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Performance analysis of the stop-and-wait automatic repeat request protocol under Markovian interruptions

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Abstract: The performance of an integrated packet voice/data multiplexer using a stop-and-wait (SW) automatic repeat request (ARQ) protocol is discussed. We assume that the input for the data traffic is exponentially distributed in increments via the Poisson process, with each data packet transmitted within an individual slot time. Another assumption is that there is only a single voice signal, which has a higher priority over the data packet, and whose traffic is given via an on-off Markov process. Whenever the voice signal is active, the output link is used and will be blocked for the data packet. We introduce the concept of buffer occupancy to simplify the analysis, and discover that data multiplexers using the SW ARQ protocol exhibit a behavior of queueing delay and buffering when the interruption signal is given via a Markov process. Simulation results verify the validity of the analytical results.

Key words: Stop-and-wait ARQ protocol; Markovian interruptions; Poisson distribution; Buffer occupancy; Waiting time
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1 Introduction

Error control techniques are used to provide reliable communications over noisy channels. The commonly used techniques are forward error correction (FEC), automatic repeat request (ARQ), or a combination, called hybrid ARQ. In ARQ, data is protected by error detecting codes. If the receiver detects errors, the corresponding frame will be retransmitted.

ARQ protocols for error control are divided into three schemes: stop-and-wait (SW), go-back-*N* (GB(*N*)), and selective repeat (SR). The SW ARQ scheme, where the transmitter sends out a data packet and stops the transmission while waiting for the corresponding acknowledgement/negative acknowledgement (ACK/NACK) signal from the receiver, is considered. If the transmitter receives the ACK signal, it will attempt to transmit the next data packet. This SW ARQ protocol is simple and can be implemented easily, but its throughput is relatively low. The performance of the SW ARQ scheme is strongly influenced by the round-trip delay. In contrast to SW, both GB(*N*) and SR protocols transmit frames continuously without waiting for acknowledgement messages. So, GB(*N*) and SR are collectively called continuous ARQ protocols. The ARQ protocol is chosen in this work for the retransmission of erroneous data packets.

In this work, a communication system that has a single time-slotted link used for the transmissions of data packets and Markovian signals is discussed. It is assumed that the voice traffic as the Markovian interruptions has precedence over data packets. When no Markovian signal is present, the data packet is sent out in a time slot if the data buffer is not empty. In this type of integrated system, data and voice traffic may be transmitted alternately through a single transmission link. Choi (2005) analyzed some queueing models in the integrated voice and data network. Gebali (2015) described the buffer behavior with

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single server queue, service time, waiting time, Poisson arrivals, synchronous transmission, and server interruptions through the on-off Markov process. The queueing analysis of packet delay within the transmitted data blocks and buffer occupancy was performed by de Munnynck et al. (2002). Also, Qin and Yang (2010) investigated the steady-state throughput of general network coding nodes, when the simple SW ARQ transmission scheme was employed. Before analysis of the performance of such a data multiplexing system under a Markovian interruption signal, research trends on the ARQ protocol and the modeling of voice traffic will be discussed.

Several models have been used to study the SW ARQ protocol. Rehman et al. (2016) analyzed both the throughput of average packet delay and end-to-end packet delay of the cognitive SW hybrid ARQ system. Their analytical approach is probability based and discrete time Markov chain based. This work was inspired by our previous studies (Khongorzul and Shinn, 2008, 2013). We investigated the achievable average buffer occupancy and the corresponding average waiting times of GB(N) ARQ in voice integrated networks. In this integrated system, the data and voice traffic are transmitted alternately in a single transmission link, and assumed that the retransmission interval time is equal to n time slots. The output link channel changed by available (A) and blocked (B) states. Then the channel enters a blocking state, which is generated by Markovian voice interruptions.

In this study, with the transmitter in the SW ARQ scheme, the sender waits until acknowledgement of signals before starting its next transmission. During this waiting time, the transmitter is not allowed to retransmit a packet. On the other hand, a simulation based study was provided by Khongorzul and Shinn (2013), in which the transmitter continuously transmitted N packets in every time slot without waiting for their acknowledgement signal during the available period.

