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Optimization of a robust collaborative-relay beamforming design for simultaneous wireless information and power transfer*

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Abstract: We investigate a collaborative-relay beamforming design for simultaneous wireless information and power transfer. A non-robust beamforming design that assumes availability of perfect channel state information (CSI) in the relay nodes is addressed. In practical scenarios, CSI errors are usually inevitable; therefore, a robust collaborative-relay beamforming design is proposed. By applying the bisection method and the semidefinite relaxation (SDR) technique, the non-convex optimization problems of both non-robust and robust beamforming designs can be solved. Moreover, the solution returned by the SDR technique may not always be rank-one; thus, an iterative sub-gradient method is presented to acquire the rank-one solution. Simulation results show that under an imperfect CSI case, the proposed robust beamforming design can obtain a better performance than the non-robust one.

Key words: Simultaneous wireless information and power transfer; Channel state information; Robust beamforming; Semidefinite relaxation; Iterative sub-gradient
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1 Introduction

Simultaneous wireless information and power transfer (SWIPT), which is an energy harvesting (EH) technique, is a promising approach for increasing the working hours of energy-constrained terminals in wireless communication networks (Varshney, 2008; Grover and Sahai, 2010; Sharma et al., 2010). In SWIPT systems, an energy receiver (ER) can harvest energy from the captured radio frequency (RF) signal. The RF signal is usually used to carry information from the transmitter to the information receiver (IR). To increase both wireless power transfer efficiency and an achievable information rate, a multi-antenna technique was implemented to exploit spatial diversity (Khandaker and Rong, 2012; Zhang and Ho, 2013). Zhang and Ho (2013) considered a three-node multiple-input multiple-output (MIMO) broadcasting system for SWIPT, in which two receivers decode the information and separately harvest energy from the signal broadcasted by the common transmitter. However, in certain resource-constrained networks, such as sensor networks, employing multiple antennas at the transmitter or receiver may not be feasible due to hardware limitations. An alternative way to improve the performance of an SWIPT system is to use relay nodes that can collaboratively form a virtual beam to increase the spatial diversity. In general, there

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are three relay methods: amplify-and-forward (AF) (Sendonaris et al., 2003), decode-and-forward (DF) (Laneman et al., 2004), and compress-and-forward (CF) (Kramer et al., 2005). In an AF relay system, the relay nodes amplify only the signal from the transmitter, and then forward it to the receiver. This method is relatively simple and very attractive. The collaborative-relay beamforming design for AF relay networks was optimized with the aid of second-order statistics from the channel state information (CSI) (Havary-Nassab et al., 2008) and perfect instantaneous CSI (Zheng et al., 2009). An optimal beamforming method with the perfect CSI for a relay SWIPT network, which consists of a single transmitter-IR pair, a multi-antenna relay node, and an ER, was proposed by Huang et al. (2014). Li et al. (2014) extended the work of Huang et al. (2014) with multiple transmitter-IR pairs to obtain the maximum sum information rate.

It is worth noting that most of the work mentioned above assumed that the perfect CSI is available. However, in reality, CSI is usually affected by quantization and feedback errors. Furthermore, the performance of a beamforming design based on the assumption of perfect CSI will be degraded in the presence of CSI errors. Hence, it is necessary to develop a robust beamforming design that provides robustness against the CSI errors. Recently, robust beamforming designs for multi-antenna SWIPT systems have been widely studied. Xiang and Tao (2012) presented a robust beamforming technique, which maximizes the worst-case harvested power at the ER while guaranteeing that the information rate at the IR is above a threshold for an MIMO broadcasting SWIPT system under the assumption of imperfect CSI at the transmitter. Xu et al. (2014) studied the robust transceiver design problem for SWIPT in MIMO underlay cognitive radio networks, where the sum harvested power at the ERs is maximized while guaranteeing the required minimum mean square error at the secondary IR and the interference constraints at the primary IRs. Khandaker and Wong (2015) investigated a robust beamforming design for maximizing the harvested energy by the ERs while maintaining the quality of service (QoS) of the IR and keeping information secure from possible wiretapping by the ERs. Almost all of the above robust designs concerned an ER and an IR that were not co-located, or assumed that a receiver worked as either an IR or an ER via a time-switching (TS) mechanism. In addition to a TS mechanism, power-splitting (PS) based robust SWIPT designs have been considered (Khandaker and Wong, 2014; Ng et al., 2014; Wang et al., 2015), where the received signal is split with an adjustable PS ratio to enable SWIPT. Ng et al. (2014) considered a multiuser multiple-input single-output (MISO) downlink system for SWIPT in the presence of potential eavesdroppers with imperfect CSI and passive eavesdroppers without CSI, and proposed a robust beamforming approach to ensure communication secrecy and facilitate efficient energy transfer for PS receivers. Khandaker and Wong (2014) studied the joint robust transmit beamforming and the receive PS problem for minimizing the transmit power of the base station subject to the signal-to-noise ratio (SNR) and energy harvesting constraints on each receiver in MISO multicasting SWIPT systems. Wang et al. (2015) investigated robust beamforming and PS designs that achieved the minimum necessary transmission power while meeting both the signal-to-interference and noise ratio (SINR) and EH requirements per user for a MISO downlink SWIPT system. However, in the work mentioned above, the relay node was not employed for SWIPT and a robust collaborative-relay beamforming design for a relay SWIPT network was not considered either.

In this study, a collaborative-relay beamforming design, consisting of a transmitter, an IR, an ER, and relay nodes, is investigated for a relay SWIPT network. First, a non-robust beamforming design is derived for the case of perfect CSI available at the relay nodes. Then a robust beamforming design is proposed for practical scenarios envisioning imperfect CSI. The bisection method and the semidefinite relaxation (SDR) technique are adopted to solve the non-convex optimization problems of both non-robust and robust beamforming designs. If the returned solution in the SDR technique is not rank-one, an iterative sub-gradient (ISG) method is proposed to effectively obtain the rank-one solution.

Here are the notations in this study. Vectors are written in lowercase boldface letters, while matrices are denoted by uppercase boldface letters. $\boldsymbol{I}_n$ is the $n \times n$ identity matrix and $\mathbf{0}$ is a zero vector or matrix. The superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ stand for conjugate, transposition, and Hermitian transposition of a complex vector or matrix, respectively. $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a complex scalar and Frobenius norm of a vector or matrix, respectively. $\odot$ denotes the (element-wise) Hadamard product, and $\mathrm{tr}(\boldsymbol{X})$ and $\mathrm{rank}(\boldsymbol{X})$ represent the trace and rank of matrix $\boldsymbol{X}$, respectively. Furthermore, $\boldsymbol{X} \succeq \boldsymbol{0}$ means $\boldsymbol{X}$ is a Hermitian positive semidefinite matrix. For a Hermitian matrix $\boldsymbol{X}$, $\lambda_i(\boldsymbol{X})$, $i = 1, 2, \ldots, N$, is its i^{th} eigenvalue, and the maximum eigenvalue $\lambda_{\max}(\boldsymbol{X})$ is defined as $\lambda_{\max}(\boldsymbol{X}) \triangleq \arg \max_{i=1,2,\ldots,N} \lambda_i(\boldsymbol{X})$. $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \ \boldsymbol{\Sigma})$ indicates that the random vector $\boldsymbol{x}$ follows the circular symmetric complex Gaussian distribution with a mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

2 System model

A two-hop AF relay SWIPT network is considered; it consists of a transmitter, an IR, an ER, and $N (N\geq 2)$ relay nodes, as shown in Fig. 1. Each node in the relay SWIPT network is equipped with a single omnidirectional antenna. The relay nodes forward signals from the transmitter to the IR, and charge the ER which harvests energy from the environment. This assumes that the direct links between the transmitter and the two receivers are sufficiently weak to be ignored. This occurs when the direct links are blocked due to long-distance path loss or obstacles.

