rho_{l,\beta}^{i'j'}R_{l,\beta}^{i'j'}, \tag{17}$$

where we define

$$R_{k,\alpha}^{ij}=\frac{1}{4}\left(C\left(\frac{p_{k,\alpha,i}|h_{k,\alpha,i}|^2}{N_0}\right)+C\left(\frac{q_{l,\beta,j'}|g_{l,\beta,j'}|^2}{N_0}\right)\right), \tag{18}$$

$$R_{l,\beta}^{i'j'}=\frac{1}{4}\left(C\left(\frac{p_{l,\beta,i'}|h_{l,\beta,i'}|^2}{N_0}\right)+C\left(\frac{q_{l,\beta,j'}|g_{l,\beta,j'}|^2}{N_0}\right)\right). \tag{19}$$

Note that in Eq. (17), we find that the information exchanged between user α in cell k and user β in cell l can be bounded by the sum rate of uplink and downlink for user α in cell k and user β in cell l, respectively. Based on this observation, we reformulate the sum rate of conventional users in the whole network as

$$\sum_{k=1}^{G}\sum_{\alpha\in\Psi_k}w_{k,\alpha}R_{k,\alpha}, \tag{20}$$

$$R_{k,\alpha}=\sum_{ij}\rho_{k,\alpha}^{ij}R_{k,\alpha}^{ij}, \tag{21}$$

$$w_{k,\alpha}=w_{k,l,\alpha,\beta}=w_{l,\beta}. \tag{22}$$

Then the original problem becomes

$$\max\sum_{k=1}^{G}\left(\sum_{\alpha\in\Psi_k}w_{k,\alpha}R_{k,\alpha}+\sum_{(\alpha,\beta)\in\Gamma_k}w_{k,k,\alpha,\beta}R_{k,k,\alpha,\beta}\right) \tag{23}$$

$$\text{s.t. Eqs. (1) and (2)}, \rho\in\{0,1\}, \tag{24}$$

$$\sum_{j=1}^{N}\left(\sum_{\alpha\in\Psi_k} q_{k,\alpha,j} + \sum_{(\alpha,\beta)\in\Gamma_k} q_{k,\alpha,\beta,j}\right) \leq \overline{q}_k \quad (25)$$
$$\forall i \text{ and } \forall k\in\{1,2,\ldots,G\},$$

$$\sum_{i=1}^{N} p_{k,\alpha,i} \leq \overline{p}_{k,\alpha},\ 0\leq p_{k,\alpha,i}\leq \overline{p}_i, \quad (26)$$
$$\forall\alpha,\ \forall i,\ \text{and } \forall k\in\{1,2,\ldots,G\}.$$

Note that for Eqs. (23)–(26) the rate for a conventional user is related only to the channel response of its corresponding BS. This greatly simplifies its complexity. The above problem can be decomposed into G parallel problems, and we will consider only one of them. For brevity, we omit the cell index k and summation of k in later sections. However, after the above transformation, the above is still a non-convex problem, requiring mixed-integer programming and non-convex rate formulation of intra-cell users. Then we will reformulate this as a convex problem. The RA problem at a BS with fixed power at users is as follows:

Since the above optimization problem is a non-convex and mixed-integer programming problem, it has intractable complexity in general. In this section, we consider fixed power at users of the above problem, which is transformed into a convex problem. Then, we solve this RA problem and obtain the optimal solution in polynomial time. To transform this problem, we first replace the downlink power for conventional users with

$$s_{\alpha,j} = q_{\alpha,j}\cdot\rho_{\alpha}^{ij}, \quad (27)$$

and the downlink power for intra-cell users with

$$s_{\alpha,\beta,j} = q_{\alpha,\beta,j}\cdot\rho_{(\alpha,\beta)}^{ij}. \quad (28)$$

Note that such a change of notations does not change the original problem, due to the fact that

$$\rho\log(1+p) = \rho\log\left(1+\frac{p}{\rho}\right),\quad \rho\in\{0,1\}. \quad (29)$$