2 System description

We compare the models of Markovian interruptions and conventional priority queue. These two models are equivalent if the characteristics of high-priority traffic are Markovian in the conventional priority queue. The purpose of this study is to derive analytical results on the proposed model, not to compare the theoretical results with those from a realistic environment. To simplify the analysis, the transmission errors of ACK (NACK) in the reverse channel are not considered, so timeout events do not occur in this ARQ model. Therefore, it is expected that the transmitter can receive the ACK (NACK) packet when the round-trip time has passed just after its original data transmission.

Fig. 1 shows the model of the SW ARQ protocol, where a packet in the input buffer Q_x may be transmitted even before receiving the acknowledgement signal for the previous packet (Khongorzul and Shinn, 2008). Here, the effects of interruption traffic are discussed. If the interruption traffic is heavy, it will be more difficult for the data traffic to have a chance of being transmitted, and the delay will be longer. If the interruption traffic is light, it will be easier for the data traffic to have a chance of being transmitted, and the delay will be shorter. The traffic pattern of a single voice can be modeled as a two-state Markov signal. If there is voice traffic, then the data traffic should be blocked, since a voice signal has a higher priority than data traffic. Therefore, this voice signal serves as an interruption signal onto data packets. Even though the retransmission of voice signals has been discussed in some research (Leong et al., 2006), the retransmission of voice signals is not considered in this study. Voice activity usually follows exponentially distributed on/off patterns. If voice and data integration in case of a single voice can be analyzed successfully, the analysis for voice and data integration in case of multiple voices can be considered next.

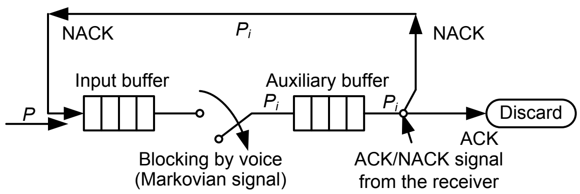

Fig. 1 System model

3 Modeling and assumptions

Fig. 2 shows an SW ARQ protocol, where the transmitter should check the ACK feedback signal before sending the next packet at the next available frame time. However, in the case of transmission error, the transmitter receives the NACK signal from the receiver and retransmits the current packet again at the next frame time after one slot. When the buffer is empty, the transmitter may check the buffer. If the buffer is not empty, the transmitter may send out the packet at the next time slot. If the receiver does not receive ACK or NACK signals due to Markovian interruptions in the feedback channel, the receiver waits for a certain timeout period and sends the frame again.

The assumptions for obtaining the buffer behavior of the system (under Markovian interruptions) adopting the SW ARQ scheme are as follows (Khongorzul and Shinn, 2013):

1. The buffer size of the transmitter is infinite, which results in no overflow of data packets.

2. The transmission is divided into a series of slot times. There is no transmission error at the feedback channel.

3. Round-trip propagation delay, r, is defined as the time delay from the end of data packet transmission time to the corresponding acknowledgment packet's instant response time (in slots).

4. Data packets are assumed to arrive at the transmitter at the beginning of a time slot. The number of data packet arrivals within a slot time is an independent and identically distributed random variable given by the Poisson process. The probability $P(j)$ with j arrivals in a time slot is given by

$$P(j) = \exp(-\lambda)\lambda^j / j!, \tag{1}$$

where λ is the average number of arrivals in a time slot. The z-transform for the number of arrivals per slot is given by

$$S(z) = \sum_{j=0}^{\infty} P(j) z^j = \exp[\lambda(z-1)]. \tag{2}$$

The z-transform of the number of arrivals $R(z)$ during the retransmission time interval is determined as

$$R(z) = \left(S(z)\right)^r, \tag{3}$$

where r represents the length of the round-trip delay in time slots.

5. Voice activity has two states: silence (available, A) and talkspurt (blocked, B) (OFF/ON). These two states of digitized voices are geometrically distributed in a time-slotted system (Gebali, 2015). Fig. 3 describes the Markov chain model that follows the discrete time available–blocked process.