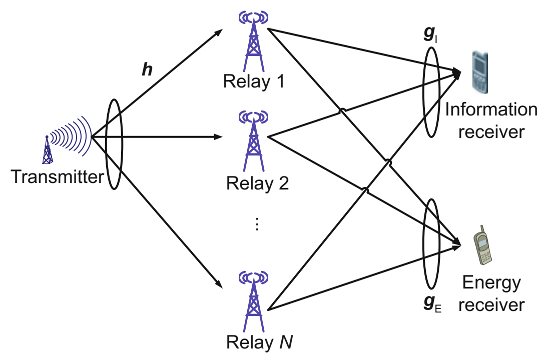

Fig. 1 System model for an amplify-and-forward relay simultaneous wireless information and power transfer network

The relay SWIPT network operates in half-duplex mode and the communication based on the relay nodes takes two time slots. In the first time slot, the transmitter broadcasts its symbol to the relay nodes. Let s denote the zero mean transmitted symbol with power $p_{\mathrm{s}}{=}E\left\{|s|^2\right\}$. The received signal at all relay nodes can be written as

$$\boldsymbol{y}_{\mathrm{R}} = \boldsymbol{h} \cdot s + \boldsymbol{z}_{\mathrm{R}}, \tag{1}$$

where $\boldsymbol{y}_{\mathrm{R}} \triangleq [y_{\mathrm{R},1}, \ y_{\mathrm{R},2}, \ \ldots, \ y_{\mathrm{R},N}]^{\mathrm{T}}$ with $y_{\mathrm{R},n}$ the received signal at the n^{th} relay node, $\boldsymbol{h} \triangleq [h_1, \ h_2, \ \ldots, \ h_N]^{\mathrm{T}}$ with h_n the channel response from the transmitter to the n^{th} relay node, and $\mathbf{z}_{\mathrm{R}} \triangleq [z_1, \ z_2, \ \ldots, \ z_N]^{\mathrm{T}} \sim \mathcal{CN}(\boldsymbol{0}, \ \sigma_{\mathrm{R}}^2 \boldsymbol{I}_N)$ with z_n the additive white Gaussian noise (AWGN) at the n^{th} relay node, $n = 1, 2, \ldots, N$.

In the second time slot, the relay nodes multiply the received signal with a complex beamforming vector to forward the weighted signal to the IR and transfer energy to the ER simultaneously. Let $\boldsymbol{w} = [w_1, \ w_2, \ \ldots, \ w_N]^{\mathrm{T}}$ be the beamforming vector, and the signal vector transmitted from all relay nodes can be written as

$$\boldsymbol{x}_{\mathrm{R}} = \boldsymbol{w} \odot \boldsymbol{y}_{\mathrm{R}} = \boldsymbol{w} \odot \boldsymbol{h} \cdot s + \boldsymbol{w} \odot \boldsymbol{z}_{\mathrm{R}}, \tag{2}$$

where $\boldsymbol{x}_{\mathrm{R}} \triangleq [x_{\mathrm{R},1}, \ x_{\mathrm{R},2}, \ \ldots, \ x_{\mathrm{R},N}]^{\mathrm{T}}$ with $x_{\mathrm{R},n}$ the transmitted signal at the n^{th} relay node, and $\boldsymbol{w} \triangleq [w_1, \ w_2, \ \ldots, \ w_N]^{\mathrm{T}}$ with w_n the beamforming coefficient at the n^{th} relay node, $n = 1, 2, \ldots, N$. The transmit power at the n^{th} relay node can be represented as

$$T_n = p_{\mathrm{s}} \left| \boldsymbol{e}_n^{\mathrm{H}} (\boldsymbol{w} \odot \boldsymbol{h}) \right|^2 + \sigma_{\mathrm{R}}^2 \left| \boldsymbol{e}_n^{\mathrm{H}} \boldsymbol{w} \right|^2, \tag{3}$$

where $\boldsymbol{e}_n$ is the n^{th} column vector of $\boldsymbol{I}_N$, $n = 1, 2, \ldots, N$. The received signal at the IR is

$$y_{\mathrm{I}}{=}\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} \boldsymbol{x}_{\mathrm{R}} + n_{\mathrm{I}}{=}\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} (\boldsymbol{w} \odot \boldsymbol{h}) s + \boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} (\boldsymbol{w} \odot \boldsymbol{z}_{\mathrm{R}}){+}z_{\mathrm{I}}, \tag{4}$$

where $\boldsymbol{g}_{\mathrm{I}} \triangleq [g_{\mathrm{I},1}, \ g_{\mathrm{I},2}, \ \ldots, \ g_{\mathrm{I},N}]^{\mathrm{T}}$ with $g_{\mathrm{I},n}$ the channel response from the n^{th} relay node to the IR, and $z_{\mathrm{I}} \sim \mathcal{CN}(0, \ \sigma_{\mathrm{I}}^2)$ is the AWGN at the IR. Then the received SNR for the IR is given by

$$\mathrm{SNR}_{\mathrm{IR}} = \frac{p_{\mathrm{s}} \left| \boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} (\boldsymbol{w} \odot \boldsymbol{h}) \right|^2}{\sigma_{\mathrm{R}}^2 \left\| \boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} \odot \boldsymbol{w} \right\|^2 + \sigma_{\mathrm{I}}^2}. \tag{5}$$

Accordingly, the achievable information rate for the IR is

$$\begin{aligned} \gamma &= \frac{1}{2} \log_2 (1 + \mathrm{SNR}_{\mathrm{IR}}) \\ &= \frac{1}{2} \log_2 \left(1 + \frac{p_{\mathrm{s}} \left| \boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} (\boldsymbol{w} \odot \boldsymbol{h}) \right|^2}{\sigma_{\mathrm{R}}^2 \left\| \boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}} \odot \boldsymbol{w} \right\|^2 + \sigma_{\mathrm{I}}^2} \right). \end{aligned} \tag{6}$$

The harvested power of the ER during the second time slot is

$$E=\eta\left(p_{\rm s}\left|\boldsymbol{g}_{\rm E}^{\rm H}(\boldsymbol{w}\odot\boldsymbol{h})\right|^2+\sigma_{\rm R}^2\left\|\boldsymbol{g}_{\rm E}^{\rm H}\odot\boldsymbol{w}\right\|^2\right), \quad (7)$$

where $\boldsymbol{g}_{\rm E}\triangleq[g_{{\rm E},1},\ g_{{\rm E},2},\ \ldots,\ g_{{\rm E},N}]^{\rm T}$ with $g_{{\rm E},n}$ the channel response from the $n^{\rm th}$ relay node to the ER, $n=1,2,\ldots,N$, and η is the EH efficiency. For convenience, we define $\eta=1$ without loss of generality.

At high training SNRs, it is reasonable to assume that $\boldsymbol{h}$ can be estimated nearly perfectly from the training sequence at the relay nodes. However, this is not true for $\boldsymbol{g}_{\rm I}$ and $\boldsymbol{g}_{\rm E}$, which in practice, have to be estimated by an IR and an ER, respectively, and fed back to the relay nodes. Then both quantization and feedback errors can occur (Shen et al., 2013). Therefore, perfect CSI for the transmitter-to-relay link and imperfect CSI for the relay-to-receiver links are assumed to be available at the relay nodes. The assumption is reasonable based on the fact that all CSI obtained at the relay nodes can be sent to a centralized processor via high-capacity backhaul links, such as optimal fiber or microwave links, without any further error (Chalise and Vandendorpe, 2010). The actual CSI $\boldsymbol{g}_{\rm I}$ and $\boldsymbol{g}_{\rm E}$ can be modeled as $\boldsymbol{g}_{\rm I}=\hat{\boldsymbol{g}}_{\rm I}+\Delta\boldsymbol{g}_{\rm I}$ and $\boldsymbol{g}_{\rm E}=\hat{\boldsymbol{g}}_{\rm E}+\Delta\boldsymbol{g}_{\rm E}$, respectively, where $\hat{\boldsymbol{g}}_{\rm I}$ and $\hat{\boldsymbol{g}}_{\rm E}$ denote the imperfect CSI known at the relays, $\Delta\boldsymbol{g}_{\rm I}$ and $\Delta\boldsymbol{g}_{\rm E}$ are the CSI error vectors and bounded by $\|\Delta\boldsymbol{g}_{\rm I}\|^2\le\rho_{\rm I}$ and $\|\Delta\boldsymbol{g}_{\rm E}\|^2\le\rho_{\rm E}$, respectively. It is worth noting that such a norm-bounded error model has been widely adopted in the literature and the result given by the spherical error model can be easily extended to the case with an ellipsoidal error region (Zheng et al., 2008).

3 Collaborative-relay beamforming design for simultaneous wireless information and power transfer

In this section, non-robust and robust beamforming designs are derived for the scenarios of perfect and imperfect CSI available at the relay nodes, respectively.

3.1 Non-robust beamforming design

Considering the case where the CSI can be perfectly obtained at the relay nodes, i.e., $\boldsymbol{g}_{\rm I}=\hat{\boldsymbol{g}}_{\rm I}$ and $\boldsymbol{g}_{\rm E}=\hat{\boldsymbol{g}}_{\rm E}$, the objective of a collaborative-relay beamforming design for SWIPT is to maximize the achievable information rate subject to harvested power requirements and peak transmit power constraints at each relay node. According to problems (3), (6), and (7), the non-robust beamforming problem can be expressed as

$$\begin{aligned}\max_{\boldsymbol{w}}\ &\gamma\\ \text{s.t.}\ &E\ge Q,\\ &T_n\le P_n,\ n=1,\ 2,\ \ldots,\ N,\end{aligned} \quad (8)$$

where Q is the predefined threshold of the harvested power of the ER, and P_n is the transmit power budget at the $n^{\rm th}$ relay node. The harvested power requirement implies that the harvested power at the ER should exceed the predefined threshold Q, and the peak transmit power constraint restricts that the consumed power at each relay node should be less than its transmit power budget.