The fact that the equality holds when $\rho=0$ is a result of the L'Hopital rule. Then, for conventional users

$$R_{\alpha}^{ij} = \frac{1}{4}\left(C\left(\frac{p_{\alpha,i}|h_{\alpha,i}|^2}{N_0}\right) + C\left(\frac{s_{\alpha,j}|g_{l,\beta,j'}|^2}{\rho_{\alpha}^{ij}N_0}\right)\right), \quad (30)$$

and for intra-cell pair users

$$R_{\alpha,\beta}^{ij} = \frac{1}{2}\left(C\left(\frac{p_{\alpha,i}|h_{\alpha,i}|^2\frac{s_{\alpha,\beta,j}}{\rho_{(\alpha,\beta)}^{ij}}|g_{\beta,j}|^2}{\left(\frac{s_{\alpha,\beta,j}}{\rho_{(\alpha,\beta)}^{ij}}|g_{\beta,j}|^2+u_{(\alpha,\beta)}^{ij}\right)N_0}\right) + C\left(\frac{p_{\beta,i}|h_{\beta,i}|^2\frac{s_{\alpha,\beta,j}}{\rho_{(\alpha,\beta)}^{ij}}|g_{\alpha,j}|^2}{\left(\frac{s_{\alpha,\beta,j}}{\rho_{(\alpha,\beta)}^{ij}}|g_{\alpha,j}|^2+u_{(\alpha,\beta)}^{ij}\right)N_0}\right)\right). \quad (31)$$

To make this problem tractable, we relax the binary variable ρ, i.e., $\rho\in[0,1]$. Then the problem becomes

$$\max\left(\sum_{\alpha\in\Psi_k} w_{\alpha}\sum_{i,j}\rho_{\alpha}^{ij}R_{\alpha}^{ij} + \sum_{(\alpha,\beta)\in\Gamma_k} w_{\alpha,\beta}\sum_{i,j}\rho_{(\alpha,\beta)}^{ij}R_{\alpha,\beta}^{ij}\right) \quad (32)$$

$$\text{s.t. Eqs. (1) and (2)},\ \rho\in[0,1], \quad (33)$$

$$\sum_{j=1}^{N}\left(\sum_{\alpha\in\Psi} s_{\alpha,j} + \sum_{(\alpha,\beta)\in\Gamma} s_{\alpha,\beta,j}\right)\leq\overline{q}, \quad (34)$$

$$s\geq 0. \quad (35)$$

We introduce a set of new variables $s=\{s_{\alpha,j}, s_{\alpha,\beta,j}\}$. Then this is jointly concave for (ρ,s) after the above redefinition, because Eqs. (9), (18), and (19) are concave functions of $q_{\alpha,j}$ and $q_{\alpha,\beta,j}$. The objective function of this problem is the perspective function of Eqs. (9), (18), and (19) and all constraints are affine. According to the Slater condition (Boyd and Vandenberghe, 2004), this becomes a convex problem. Note that the convex reformulation is necessary because it guarantees a duality gap close to zero. Using continuous relaxation on the integer programming problem is not a new technique. However, by doing this, it typically leads only to heuristics or approximation, which is usually a bound for the original problem and cannot give an insight into SA and SP. In particular, Eqs. (32)–(35) do not necessarily give a binary ρ_{α}^{ij} and $\rho_{\alpha,\beta}^{ij}$ required for Eqs. (11) and (23). In this section, we will solve Eq. (32) with optimal PA and binary indicator results for SA and SP.

We consider the Lagrangian function of Eqs. (32)–(35) as

$$\mathrm{L}(s,\rho,\lambda)=\sum_{j=1}^{N}\left(\mathrm{L}_{\alpha,j}(s_{\alpha,j},\rho,\lambda)+\mathrm{L}_{\alpha,\beta,j}(s_{\alpha,\beta,j},\rho,\lambda)\right)+\lambda\bar{q}, \tag{36}$$

where

$$\mathrm{L}_{\alpha,j}(s_{\alpha,j},\rho,\lambda)=\sum_{\alpha\in\Psi}w_\alpha\sum_{i=1}^{N}\rho_\alpha^{ij}R_\alpha^{ij}-\lambda\sum_{\alpha\in\Psi}\sum_{i=1}^{N}s_{k,\alpha,j} \tag{37}$$

and

$$\mathrm{L}_{\alpha,\beta,j}(s_{\alpha,\beta,j},\rho)=\sum_{(\alpha,\beta)\in\Gamma}w_{\alpha,\beta}\sum_{i=1}^{N}\rho_{(\alpha,\beta)}^{ij}R_{(\alpha,\beta)}^{ij}-\sum_{(\alpha,\beta)\in\Gamma}\sum_{i=1}^{N}s_{\alpha,\beta,j}. \tag{38}$$

The dual function is therefore

$$g(\lambda)=\max_{s,\rho}\mathrm{L}(s,\rho,\lambda). \tag{39}$$

The above maximization of the Lagrangian function can be implemented step by step. First, with a fixed ρ, we can find the optimal power by the Karush-Kuhn-Tucker (KKT) conditions. It is suggested that it can be decomposed into $N\times N\times(K+M/2)$ independent sub-problems, where N represents the number of subcarriers, K represents the cardinality of the conventional user set, and M represents cardinality of the intra-cell user pair set.