The state transition probabilities (α_s, β_s) during one slot time in Fig. 3 are defined as follows:

1. α_s is the probability that the system remains in state A (available) at the next time slot interval when the system was in state A at the preceding time slot.

2. β_s is the probability that the system remains in state B (blocked) at the next time slot interval when the system was in state B at the preceding time slot.

The probability $P(A)$ that no Markovian interruptions exist (so the output channel may be available for data transmission) is given by

$$P(A) = \frac{1-\beta_s}{(1-\alpha_s)+(1-\beta_s)} = \frac{1-\beta_s}{2-\alpha_s-\beta_s}. \tag{4}$$

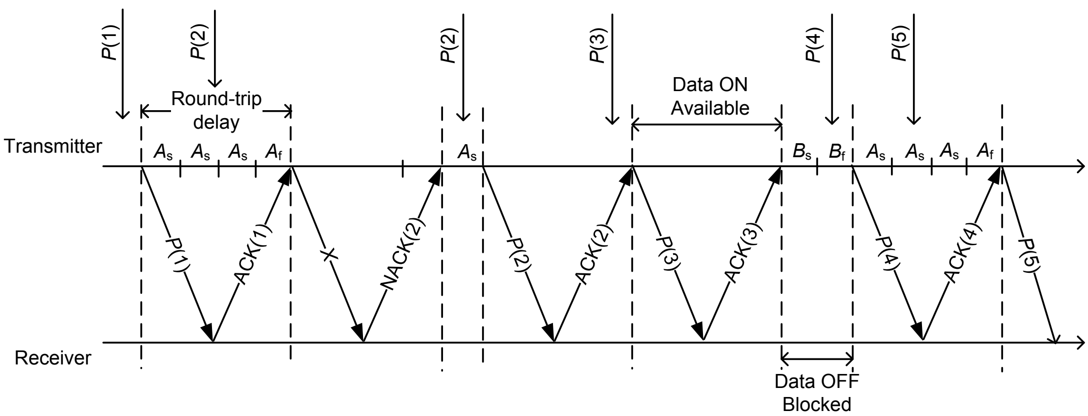

Fig. 2 Transmission diagram for the SW ARQ protocol (round trip time r=four slots)

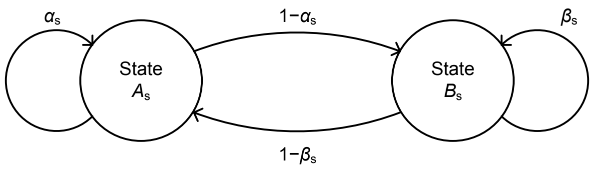

Fig. 3 Markov chain for modeling voice activity

To find the approximate upper limit of traffic intensity for each packet transmission error, the r packets in the maximum including the erroneous packet itself will be retransmitted. Then the traffic intensity will increase. However, it should be less than $P(A)$ in any case. The probability for the occurrence of i transmission errors is given by $p^i(1-p)$. The corresponding maximum possible number of transmissions is $1+i$. Therefore, the increased traffic intensity is given by

$$\lambda' = \lambda \sum_{i=0}^{\infty} (1+i) p^i (1-p) = \frac{\lambda}{1-p}$$

and will not exceed the available probability of $P(A)$. This will lead to the stable criterion with respect to the arrival rate in the following: since $\lambda_{\max}$ should be smaller than $P(A)$ (i.e., $\lambda_{\max}<P(A)$), λ will be given by

$$\lambda \le (1-p)\frac{1-\beta_s}{2-\alpha_s-\beta_s}. \tag{5}$$

4 Analysis of the stop-and-wait ARQ protocol

4.1 Analysis of queueing behavior

Fig. 4 illustrates the state transition diagram of the SW schemes under the Markovian interruptions. The availability period is a random variable, which follows the geometric probability density function where a time-slotted output channel is assumed. Therefore, parameters α_s and β_s are obtained from Gebali (2015) as follows:

$$P_A(m) = (1-\alpha)\alpha^{m-1}, \quad P_B(m) = (1-\beta)\beta^{m-1}.$$

The average value of the available period is obtained as follows: The probability that system stays in an available state for m consecutive slots successfully is given by $(1-\alpha)\alpha^{m-1}$. The average value is given by

$$\overline{A} = \sum_{m=1}^{\infty} m(1-\alpha)\alpha^{m-1} = \frac{1}{1-\alpha}. \tag{6}$$