By introducing an auxiliary variable τ, problem (8) can be reformulated as

$$\begin{aligned}\max_{\boldsymbol{w},\tau}\ &\frac{1}{2}\log_2(1+\tau)\\ \text{s.t.}\ &\text{SNR}_{\rm IR}\ge\tau,\\ &E\ge Q,\\ &T_n\le P_n,\ n=1,\ 2,\ \ldots,\ N.\end{aligned} \quad (9)$$

Considering that the SNR requirement in problem (9) is a fractional quadratic constraint, problem (9) can be solved using a binary search over τ. Specifically, for a given τ, the objective is to find a feasible solution which satisfies all the constraints in problem (9), i.e.,

$$\begin{aligned}\text{find}\ &\boldsymbol{w}\\ \text{s.t.}\ &\text{SNR}_{\rm IR}\ge\tau,\\ &E\ge Q,\\ &T_n\le P_n,\ n=1,\ 2,\ \ldots,\ N.\end{aligned} \quad (10)$$

By employing $(\boldsymbol{a}\odot\boldsymbol{b})(\boldsymbol{a}\odot\boldsymbol{b})^{\rm H}=(\boldsymbol{a}\boldsymbol{a}^{\rm H})\odot(\boldsymbol{b}\boldsymbol{b}^{\rm H})$, the SNR requirement in problem (10) can be reformulated as

$$\frac{\operatorname{tr}\left(\left(p_{\rm s}(\boldsymbol{h}^*\boldsymbol{h}^{\rm T})\odot(\hat{\boldsymbol{g}}_{\rm I}\hat{\boldsymbol{g}}_{\rm I}^{\rm H})\right)\boldsymbol{w}\boldsymbol{w}^{\rm H}\right)}{\operatorname{tr}\left(\left(\sigma_{\rm R}^2\boldsymbol{I}\odot(\hat{\boldsymbol{g}}_{\rm I}\hat{\boldsymbol{g}}_{\rm I}^{\rm H})\right)\boldsymbol{w}\boldsymbol{w}^{\rm H}\right)+\sigma_{\rm I}^2}\ge\tau. \quad (11)$$

The harvested power requirement can be expressed as $\operatorname{tr}\Big(\big(p_{\rm s}(\boldsymbol{h}^*\boldsymbol{h}^{\rm T})\odot(\hat{\boldsymbol{g}}_{\rm E}\hat{\boldsymbol{g}}_{\rm E}^{\rm H})+\sigma_{\rm R}^2\boldsymbol{I}\odot(\hat{\boldsymbol{g}}_{\rm E}\hat{\boldsymbol{g}}_{\rm E}^{\rm H})\big)\boldsymbol{w}\boldsymbol{w}^{\rm H}\Big)\ge Q$. The $n^{\rm th}$ relay transmit power constraint can be expressed as $\mathrm{tr}\Big(\big(p_\mathrm{s}(\boldsymbol{h}\boldsymbol{h}^\mathrm{H})\odot(\boldsymbol{e}_n\boldsymbol{e}_n^\mathrm{H})+\sigma_\mathrm{R}^2\boldsymbol{e}_n\boldsymbol{e}_n^\mathrm{H}\big)\boldsymbol{w}\boldsymbol{w}^\mathrm{H}\Big)\leq P_n$.

Define $\boldsymbol{W}\triangleq\boldsymbol{w}\boldsymbol{w}^\mathrm{H}$. Then problem (10) can be reformulated as

$$\begin{aligned}\underset{\boldsymbol{W}\succeq\boldsymbol{0}}{\mathrm{find}}\ &\boldsymbol{W}\\ \mathrm{s.t.}\ &\mathrm{tr}\left[(\boldsymbol{A}_1-\tau\boldsymbol{A}_2)\boldsymbol{W}\right]\geq\tau\sigma_\mathrm{I}^2,\\ &\mathrm{tr}(\boldsymbol{B}\boldsymbol{W})\geq Q,\\ &\mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W})\leq P_n,\ \forall n,\\ &\mathrm{rank}(\boldsymbol{W})=1,\end{aligned}\tag{12}$$

where

$$\begin{aligned}\boldsymbol{A}_1&=p_\mathrm{s}(\boldsymbol{h}^*\boldsymbol{h}^\mathrm{T})\odot(\hat{\boldsymbol{g}}_\mathrm{I}\hat{\boldsymbol{g}}_\mathrm{I}^\mathrm{H}),\\ \boldsymbol{A}_2&=\sigma_\mathrm{R}^2\boldsymbol{I}\odot(\hat{\boldsymbol{g}}_\mathrm{I}\hat{\boldsymbol{g}}_\mathrm{I}^\mathrm{H}),\\ \boldsymbol{B}&=p_\mathrm{s}(\boldsymbol{h}^*\boldsymbol{h}^\mathrm{T})\odot(\hat{\boldsymbol{g}}_\mathrm{E}\hat{\boldsymbol{g}}_\mathrm{E}^\mathrm{H})+\sigma_\mathrm{R}^2\boldsymbol{I}\odot(\hat{\boldsymbol{g}}_\mathrm{E}\hat{\boldsymbol{g}}_\mathrm{E}^\mathrm{H}),\\ \boldsymbol{C}_n&=p_\mathrm{s}(\boldsymbol{h}\boldsymbol{h}^\mathrm{H})\odot(\boldsymbol{e}_n\boldsymbol{e}_n^\mathrm{H})+\sigma_\mathrm{R}^2\boldsymbol{e}_n\boldsymbol{e}_n^\mathrm{H}.\end{aligned}\tag{13}$$

Due to the rank-one constraint, problem (12) is not tractable. To solve this problem, the SDR technique is adopted to relax it into the following convex semidefinite programming (SDP) problem:

$$\begin{aligned}\underset{\boldsymbol{W}\succeq\boldsymbol{0}}{\mathrm{find}}\ &\boldsymbol{W}\\ \mathrm{s.t.}\ &\mathrm{tr}\left[(\boldsymbol{A}_1-\tau\boldsymbol{A}_2)\boldsymbol{W}\right]\geq\tau\sigma_\mathrm{I}^2,\\ &\mathrm{tr}(\boldsymbol{B}\boldsymbol{W})\geq Q,\\ &\mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W})\leq P_n,\ \forall n,\end{aligned}\tag{14}$$

by removing the rank-one constraint. Problem (14) can be efficiently solved using a standard inner-point method (Boyd and Vandenberghe, 2004). If the solution $\boldsymbol{W}_\mathrm{o}$ returned by the SDR technique is of rank-one, then it is also a feasible solution to problem (10) and can be decomposed as $\boldsymbol{W}_\mathrm{o}=\boldsymbol{w}_\mathrm{o}\boldsymbol{w}_\mathrm{o}^\mathrm{H}$, where $\boldsymbol{W}_\mathrm{o}$ is a feasible solution to problem (10). Otherwise, consider that for any $\mathbf{W}\succeq\mathbf{0}$, $\mathrm{tr}(\boldsymbol{W})=\sum\limits_{i=1}^{N}\lambda_i(\boldsymbol{W})\geq\lambda_{\max}(\boldsymbol{W})$ always holds. If the rank of $\boldsymbol{W}$ is larger than one, we can have $\mathrm{tr}(\boldsymbol{W})-\lambda_{\max}(\boldsymbol{W})>0$. To satisfy the rank-one constraint, the value of $\mathrm{tr}(\boldsymbol{W})-\lambda_{\max}(\boldsymbol{W})$ should be equal to zero, which is also the minimum value of $\mathrm{tr}(\boldsymbol{W})-\lambda_{\max}(\boldsymbol{W})$ for any $\mathbf{W}\succeq\mathbf{0}$. Based on this idea, we can obtain

$$\begin{aligned}\min_{\boldsymbol{W}\succeq\boldsymbol{0}}\ &\mathrm{tr}(\boldsymbol{W})-\lambda_{\max}(\boldsymbol{W})\\ \mathrm{s.t.}\ &\mathrm{tr}\left[(\boldsymbol{A}_1-\tau\boldsymbol{A}_2)\boldsymbol{W}\right]\geq\tau\sigma_\mathrm{I}^2,\\ &\mathrm{tr}(\boldsymbol{B}\boldsymbol{W})\geq Q,\\ &\mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W})\leq P_n,\ \forall n.\end{aligned}\tag{15}$$

If problem (12) is feasible, the optimal solution to problem (15) always satisfies $\mathrm{tr}(\boldsymbol{W})-\lambda_{\max}(\boldsymbol{W})=0$, which means that the rank of $\boldsymbol{W}$ is one. Then $\boldsymbol{W}$ is also a feasible solution to problem (12). Unfortunately, problem (15) is a non-smooth concave optimization problem since the objective function is the difference between a linear function and a non-smooth convex function ($\lambda_{\max}(\boldsymbol{W})$ is convex but un-differentiable). Therefore, problem (15) cannot be solved by convex optimization.