1. For fixed $\rho_\alpha^{ij}=1$

We have

$$\max_{s_{\alpha,j}}\left[\frac{w_\alpha}{4}\rho_\alpha^{ij}\left(C\left(\frac{|h_{\alpha,i}|^2p_{\alpha,i}}{N_0}\right)+C\left(\frac{|g_{\alpha,j}|^2s_{\alpha,j}}{N_0}\right)\right)-\lambda s_{\alpha,j}\right]. \tag{40}$$

According to the KKT conditions, the solution can be expressed as

$$s_{\alpha,j}=\left(\frac{1}{4}\frac{w_\alpha\,|g_{\alpha,j}|}{\lambda N_0\ln 2}-1\right)^{+}N_0|g_{\alpha,j}|^{-1}\rho_\alpha^{ij}. \tag{41}$$

2. For fixed $\rho_{(\alpha,\beta)}^{ij}=1$

With a similar process, $s_{\alpha,\beta,j}$ is the larger and non-negative root of the quadratic equation $\mathrm{d}(R_{(\alpha,\beta)}^{ij})/\mathrm{d}(s_{\alpha,\beta,j})=0$. After PA, we consider SA and pairing. We define $A_\alpha^{ij}(\lambda)$ and $A_{(\alpha,\beta)}^{ij}(\lambda)$ as

$$A_\alpha^{ij}(\lambda)=\frac{1}{\rho_\alpha^{ij}}\mathrm{L}_{\alpha,j}(\rho_\alpha^{ij},s_{\alpha,j},\lambda), \tag{42}$$

$$A_{(\alpha,\beta)}^{ij}(\lambda)=\frac{1}{\rho_{(\alpha,\beta)}^{ij}}\mathrm{L}_{\alpha,\beta,j}(\rho_{(\alpha,\beta)}^{ij},s_{\alpha,\beta,j},\lambda). \tag{43}$$

Note that A_α^{ij} and $A_{(\alpha,\beta)}^{ij}$ are independent of ρ_α^{ij} or $\rho_{(\alpha,\beta)}^{ij}$. Because of the multiplication form of Eq. (41) and solution of $s_{\alpha,\beta,j}$, Eq. (39) can be determined by the following optimization problem:

$$\max\sum_{ij}\left(\sum_{\alpha\in\Psi}\rho_\alpha^{ij}A_\alpha^{ij}(\lambda)+\sum_{(\alpha,\beta)\in\Gamma}\rho_{(\alpha,\beta)}^{ij}A_{(\alpha,\beta)}^{ij}(\lambda)\right) \tag{44}$$

$$\text{s.t. Eq. (33).} \tag{45}$$

The matrix $\boldsymbol{\rho}\triangleq[\rho_\alpha^{ij},\rho_{(\alpha,\beta)}^{ij}]_{N\times N\times(K+M/2)}$, according to Boyd and Vandenberghe (2004), can be decomposed into one matrix $\boldsymbol{X}=[x^{ij}]_{N\times N}$ and N^2 vectors $\boldsymbol{y}^{ij}=[y_\alpha^{ij},y_{(\alpha,\beta)}^{ij}]_{1\times(K+M/2)}$, $[\rho_\alpha^{ij},\rho_{(\alpha,\beta)}^{ij}]=[x^{ij}y_\alpha^{ij},x^{ij}y_{(\alpha,\beta)}^{ij}]$, and they satisfy the following conditions:

$$\sum_{i=1}^{N}x^{ij}=1,\ \forall j, \tag{46}$$

$$\sum_{j=1}^{N}x^{ij}=1,\ \forall i, \tag{47}$$

$$\sum_{\alpha\in\Psi}y_\alpha^{ij}+\sum_{(\alpha,\beta)\in\Gamma}y_{(\alpha,\beta)}^{ij}=1. \tag{48}$$