In a similar way, the average value of a blocked period is given by

$$\overline{B} = \sum_{m=1}^{\infty} m(1-\beta)\beta^{m-1} = \frac{1}{1-\beta}. \tag{7}$$

Fig. 4 shows the transition diagram among these states. (α_r, β_r) are defined as follows:

1. α_r is the transition probability from state A to state A during the round-trip delay time, i.e., r [slots].

2. β_r is the transition probability from state B to state B during the round-trip delay time, i.e., r [slots].

Then (α_r, β_r) can be found from (α_s, β_s) as

$$\begin{bmatrix} \alpha_r & 1-\alpha_r \\ 1-\beta_r & \beta_r \end{bmatrix} = \begin{bmatrix} \alpha_s & 1-\alpha_s \\ 1-\beta_s & \beta_s \end{bmatrix}^r. \tag{8}$$

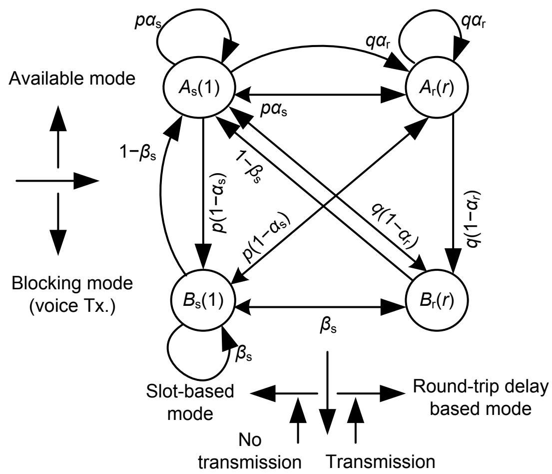

Fig. 4 State transition diagram of the data multiplexer under Markovian interruptions for the SW ARQ protocol
The four states are A_s (slot-based, available), A_r (round-trip delay based, available), B_s (slot-based, blocked), and B_r (round-trip delay based, blocked)

Also, in Fig. 4, the system state changes from A_s to B_s if the channel is changed to blocked state B because of the interruption signal. In state A_s the channel is available. One data packet will be transmitted if the buffer is not empty. If this transmission is successful, then the transmitter is eligible to transmit a packet at the next time slot. The system state at the next time slot will be A_s with probability $q\alpha_s$ or B_s

with probability $q(1-\alpha_s)$. If the transmission is successful, state A_r occurs when the ACK signal is received and the channel is available. State B_r occurs when the NACK signal is received and the channel is changed to the blocked state. State B_s can also be entered during a slot time from state A_r or B_r if the output link is blocked during the time slot period. Now $A_s(z)$, $A_r(z)$, $B_s(z)$, and $B_r(z)$ are defined to be the distribution of buffer occupancy at states A_s, A_r, B_s, and B_r, respectively. The probability of a packet error in transmission is given by p, and the probability of no packet error in transmission is given by $q=1-p$.

4.2 Average buffer occupancy

In this subsection, the buffer behavior of the SW ARQ protocol will be examined. In this protocol, the sender waits until the reception of ACK/NACK signals before starting its next transmission. Thus, the minimum time interval for consecutive transmission is contained for the frame time in slots.

The overall systems are separated into four states based on the following two observations:

1. If the packet is transmitted in error, the packet will be retransmitted after the round-trip delay, which is equal to r slots. If the packet is transmitted in success, on the contrary, the buffer occupancy will be decremented by one after the frame time. Therefore, the time epochs of interest are divided into two classifications: slot- and frame-based time epochs.