The sub-gradient (SG) method (Tuan et al., 2000) is adopted to iteratively solve non-smooth problem (15). Assume $\widehat{\boldsymbol{W}}_\mathrm{o}$ is a feasible solution to problem (15) and $\hat{\boldsymbol{w}}_\mathrm{o}$ is the principal component of $\widehat{\boldsymbol{W}}_\mathrm{o}$ with a unit norm, i.e., the eigenvector corresponding to the maximum eigenvalue. Then an SG of $\lambda_{\max}(\boldsymbol{W})$ at $\widehat{\boldsymbol{W}}_\mathrm{o}$ is $\hat{\boldsymbol{w}}_\mathrm{o}\hat{\boldsymbol{w}}_\mathrm{o}^\mathrm{H}$, since for any $\boldsymbol{W}$,

$$\begin{aligned}&\mathrm{tr}\left((\boldsymbol{W}-\widehat{\boldsymbol{W}}_\mathrm{o})\hat{\boldsymbol{w}}_\mathrm{o}\hat{\boldsymbol{w}}_\mathrm{o}^\mathrm{H}\right)=\hat{\boldsymbol{w}}_\mathrm{o}^H\boldsymbol{W}\hat{\boldsymbol{w}}_\mathrm{o}-\hat{\boldsymbol{w}}_\mathrm{o}^\mathrm{H}\hat{\boldsymbol{W}}_\mathrm{o}\hat{\boldsymbol{w}}_\mathrm{o}\\ &\leq\lambda_{\max}(\boldsymbol{W})-\lambda_{\max}(\hat{\boldsymbol{W}}_\mathrm{o})\end{aligned}\tag{16}$$

always holds. Based on problem (16), we can obtain

$$\begin{aligned}\min_{\boldsymbol{W}\succeq\boldsymbol{0}}\ &\mathrm{tr}(\boldsymbol{W})-\Big(\lambda_{\max}(\widehat{\boldsymbol{W}}_\mathrm{o})+\mathrm{tr}\big((\boldsymbol{W}-\widehat{\boldsymbol{W}}_\mathrm{o})\hat{\boldsymbol{w}}_\mathrm{o}\hat{\boldsymbol{w}}_\mathrm{o}^\mathrm{H}\big)\Big)\\ \mathrm{s.t.}\ &\mathrm{tr}\left((\boldsymbol{A}_1-\tau\boldsymbol{A}_2)\boldsymbol{W}\right)\geq\tau\sigma_\mathrm{I}^2,\\ &\mathrm{tr}(\boldsymbol{B}\boldsymbol{W})\geq Q,\\ &\mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W})\leq P_n,\ \forall n.\end{aligned}\tag{17}$$

Assuming that $\widetilde{\boldsymbol{W}}_\mathrm{o}$ is the optimal solution to problem (17), then it is a better solution than $\widehat{\boldsymbol{W}}_\mathrm{o}$ to problem (15), since

$$\begin{aligned}&\mathrm{tr}(\widetilde{\boldsymbol{W}}_\mathrm{o})-\lambda_{\max}(\widetilde{\boldsymbol{W}}_\mathrm{o})\\ &\leq\mathrm{tr}(\widetilde{\boldsymbol{W}}_\mathrm{o})-\Big(\lambda_{\max}(\widehat{\boldsymbol{W}}_\mathrm{o})+\mathrm{tr}\left((\widetilde{\boldsymbol{W}}_\mathrm{o}-\widehat{\boldsymbol{W}}_\mathrm{o})\hat{\boldsymbol{w}}_\mathrm{o}\hat{\boldsymbol{w}}_\mathrm{o}^\mathrm{H}\right)\Big)\\ &\leq\mathrm{tr}(\widehat{\boldsymbol{W}}_\mathrm{o})-\lambda_{\max}(\widehat{\boldsymbol{W}}_\mathrm{o})\end{aligned}\tag{18}$$

always holds. As a result, initialized by a feasible solution $\widehat{\boldsymbol{W}}_\mathrm{o}$, a sequence of improved solutions to problem (15) can be obtained by iteratively solving problem (17). The initial solution $\widehat{\boldsymbol{W}}_\mathrm{o}$ can be directly obtained by solving problem (14). The iterative procedure terminates until $\widetilde{\boldsymbol{W}}_\mathrm{o}\approx\widehat{\boldsymbol{W}}_\mathrm{o}$, which means that no improvement will be achieved by the ISG method. In practice, once $\mathrm{rank}(\widetilde{\boldsymbol{W}}_\mathrm{o})$ is equal to 1, the procedure could also stop since a rank-one solution to problem (14), i.e., a feasible solution to problem (12), has been found.

To employ the bisection method in solving the beamforming in problem (9), we should determine the lower bound and upper bound of τ. For convenience, the lower bound can be set to zero. For the upper bound, we consider

$$\begin{aligned}\tau &\leq \frac{\operatorname{tr}\Big(\big(p_{\mathrm{s}}(\boldsymbol{h}^{*}\boldsymbol{h}^{\mathrm{T}}) \odot (\hat{\boldsymbol{g}}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}})\big)(\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}})\Big)}{\operatorname{tr}\Big(\big(\sigma_{\mathrm{R}}^{2}\boldsymbol{I} \odot (\hat{\boldsymbol{g}}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}})\big)(\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}})\Big) + \sigma_{\mathrm{I}}^{2}} \\ &< \frac{p_{\mathrm{s}}}{\sigma_{\mathrm{R}}^{2}} \cdot \frac{(\boldsymbol{w} \odot \hat{\boldsymbol{g}}_{\mathrm{I}})^{\mathrm{H}}(\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}})(\boldsymbol{w} \odot \hat{\boldsymbol{g}}_{\mathrm{I}})}{(\boldsymbol{w} \odot \hat{\boldsymbol{g}}_{\mathrm{I}})^{\mathrm{H}}(\boldsymbol{w} \odot \hat{\boldsymbol{g}}_{\mathrm{I}})}.\end{aligned} \tag{19}$$

Based on the Rayleigh-Ritz theorem (Golub and van Loan, 2012), we obtain $\tau < p_{\mathrm{s}}\lambda_{\max}(\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}})/\sigma_{\mathrm{R}}^{2} = p_{\mathrm{s}}\boldsymbol{h}^{\mathrm{H}}\boldsymbol{h}/\sigma_{\mathrm{R}}^{2}$. Therefore, the upper bound can be set to $p_{\mathrm{s}}\boldsymbol{h}^{\mathrm{H}}\boldsymbol{h}/\sigma_{\mathrm{R}}^{2}$.

The initial value of τ is set to be the midpoint $(\tau_{\mathrm{l}} + \tau_{\mathrm{u}})/2$, where τ_{l} and τ_{u} represent the lower and upper bounds of τ, repectively. If a rank-one solution of the SDP to problem (14) can be obtained for a given τ, the lower bound τ_{l} is increased by setting it to the current given τ; otherwise, the upper bound τ_{u} is decreased by setting it to the current given τ. The binary search procedure stops when the desired accuracy given by $\tau_{\mathrm{u}} - \tau_{\mathrm{l}} \leq \tau_{0}$ is achieved. τ_{0} can be set to be $0.001\tau_{\mathrm{l}}$ at each iteration of the binary search procedure. The non-robust collaborative-relay beamforming design for SWIPT is shown in Algorithm 1.