Maximizing $\boldsymbol{\rho}$ in this problem is equivalent to maximizing it over $\boldsymbol{X}$ and $\boldsymbol{y}$. The equivalent problem is

$$\max\sum_{ij}x^{ij}\left(\sum_{\alpha\in\Psi}y_\alpha^{ij}A_\alpha^{ij}(\lambda)+\sum_{(\alpha,\beta)\in\Gamma}y_{(\alpha,\beta)}^{ij}A_{(\alpha,\beta)}^{ij}(\lambda)\right) \tag{49}$$

$$\text{s.t.}\ \sum_{i=1}^{\mathrm{N}}x^{ij}=1,\ \forall_j,\ \sum_{j=1}^{N}x^{ij}=1,\ \forall_i,\ 0\le x_{ij}\le 1, \tag{50}$$

$$\sum_{\alpha\in\Psi}y_\alpha^{ij}+\sum_{(\alpha,\beta)\in\Gamma}y_{(\alpha,\beta)}^{ij}=1,\ y_\alpha^{ij},\ y_{(\alpha,\beta)}^{ij}\in[0,1]. \tag{51}$$

This problem can be divided into two steps. The inner-sum term is maximized over y_α^{ij} and $y_{(\alpha,\beta)}^{ij}$ for each subcarrier pair (i, j), i.e.,

$$\max\left(\sum_{\alpha\in\Psi} y_\alpha^{ij} A_\alpha^{ij}(\lambda) + \sum_{(\alpha,\beta)\in\Gamma} y_{(\alpha,\beta)}^{ij} A_{(\alpha,\beta)}^{ij}(\lambda)\right) \tag{52}$$

$$\text{s.t.} \sum_{\alpha\in\Psi} y_\alpha^{ij} + \sum_{(\alpha,\beta)\in\Gamma} y_{(\alpha,\beta)}^{ij} = 1,\ \ y_\alpha^{ij},\ \ y_{(\alpha,\beta)}^{ij} \in [0,1]. \tag{53}$$

The solution is readily obtained as

$$\begin{cases} y_{\alpha^*}^{ij} = 1, & \alpha^* = \arg\max Q, \\ y_{(\alpha^*,\beta^*)}^{ij} = 1, & (\alpha^*,\beta^*) = \arg\max Q, \end{cases} \tag{54}$$

where

$$Q = \sum_{\alpha\in\Psi} y_\alpha^{ij} A_\alpha^{ij}(\lambda) + \sum_{(\alpha,\beta)\in\Gamma} y_{(\alpha,\beta)}^{ij} A_{(\alpha,\beta)}^{ij}(\lambda).$$

Then, the problem becomes

$$\max \sum_{ij} x^{ij}\left(y_{\alpha^*}^{ij} A_{\alpha^*}^{ij}(\lambda) + y_{(\alpha^*,\beta^*)}^{ij} A_{(\alpha^*,\beta^*)}^{ij}(\lambda)\right) \tag{55}$$

$$\text{s.t. Eq. (50).} \tag{56}$$

It is well known that Eq. (55) is a two-dimensional integer-programming problem. The Hungarian algorithm can solve this problem efficiently. The optimal solution x_*^{ij} is binary. Finally, we can obtain the optimal solution, which satisfies the original constraints Eqs. (1) and (2):

$$\rho_{\alpha^*}^{ij}(\lambda) = x_*^{ij}(\lambda) y_{\alpha^*}^{ij}(\lambda), \tag{57}$$

$$\rho_{(\alpha^*,\beta^*)}^{ij}(\lambda) = x_*^{ij}(\lambda) y_{(\alpha^*,\beta^*)}^{ij}(\lambda). \tag{58}$$

The last step is to solve the Lagrange dual problem $g(\lambda)$:

$$\min g(\lambda) \tag{59}$$

$$\text{s.t. } \lambda \geq 0. \tag{60}$$

The standard sub-gradient and ellipsoid method can be used to search for the optimal λ^*. The process of searching for the optimal binary $\boldsymbol{\rho}$ and PA $\boldsymbol{p}$ is as follows:

1. Initiate λ^0.

2. For given λ^n, where n represents the iteration number, we obtain the PA $s(\lambda^n)$, $\rho_\alpha^{ij}(\lambda^n)$, and $\rho_{(\alpha,\beta)}^{ij}(\lambda^n)$.

3. Based on the sub-gradient or ellipsoid method, update $\lambda^{n+1} = [\lambda^n - \Delta(\lambda^n)e^n]^+$, where Δ represents the step size of iteration and e represents the sub-gradient.

4. Repeat 2 and 3 until it satisfies the stopping criterion.

The complexity of Eq. (52) is $O(K + M/2)$. The complexity of the Hungarian algorithm is $O(N^3)$. The complexity of Eq. (44) is $O(K + M/2 + N^3)$ for each iteration, which is polynomial. The total complexity for all cells is $O(\sum_{k=1}^{G}(M_k/2 + K_k + N^3))$ for each iteration, which is also polynomial.