2. If there is a voice signal occurrence, it will be transmitted first, since the voice signal is real-time traffic with a higher priority over the data packet. During the transmission of the voice signal, data traffic will be blocked. Depending on the existence of a voice signal, the system will be divided into two groups: A or B state.

In the state transition diagram in Fig. 4, the buffer occupancy is derived in the following paragraph. The z-transform of the buffer occupancy $A_s(z)$, $A_r(z)$, $B_s(z)$, and $B_r(z)$ can be given by

$$\begin{aligned} A_s(z) &= \alpha_s\left(A_s(0)+A_r(0)\right)S(z) \\ &\quad +(1-\beta_s)\left(B_s(z)+B_r(z)\right)S(z), \end{aligned} \tag{9}$$

$$\begin{aligned} A_r(z) &= (qz^{-1}+p)\alpha_r\left(A_r(z)-A_r(0)\right)R(z) \\ &\quad +(qz^{-1}+p)\alpha_r\left(A_s(z)-A_s(0)\right)R(z), \end{aligned} \tag{10}$$

$$\begin{aligned} B_s(z) &= (1-\alpha_s)\left(A_s(0)+A_r(0)\right)S(z) \\ &\quad +\beta_s\left(B_s(z)+B_r(z)\right)S(z), \end{aligned} \tag{11}$$

$$\begin{aligned} B_r(z) &= (qz^{-1}+p)(1-\alpha_r)\left(A_r(z)-A_r(0)\right)R(z) \\ &\quad +(qz^{-1}+p)(1-\alpha_r)\left(A_s(z)-A_s(0)\right)R(z), \end{aligned} \tag{12}$$

$$\frac{1-\beta_s}{2-\alpha_s-\beta_s}-\frac{\lambda}{1-p}=A_s(0)+A_r(0), \tag{13}$$

where $S(z)$ and $R(z)$ are the distributions of new arrivals during the slot time and frame time respectively and are given by Eqs. (2) and (3) respectively. Symbols $A_s(z)$ and $A_r(z)$ represent the buffer behavior when the output link is available. Likewise, $B_s(z)$ and $B_r(z)$ are the buffer behavior while the output link is blocked due to Markovian interruptions of voice, given by Eqs. (11) and (12), respectively. The first line of $A_r(z)$ represents the new buffer occupancies one round-trip delay time later, after a data packet is transmitted, because it takes one frame time for the transmitter to receive the corresponding answers (ACK or NACK) from the receiver. The term of (qz^{-1}) acknowledges that the transmission is successful with probability q and thus buffer occupancy is decremented by one. The term of p represents the probability that there is a transmission error; therefore, buffer occupancy is not decremented. α_r in Eq. (10) is the probability that the system stays in the available state for the next time frame on the condition that the system stayed in state A at the previous frame time. Solving Eqs. (9)–(12) for $A_s(z)$, $A_r(z)$, $B_s(z)$, and $B_r(z)$, we obtain $X_1(z)$, $X_2(z)$, $N_{AS}(z)$, and $D_{AS}(z)$ as follows:

$$X_1(z)=\alpha_r(qz^{-1}+p)R(z), \tag{14}$$

$$X_2(z)=(1-\alpha_r)(1-\beta_s)(qz^{-1}+p)S(z)R(z), \tag{15}$$

$$\begin{aligned} N_{AS}(z) &= (1-\alpha_s-\beta_s)S^2(z)\left(1-X_1(z)\right) \\ &\quad +\alpha_s S(z)\left(1-X_1(z)\right)-X_2(z), \end{aligned} \tag{16}$$

$$D_{AS}(z)=\left(1-\beta_s S(z)\right)\left(1-X_1(z)\right)-X_2(z). \tag{17}$$

Then from Eq. (A11) in the Appendix, $A_s(z)$ will be given as follows:

$$\begin{aligned} A_s(z) &= \frac{\alpha_s S(z)+(1-\alpha_s-\beta_s)S^2(z)}{\left(1-\beta_s S(z)\right)\left(1-X_1(z)\right)-X_2(z)} \\ &\quad \cdot\left(1-X_1(z)\right)-X_2(z)\left(A_s(0)+A_r(0)\right) \\ &= \frac{N_{AS}(z)}{D_{AS}(z)}\left(A_s(0)+A_r(0)\right), \end{aligned} \tag{18}$$

where $N_{\mathrm{AS}}(z)$ and $D_{\mathrm{AS}}(z)$ are given by Eqs. (16) and (17), respectively.