Algorithm 1 Non-robust collaborative-relay beamforming design for SWIPT

1: **Initialize**: set $\tau_{\mathrm{l}} = 0$, $\tau_{\mathrm{u}} = p_{\mathrm{s}}\boldsymbol{h}^{\mathrm{H}}\boldsymbol{h}/\sigma_{\mathrm{R}}^{2}$, $\delta = 0.001$, and $\boldsymbol{w}_{\mathrm{o}} = \boldsymbol{0}$
2: **while** $\tau_{\mathrm{u}} - \tau_{\mathrm{l}} > \tau_{0}$ **do**
3: Compute $\tau = (\tau_{\mathrm{l}} + \tau_{\mathrm{u}})/2$ and solve the SDP problem (14) to obtain $\boldsymbol{W}_{\mathrm{o}}$
4: **if** problem (14) is not feasible **then**
5: Set $\tau_{\mathrm{u}}=\tau$ and go back to step 2
6: **else if** $\operatorname{rank}(\boldsymbol{W}_{\mathrm{o}}) = 1$ **then**
7: Decompose $\boldsymbol{W}_{\mathrm{o}} = \boldsymbol{w}_{\mathrm{o}}\boldsymbol{w}_{\mathrm{o}}^{\mathrm{H}}$, update $\boldsymbol{w}_{\mathrm{o}}$, set $\tau_{\mathrm{l}} = \tau$, and go back to step 2
8: **else**
9: Set $\widehat{\boldsymbol{W}}_{\mathrm{o}}=\boldsymbol{W}_{\mathrm{o}}$
10: Calculate the principal component of $\widehat{\boldsymbol{W}}_{\mathrm{o}}$, and solve problem (17) to obtain $\widetilde{\boldsymbol{W}}_{\mathrm{o}}$
11: **if** $\operatorname{rank}(\widetilde{\boldsymbol{W}}_{\mathrm{o}}) = 1$ **then**
12: Calculate $\widetilde{\boldsymbol{W}}_{\mathrm{o}} = \tilde{\boldsymbol{w}}_{\mathrm{o}}\tilde{\boldsymbol{w}}_{\mathrm{o}}^{\mathrm{H}}$, update $\boldsymbol{w}_{\mathrm{o}} = \tilde{\boldsymbol{w}}_{\mathrm{o}}$, set $\tau_{\mathrm{l}} = \tau$, and go back to step 2
13: **else if** $\left\|\widetilde{\boldsymbol{W}}_{\mathrm{o}} - \widehat{\boldsymbol{W}}_{\mathrm{o}}\right\| > \delta \left\|\widehat{\boldsymbol{W}}_{\mathrm{o}}\right\|$ **then**
14: Set $\widehat{\boldsymbol{W}}_{\mathrm{o}} = \widetilde{\boldsymbol{W}}_{\mathrm{o}}$ and go back to step 10
15: **else**
16: Set $\tau_{\mathrm{u}}=\tau$ and go back to step 2
17: **end if**
18: **end if**
19: **end while**
20: **Output**: $\boldsymbol{w}_{\mathrm{o}}$ and τ_{l}

3.2 Robust beamforming design

When the CSI which is available at the relay nodes is imperfect, the non-robust beamforming design results in performance degradation. To overcome this drawback of the non-robust design, a robust beamforming design based on the worst-case criterion is proposed. The robust beamforming problem can be expressed as

$$\begin{aligned}&\max_{\boldsymbol{w}} \min_{\|\Delta\boldsymbol{g}_{\mathrm{I}}\|^{2}\leq\rho_{\mathrm{I}}} \gamma \\ &\ \text{s.t.} \min_{\|\Delta\boldsymbol{g}_{\mathrm{E}}\|^{2}\leq\rho_{\mathrm{E}}} E \geq Q, \\ &\qquad T_{n} \leq P_{n},\ n = 1,\ 2,\ \ldots,\ N.\end{aligned} \tag{20}$$

The worst-case harvested power requirement implies that the worst-case harvested power at the ER should exceed the predefined threshold Q. By introducing an auxiliary variable τ, the max-min problem (20) can be equivalently expressed as

$$\begin{aligned}&\max_{\boldsymbol{w},\tau} \ \frac{1}{2}\log_{2}(1+\tau) \\ &\ \text{s.t.} \min_{\|\Delta\boldsymbol{g}_{\mathrm{I}}\|^{2}\leq\rho_{\mathrm{I}}} \mathrm{SNR}_{\mathrm{IR}} \geq \tau, \\ &\qquad \min_{\|\Delta\boldsymbol{g}_{\mathrm{E}}\|^{2}\leq\rho_{\mathrm{E}}} E \geq Q, \\ &\qquad T_{n} \leq P_{n},\ n = 1,\ 2,\ \ldots,\ N.\end{aligned} \tag{21}$$

Similar to problem (9), problem (21) can be solved using the binary search over τ. Specifically, for a given τ, the objective is to find a feasible solution satisfying all the constraints in problem (21), which can be expressed as

$$\begin{aligned}&\text{find } \boldsymbol{w} \\ &\ \text{s.t.} \min_{\|\Delta\boldsymbol{g}_{\mathrm{I}}\|^{2}\leq\rho_{\mathrm{I}}} \mathrm{SNR}_{\mathrm{IR}} \geq \tau, \\ &\qquad \min_{\|\Delta\boldsymbol{g}_{\mathrm{E}}\|^{2}\leq\rho_{\mathrm{E}}} E \geq Q, \\ &\qquad T_{n} \leq P_{n},\ n = 1,\ 2,\ \ldots,\ N.\end{aligned} \tag{22}$$

By employing $(\boldsymbol{a}\odot\boldsymbol{b})(\boldsymbol{a}\odot\boldsymbol{b})^{\mathrm{H}} = (\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}})\odot(\boldsymbol{b}\boldsymbol{b}^{\mathrm{H}})$, the worst-case SNR requirement in problem (22) can be expressed as

$$\min_{||\Delta\boldsymbol{g}_{\mathrm{I}}||^{2}\leq\rho_{\mathrm{I}}} \frac{\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}}\big(p_{\mathrm{s}}(\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}) \odot (\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}})\big)\boldsymbol{g}_{\mathrm{I}}}{\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}}\big(\sigma_{\mathrm{R}}^{2}\boldsymbol{I} \odot (\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}})\big)\boldsymbol{g}_{\mathrm{I}} + \sigma_{\mathrm{I}}^{2}} \geq \tau. \tag{23}$$

The worst-case harvested power requirement can be expressed as

$$\min_{\|\Delta \boldsymbol{g}_{\mathrm{E}}\|^2 \le \rho_{\mathrm{E}}} \boldsymbol{g}_{\mathrm{E}}^{\mathrm{H}}\big((p_{\mathrm{s}}\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}} + \sigma_{\mathrm{R}}^2 \boldsymbol{I}) \odot (\boldsymbol{w}\boldsymbol{w}^{\mathrm{H}})\big)\boldsymbol{g}_{\mathrm{E}} \ge Q. \tag{24}$$

Define $\boldsymbol{W} \triangleq \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$. Then problem (22) can be transformed into

$$\begin{aligned}
&\underset{\boldsymbol{W}\succeq \mathbf{0}}{\text{find}} \ \boldsymbol{W}\\
&\text{s.t.} \ \min_{\|\Delta \boldsymbol{g}_{\mathrm{I}}\|^2 \le \rho_{\mathrm{I}}} \frac{\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}}\big(p_{\mathrm{s}}(\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}}) \odot \boldsymbol{W}\big)\boldsymbol{g}_{\mathrm{I}}}{\boldsymbol{g}_{\mathrm{I}}^{\mathrm{H}}(\sigma_{\mathrm{R}}^2\boldsymbol{I} \odot \boldsymbol{W})\boldsymbol{g}_{\mathrm{I}} + \sigma_{\mathrm{I}}^2} \ge \tau,\\
&\quad \min_{\|\Delta \boldsymbol{g}_{\mathrm{E}}\|^2 \le \rho_{\mathrm{E}}} \boldsymbol{g}_{\mathrm{E}}^{\mathrm{H}}\big((p_{\mathrm{s}}\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}} + \sigma_{\mathrm{R}}^2\boldsymbol{I}) \odot \boldsymbol{W}\big)\boldsymbol{g}_{\mathrm{E}} \ge Q,\\
&\quad \mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W}) \le P_n, \ \forall n,\\
&\quad \mathrm{rank}(\boldsymbol{W}) = 1.
\end{aligned} \tag{25}$$

By applying the S-procedure (Beck and Eldar, 2006), problem (25) is transformed into

$$\begin{aligned}
&\underset{\boldsymbol{W}\succeq \mathbf{0}}{\text{find}} \ \boldsymbol{W}\\
&\text{s.t.} \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} - \tau\sigma_{\mathrm{I}}^2 - s_{\mathrm{I}}\rho_{\mathrm{I}} & \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\\ \boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} & \boldsymbol{Q}_{\mathrm{I}} + s_{\mathrm{I}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{I}} \ge 0,\\
&\quad \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} - Q - s_{\mathrm{E}}\rho_{\mathrm{E}} & \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\\ \boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} & \boldsymbol{Q}_{\mathrm{E}} + s_{\mathrm{E}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{E}} \ge 0,\\
&\quad \mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W}) \le P_n, \ \forall n,\\
&\quad \mathrm{rank}(\boldsymbol{W}) = 1,
\end{aligned} \tag{26}$$

where $\boldsymbol{Q}_{\mathrm{E}} = (p_{\mathrm{s}}\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}} + \sigma_{\mathrm{R}}^2\boldsymbol{I}) \odot \boldsymbol{W}$ and $\boldsymbol{Q}_{\mathrm{I}} = (p_{\mathrm{s}}\boldsymbol{h}\boldsymbol{h}^{\mathrm{H}} - \tau\sigma_{\mathrm{R}}^2\boldsymbol{I}) \odot \boldsymbol{W}$.