4 Numerical results

In this section, we will present some simulation results. Because of decomposition, we consider only a single cell with normalized radius. The relative delay is 0, 30, 70, 80, 110, 190, and 410 ns. The relative power is 0, 1, 2, 3, 8, 17.2, and 20.8 dB. We consider two situations:

1. one conventional user and one intra-cell communication pair with 8 subcarriers;

2. two conventional users and two intra-cell communication pairs with 16 subcarriers.

For comparison, we present results on two suboptimal schemes and one conventional transmission scheme:

1. Equal PA: power is uniformly allocated to each subcarrier.

2. No SP: SA and PA are jointly optimized. However, no SP is performed, which means that the same subcarrier index is allocated to a conventional user or intra-cell communication pair for uplink and downlink.

3. Conventional transmission scheme: For intra-cell communication pair (α, β), a BS decodes the information of user α, and then transmits it to user β forward. In the next two slots, it decodes information from user β and transmits it to α forward.

We consider the case where the user power is fixed at 10 dB on each subcarrier in Fig. 2. In Fig. 3, we consider that conventional users and intra-cell users have unequal weights. The upper bound of the primal problem is presented in Figs. 2 and 3. It can be observed that the proposed optimal algorithm using the dual-method approaches the upper bound closely.

From Fig. 2, we find that our proposed scheme outperforms other schemes especially in the high dB region. At 25 dB, our proposed scheme improves the sum rate by 30% and 25% over conventional transmission schemes for 8 and 16 subcarriers, respectively. It is also observed that the sum rate increases when the number of subcarriers increases. In Fig. 2, we also note that the equal PA scheme suffers from a limited performance loss, as compared with our proposed scheme. Based on this observation, we conclude that the equal power transmission scheme may be preferable in a practical system due to its low complexity. Furthermore, we find that the scheme without SP suffers from huge performance loss when the number of subcarriers increases in the high dB region. When the number of subcarriers increases, SP can fully explore the spectrum diversity to improve the performance. Specifically, we set intra-cell communication pairs with weight $w_{\alpha,\beta} = 1.25$, and the conventional user with weight $w_\alpha = 1$. From Fig. 3, we find that the weighted sum rate is improved when the power constraint is in the high dB region. However, the sum rate does not have much difference in the lower dB region. This is because more resource is allocated to conventional users when the power constraint is in the low dB region. More resource is allocated to intra-cell communication pairs when the power constraint is in the high dB region.

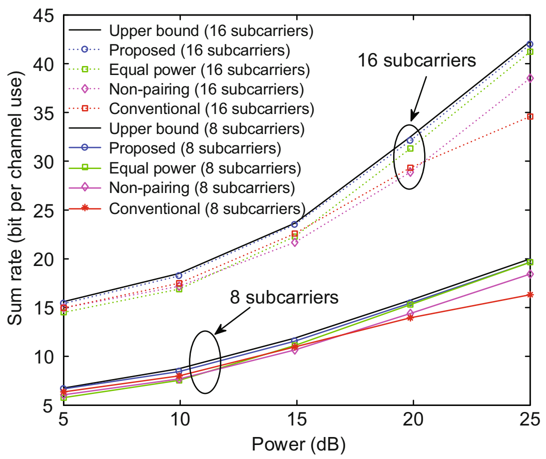

Fig. 2 Sum rate versus power constraint with equal weights

5 Conclusions

Inspired by multi-way relay and intra-cell communication, we proposed that the BS can be considered as a TW DF relay to assist cellular communication. Joint uplink and downlink resource optimization for the OFDMA system was investigated. These problems led to a non-convex optimization problem

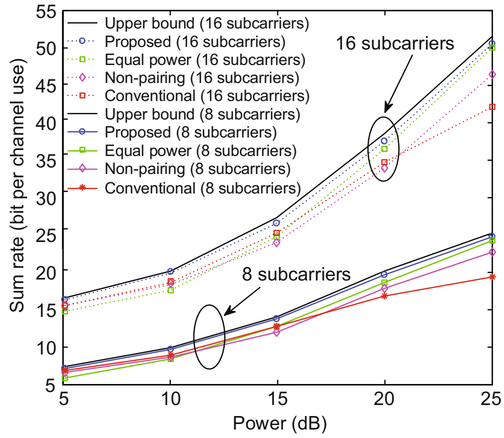

Fig. 3 Sum rate versus power constraint with unequal weights

with high complexity. Since they were intractable, we transformed these problems into less complicated ones, and proposed an efficient algorithm. Finally, numerical results showed that BS serving as a TW relay improves intra-cell communication, especially when the power constraint is in the high dB region.

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