Due to the rank-one constraint, problem (26) is not tractable. Therefore, the SDR technique is adopted to solve problem (26) by removing the rank-one constraint, which can be expressed as

$$\begin{aligned}
&\underset{\boldsymbol{W}\succeq \mathbf{0}}{\text{find}} \ \boldsymbol{W}\\
&\text{s.t.} \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} - \tau\sigma_{\mathrm{I}}^2 - s_{\mathrm{I}}\rho_{\mathrm{I}} & \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\\ \boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} & \boldsymbol{Q}_{\mathrm{I}} + s_{\mathrm{I}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{I}} \ge 0,\\
&\quad \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} - Q - s_{\mathrm{E}}\rho_{\mathrm{E}} & \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\\ \boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} & \boldsymbol{Q}_{\mathrm{E}} + s_{\mathrm{E}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{E}} \ge 0,\\
&\quad \mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W}) \le P_n, \ \forall n.
\end{aligned} \tag{27}$$

The solution to the SDP problem (27) can be found by a standard inner-point method. If the solution $\boldsymbol{W}_{\mathrm{r}}$ returned by the SDR technique is of rank-one, then it is also a feasible solution to problem (26) and can be denoted as $\boldsymbol{W}_{\mathrm{r}} = \boldsymbol{w}_{\mathrm{r}}\boldsymbol{w}_{\mathrm{r}}^{\mathrm{H}}$, where $\boldsymbol{w}_{\mathrm{r}}$ is a a feasible solution to problem (22). Otherwise, the ISG method is also adopted to obtain the rank-one solution. Assuming that $\widehat{\boldsymbol{W}}_{\mathrm{r}}$ is a feasible solution to problem (27) and $\hat{\boldsymbol{w}}_{\mathrm{r}}$ the principal component of $\widehat{\boldsymbol{W}}_{\mathrm{r}}$, the optimization problem at each iteration can be expressed as

$$\begin{aligned}
&\min_{\boldsymbol{W}\succeq \mathbf{0}} \ \mathrm{tr}(\boldsymbol{W}) - \Big(\lambda_{\max}(\widehat{\boldsymbol{W}}_{\mathrm{r}}) + \mathrm{tr}\big((\boldsymbol{W} - \widehat{\boldsymbol{W}}_{\mathrm{r}})\hat{\boldsymbol{w}}_{\mathrm{r}}\hat{\boldsymbol{w}}_{\mathrm{r}}^{\mathrm{H}}\big)\Big)\\
&\text{s.t.} \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} - \tau\sigma_{\mathrm{I}}^2 - s_{\mathrm{I}}\rho_{\mathrm{I}} & \hat{\boldsymbol{g}}_{\mathrm{I}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{I}}\\ \boldsymbol{Q}_{\mathrm{I}}\hat{\boldsymbol{g}}_{\mathrm{I}} & \boldsymbol{Q}_{\mathrm{I}} + s_{\mathrm{I}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{I}} \ge 0,\\
&\quad \begin{bmatrix} \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} - Q - s_{\mathrm{E}}\rho_{\mathrm{E}} & \hat{\boldsymbol{g}}_{\mathrm{E}}^{\mathrm{H}}\boldsymbol{Q}_{\mathrm{E}}\\ \boldsymbol{Q}_{\mathrm{E}}\hat{\boldsymbol{g}}_{\mathrm{E}} & \boldsymbol{Q}_{\mathrm{E}} + s_{\mathrm{E}}\boldsymbol{I}\end{bmatrix} \succeq \mathbf{0},\\
&\quad s_{\mathrm{E}} \ge 0,\\
&\quad \mathrm{tr}(\boldsymbol{C}_n\boldsymbol{W}) \le P_n, \ \forall n.
\end{aligned} \tag{28}$$

A sequence of the improved solutions $\widetilde{\boldsymbol{W}}_{\mathrm{r}}$ can be obtained by iteratively solving the convex SDP problem (28). The detailed process of the binary search procedure for robust beamforming design is similar to that of the non-robust one. The robust collaborative-relay beamforming design for SWIPT is shown in Algorithm 2.

3.3 Complexity comparison

The main difference between Algorithms 1 and 2 is that different SDP problems are solved in steps 3 and 10, respectively. As Polik and Terlaky (2010) proposed, the computational complexity for solving an SDP problem within a tolerance ε is $O((mn^{3.5} + m^2n^{2.5} + m^3n^{0.5}) \cdot \log(1/\varepsilon))$, where n is the dimension of the semidefinite cone and m is the number of linear constraints. As shown in Algorithm 1, the computational complexity of non-robust beamforming design is mainly due to the computation of the SDP problems (14) and (17), both of which can be solved within a complexity of $O(((N+2)N^7 + (N+2)^2N^5 + (N+2)^3N) \cdot \log(1/\varepsilon)) = O(N^8 \cdot \log(1/\varepsilon))$. Similarly, the computational complexity of the robust design is mainly due to the computation of SDP problems (27) and (28), both of which can be solved within a complexity of $O(((N^2+2)(3N+4)^7 + (N^2+2)^2(3N+4)^5 +$

$(N^2+2)^3(3N+4))\cdot\log(1/\varepsilon)) = O(3^7N^9\cdot\log(1/\varepsilon))$. Therefore, in general, the complexity of the robust design is higher than that of the non-robust one, but it will be apparent that a substantial performance gain can be achieved with the robust design for the case of imperfect CSI, because the robust design takes CSI errors into consideration.

Algorithm 2 Robust collaborative-relay beamforming design for SWIPT

1: **Initialize**: set $\tau_\mathrm{l}=0$, $\tau_\mathrm{u}=p_\mathrm{s}\boldsymbol{h}^\mathrm{H}\boldsymbol{h}/\sigma_\mathrm{R}^2$, $\delta=0.001$, and $\boldsymbol{w}_\mathrm{r}=\boldsymbol{0}$
2: **while** $\tau_\mathrm{u}-\tau_\mathrm{l}>\tau_0$ **do**
3: Compute $\tau=(\tau_\mathrm{l}+\tau_\mathrm{u})/2$ and solve the SDP problem (27) to obtain $\boldsymbol{W}_\mathrm{r}$
4: **if** problem (27) is not feasible **then**
5: Set $\tau_\mathrm{u}=\tau$ and go back to step 2
6: **else if** $\mathrm{rank}(\boldsymbol{W}_\mathrm{r})=1$ **then**
7: Decompose $\boldsymbol{W}_\mathrm{r}=\boldsymbol{w}_\mathrm{r}\boldsymbol{w}_\mathrm{r}^\mathrm{H}$, update $\boldsymbol{w}_\mathrm{r}$, set $\tau_\mathrm{l}=\tau$, and go back to step 2
8: **else**
9: Set $\widehat{\boldsymbol{W}}_\mathrm{r}=\boldsymbol{W}_\mathrm{r}$
10: Calculate the principal component of $\widehat{\boldsymbol{W}}_\mathrm{r}$ and solve problem (28) to obtain $\widetilde{\boldsymbol{W}}_\mathrm{r}$
11: **if** $\mathrm{rank}(\widetilde{\boldsymbol{W}}_\mathrm{r})=1$ **then**
12: Calculate $\widetilde{\boldsymbol{W}}_\mathrm{r}=\widetilde{\boldsymbol{w}}_\mathrm{r}\widetilde{\boldsymbol{w}}_\mathrm{r}^\mathrm{H}$, update $\boldsymbol{w}_\mathrm{r}=\widetilde{\boldsymbol{w}}_\mathrm{r}$, set $\tau_\mathrm{l}=\tau$, and go back to step 2
13: **else if** $\left\|\widetilde{\boldsymbol{W}}_\mathrm{r}-\widehat{\boldsymbol{W}}_\mathrm{r}\right\|>\delta\left\|\widehat{\boldsymbol{W}}_\mathrm{r}\right\|$ **then**
14: Set $\widehat{\boldsymbol{W}}_\mathrm{r}=\widetilde{\boldsymbol{W}}_\mathrm{r}$ and go back to step 10
15: **else**
16: Set $\tau_\mathrm{u}=\tau$ and go back to step 2
17: **end if**
18: **end if**
19: **end while**
20: **Output**: $\boldsymbol{w}_\mathrm{r}$ and τ_l

4 Simulation results

The performance of the proposed robust collaborative-relay beamforming design for SWIPT is demonstrated through numerical simulations. For convenience, the transmit power budget at each relay node is assumed to be $P_n=P$, $n=1, 2, \ldots, N$, the noise power at all receive nodes, including relay nodes and the IR, is assumed to be $\sigma_\mathrm{R}^2=\sigma_\mathrm{I}^2=\sigma^2$, and the CSI error bounds are assumed to be $\rho_\mathrm{I}=\rho_\mathrm{E}=\rho$. The average SNR from the transmitter to the relay nodes p_s/σ^2 is set to be 10 dB. The channel vectors $\boldsymbol{h}$, $\hat{\boldsymbol{g}}_\mathrm{I}$, and $\hat{\boldsymbol{g}}_\mathrm{E}$ are assumed to be Rayleigh flat fading and follow a complex Gaussian distribution $\mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$. The CVX toolbox (Grant and Boyd, 2015) is used to numerically solve the SDP problems. All simulation results are averaged over 1000 independent randomly generated channel realizations.

The performance of the proposed robust beamforming design under the imperfect CSI case is illustrated in terms of feasibility rate and worst-case achievable information rate. To verify the effectiveness of the proposed robust beamforming design, the performance of both the upper and lower bounds of the robust design is also simulated by solving problem (26) with only the SDR technique at each iteration of the binary search procedure. The upper bound is obtained by setting τ_l to the current given τ in Algorithm 2 once the SDP problem (27) is feasible during the iterations, while the lower bound is obtained by setting τ_l to the current given τ if and only if the solution returned by the SDR technique is of rank-one.

Fig. 2 shows the feasibility rates for both robust and non-robust beamforming designs versus the relay transmit power budget to the noise power ratio for $Q/\sigma^2=10$ dB, $N=4$, and $\rho=0.04, 0.2$. The robust beamforming design is considered feasible for channel realization if a non-zero τ_l is returned by Algorithm 2, while the non-robust beamforming design is considered feasible if the worst-case harvested power, denoted as Q', exceeds the predefined threshold Q. For a solution obtained in non-robust beamforming design $\boldsymbol{w}_\mathrm{o}$, Q' can be obtained by solving

$$\begin{aligned}\max_{Q'}\ & Q'\\ \text{s.t.}\ & \begin{bmatrix}\hat{\boldsymbol{g}}_\mathrm{E}^\mathrm{H}\boldsymbol{Q}_\mathrm{E}\hat{\boldsymbol{g}}_\mathrm{E}-Q'-s_\mathrm{E}\rho_\mathrm{E} & \hat{\boldsymbol{g}}_\mathrm{E}^\mathrm{H}\boldsymbol{Q}_\mathrm{E}\\ \boldsymbol{Q}_\mathrm{E}\hat{\boldsymbol{g}}_\mathrm{E} & \boldsymbol{Q}_\mathrm{E}+s_\mathrm{E}\boldsymbol{I}\end{bmatrix}\succeq\boldsymbol{0},\\ & s_\mathrm{E}\geq 0,\end{aligned}\tag{29}$$

where $\boldsymbol{Q}_\mathrm{E}=(p_\mathrm{s}\boldsymbol{h}\boldsymbol{h}^\mathrm{H}+\sigma_\mathrm{R}^2\boldsymbol{I})\odot(\boldsymbol{w}_\mathrm{o}\boldsymbol{w}_\mathrm{o}^\mathrm{H})$. The ideal case with perfect CSI is simulated as a benchmark.

Both the upper and lower bounds of the feasibility rate for the robust beamforming design are also presented in Fig. 2. It can be observed that the feasibility rate for the robust beamforming design increases with the increase of P since a higher P results in a larger feasibility region for the robust beamforming problem (20). The feasibility rate for the non-robust beamforming design also increases with the increase of P, but it is always lower than that of the robust method. Besides, the feasibility rate for the robust method decreases with an increase in the error norm bound, especially in the low P region.

The feasibility rates versus the harvested power threshold to the noise power ratio for $P/\sigma^2 = 20$ dB, $N = 4$, and $\rho = 0.04, 0.2$ are depicted in Fig. 3. It can be seen from Fig. 3 that the feasibility rate for the robust beamforming design decreases with the increase of Q because increasing Q reduces the feasible region of problem (20). In addition, when Q is high, the feasibility rate for the robust method obviously decreases with the increase in the CSI error bound. Fig. 4 presents the feasibility rates versus the number of relay nodes for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 20$ dB, and $\rho = 0.04, 0.2$. We can see from Fig. 4 that the feasibility rate for the robust beamforming design increases with the increase of N, but decreases with an increase in the CSI error bound when N is less than nine. Moreover, the feasibility rate for the non-robust beamforming design is much lower than that of the robust method as N increases from 4 to 10. The feasibility rates versus the CSI error bound for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 10$ dB, and $N = 4$ are shown in Fig. 5. It can be seen that the feasibility rate for the robust beamforming design decreases with an increase in ρ. From Figs. 2–5, we can also find that the feasibility rate for the robust beamforming design is close to its upper bound, which demonstrates that when the feasible region of problem (20) is nonempty for a channel realization, the proposed robust design can almost always find a solution. Furthermore, the feasibility rate for the robust beamforming design is higher than its lower bound. This is because the ISG method can increase the chance that the rank-one solution to problem (27) is found. Fig. 6 illustrates the rank-one solution proportion obtained by both the SDR technique with the ISG method (denoted as "SDR+ISG" in the legend) and by only the SDR technique (denoted as "SDR"). It can be observed that the rank-one solution proportion obtained by the former is above 90%, which is much higher than that obtained by the latter in most of these cases.

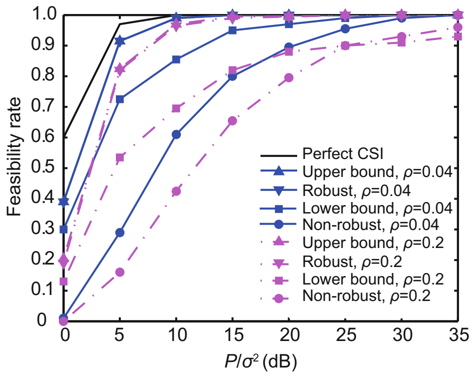

Fig. 2 Feasibility rates versus P/σ^2 for $Q/\sigma^2 = 10$ dB, $N = 4$, and $\rho = 0.04, 0.2$

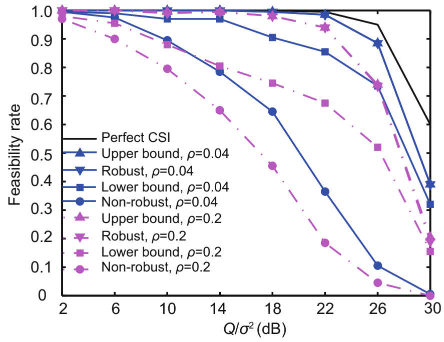

Fig. 3 Feasibility rates versus Q/σ^2 for $P/\sigma^2 = 20$ dB, $N = 4$, and $\rho = 0.04, 0.2$

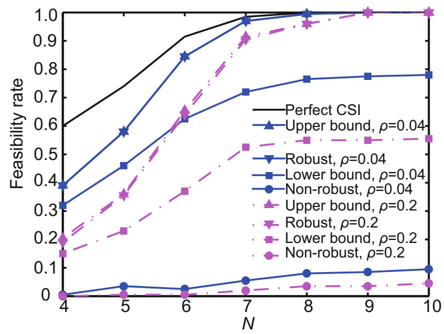

Fig. 4 Feasibility rates versus N for P/σ^2=10 dB, Q/σ^2=20 dB, and ρ=0.04, 0.2

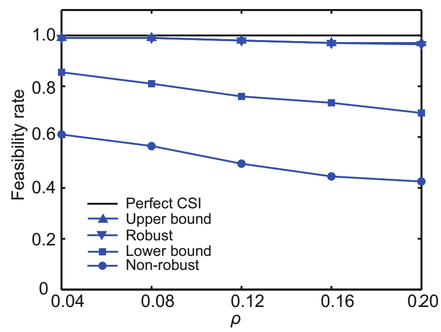

Fig. 5 Feasibility rates versus ρ for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 10$ dB, and $N = 4$

Fig. 7 shows the average worst-case achievable information rates of both robust and non-robust methods versus the relay transmit power budget to the noise power ratio for $Q/\sigma^2 = 10$ dB, $N = 4$, and $\rho = 0.04, 0.2$. For a solution obtained for non-robust

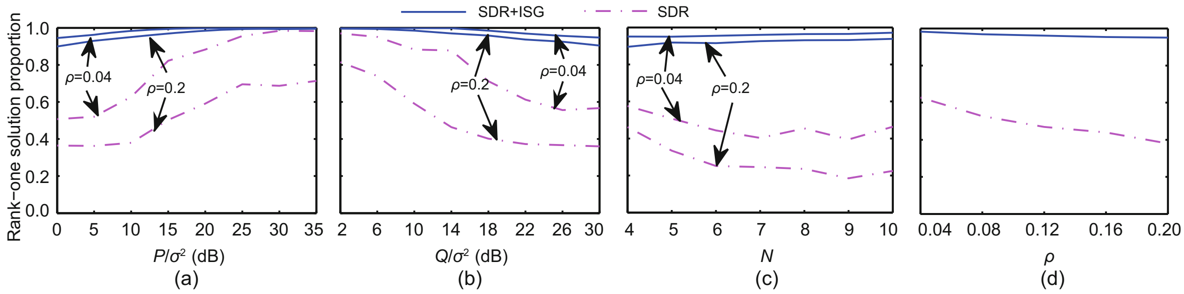

Fig. 6 Rank-one solution proportion: (a) P/σ^2 with $Q/\sigma^2 = 10$ dB, $N = 4$, and $\rho = 0.04, 0.2$; (b) Q/σ^2 with $P/\sigma^2 = 20$ dB, $N = 4$, and $\rho = 0.04, 0.2$; (c) N with $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 20$ dB, and $\rho = 0.04, 0.2$; (d) ρ with $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 10$ dB, and $N = 4$

beamforming design $\boldsymbol{w}_{\rm o}$, the worst-case achievable information rate can be obtained by solving

$$\begin{aligned} &\max_{\tau} \ \tau \\ &\text{s.t.} \ \begin{bmatrix} \hat{\boldsymbol{g}}_{\rm I}^{\rm H}\boldsymbol{Q}_{\rm I}\hat{\boldsymbol{g}}_{\rm I} - \tau\sigma_{\rm I}^2 - s_{\rm I}\rho_{\rm I} & \hat{\boldsymbol{g}}_{\rm I}^{\rm H}\boldsymbol{Q}_{\rm I} \\ \boldsymbol{Q}_{\rm I}\hat{\boldsymbol{g}}_{\rm I} & \boldsymbol{Q}_{\rm I} + s_{\rm I}\boldsymbol{I} \end{bmatrix} \succeq \boldsymbol{0}, \\ &\quad\ s_{\rm I} \geq 0, \end{aligned} \tag{30}$$

where $\boldsymbol{Q}_{\rm I}=(p_{\rm s}\boldsymbol{h}\boldsymbol{h}^{\rm H} - \tau\sigma_{\rm R}^2\boldsymbol{I}) \odot (\boldsymbol{w}_{\rm o}\boldsymbol{w}_{\rm o}^{\rm H})$ and the worst-case achievable information rate is $1/2\log_2(1+\tau)$.

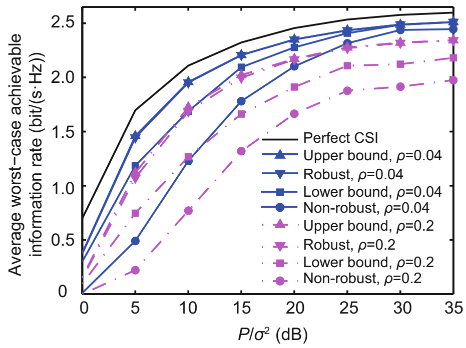

Fig. 7 Average worst-case achievable information rate versus P/σ^2 for $Q/\sigma^2 = 10$ dB, N= 4, and $\rho = 0.04$, 0.2

Both the upper and lower bounds of the worst-case achievable information rate for the robust beamforming design are also displayed in Fig. 7. It can be seen that the average worst-case achievable information rate for the robust beamforming design increases with an increase in P as expected and is higher than that of the non-robust method, especially with a large CSI error bound. The average worst-case achievable information rates versus the harvested power threshold to the noise power ratio for $P/\sigma^2 = 20$ dB, $N = 4$, and $\rho = 0.04, 0.2$ are depicted in Fig. 8. We can find that the average worst-case achievable information rates decrease with the increase of Q, which illustrates the tradeoff between the performance of the IR and ER. Fig. 9 presents the average worst-case achievable information rates versus the number of relay nodes for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 20$ dB, and $\rho = 0.04, 0.2$. It can be observed that the robust beamforming design performs much better than the non-robust method as N increases from 4 to 10. The average worst-case achievable information rates versus the CSI error bound for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 10$ dB, and $N = 4$ are shown in Fig. 10. As can be seen from Fig. 10, the performance of both robust and non-robust methods degrades with the increase in the error bound since a larger error bound would decrease the lowest SNR at the IR. Moreover, we can observe from Figs. 7–10 that the average worst-case achievable information rate for the robust beamforming design approaches its upper bound, which illustrates that the proposed Algorithm 2 could achieve a near-optimal solution to problem (20). Besides, the robust beamforming design outperforms its lower bound, which illustrates the performance gain by exploiting the SDR technique with the ISG method to obtain the rank-one solution to problem (26).

5 Conclusions

In this paper, a collaborative-relay beamforming design for a relay SWIPT network has been investigated. Under the scenarios of perfect and imperfect CSI available at the relay nodes, non-robust and robust beamforming designs have been proposed, respectively. The non-convex optimization problems of the proposed beamforming designs can be solved

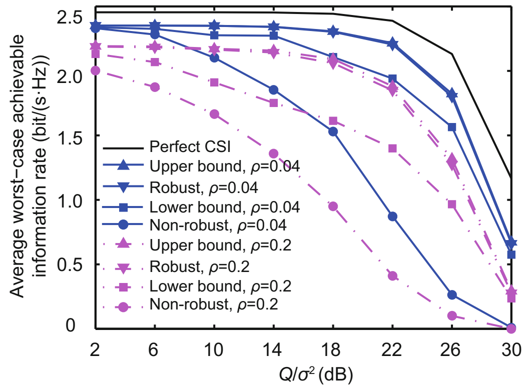

Fig. 8 Average worst-case achievable information rate versus Q/σ^2 for $P/\sigma^2 = 20$ dB, $N = 4$, and $\rho = 0.04$, 0.2

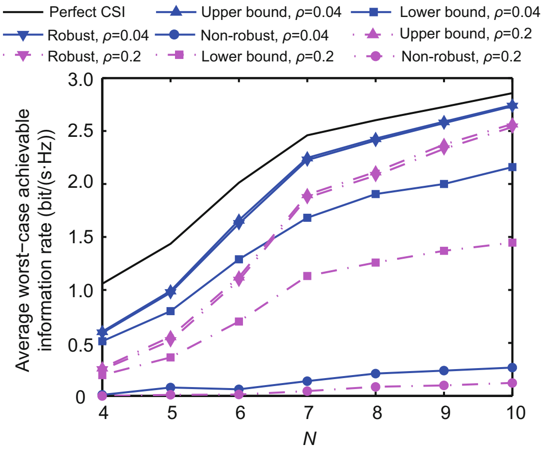

Fig. 9 Average worst-case achievable information rate versus N for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 20$ dB, and $\rho =$ 0.04, 0.2

by the bisection method and SDR technique. When the solution returned through the SDR technique is not of rank-one, an ISG method has also been presented to obtain the rank-one solution. The simulation results have verified the improved performance of the robust beamforming design compared to the non-robust one in the presence of imperfect CSI.

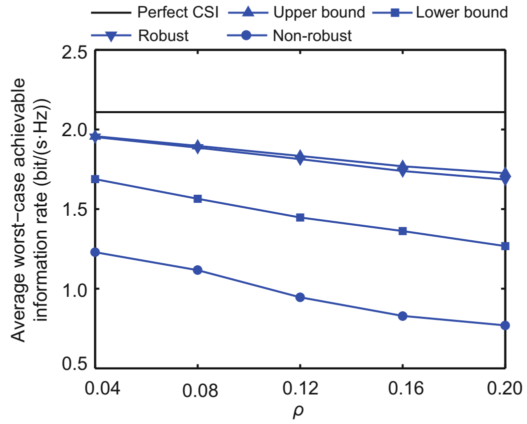

Fig. 10 Average worst-case achievable information rate versus ρ for $P/\sigma^2 = 10$ dB, $Q/\sigma^2 = 10$ dB, and $N = 4$

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