Using Eq. (18) we can derive the following:

$$\begin{aligned} A_{\mathrm{r}}(z) &= \frac{X_1(z)}{1-X_1(z)}\left(\frac{N_{\mathrm{AS}}(z)}{D_{\mathrm{AS}}(z)}-1\right)\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right) \\ &= \frac{N_{\mathrm{AR}}(z)}{D_{\mathrm{AR}}(z)}\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right), \end{aligned} \tag{19}$$

$$\begin{aligned} B_{\mathrm{s}}(z) &= \frac{S(z)}{1-\beta_{\mathrm{s}}S(z)}\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right)\Bigg((1-\alpha_{\mathrm{s}}) \\ &\quad -\frac{(qz^1+p)(1-\alpha_{\mathrm{r}})\beta_{\mathrm{s}}\left(N_{\mathrm{AS}}(z)-D_{\mathrm{AS}}(z)\right)}{(1-X_1(z))D_{\mathrm{AS}}(z)}\Bigg) \\ &= \frac{N_{\mathrm{BS}}(z)}{D_{\mathrm{BS}}(z)}\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right), \end{aligned} \tag{20}$$

$$\begin{aligned} B_{\mathrm{r}}(z) &= \frac{(1-\alpha_{\mathrm{r}})(qz^{-1}+p)R(z)}{\left(1-X_1(z)\right)D_{\mathrm{AS}}(z)}\left(N_{\mathrm{AS}}(z)-D_{\mathrm{AS}}(z)\right) \\ &\quad \cdot\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right)=\frac{N_{\mathrm{BR}}(z)}{D_{\mathrm{BR}}(z)}\left(A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)\right), \end{aligned} \tag{21}$$

where $N_{\mathrm{AR}}(z)$, $D_{\mathrm{AR}}(z)$, $N_{\mathrm{BS}}(z)$, $D_{\mathrm{BS}}(z)$, $N_{\mathrm{BR}}(z)$, and $D_{\mathrm{BR}}(z)$ are given in Eqs. (A17)–(A22) in the Appendix.

From now on, we are going to derive the solutions: first $A_{\mathrm{s}}(1)$ can be given by $A_{\mathrm{s}}(1)=\lim_{z\to 1}A_{\mathrm{s}}(z)= A(0)\lim_{z\to 1}\frac{N_{\mathrm{AS}}(z)}{D_{\mathrm{AS}}(z)}$, where $A(0)$ is given by $A(0)=A_{\mathrm{s}}(0)+A_{\mathrm{r}}(0)$. Since numerator $N_{\mathrm{AS}}(z)$, $N_{\mathrm{AR}}(z)$, $N_{\mathrm{BS}}(z)$, or $N_{\mathrm{BR}}(z)$, and denominator $D_{\mathrm{AS}}(z)$, $D_{\mathrm{AR}}(z)$, $D_{\mathrm{BS}}(z)$, or $D_{\mathrm{BR}}(z)$ go to zero when z goes to one infinitesimally, the L'Hopital theorem can be used in Eqs. (18)–(21):

$$A_{\mathrm{s}}(1)=A(0)\lim_{z\to 1}\frac{N'_{\mathrm{AS}}(z)}{D'_{\mathrm{AS}}(z)}, \tag{22}$$

$$B_{\mathrm{s}}(1)=A(0)\lim_{z\to 1}\frac{N'_{\mathrm{BS}}(z)}{D'_{\mathrm{BS}}(z)}, \tag{23}$$

$$A_{\mathrm{r}}(1)=A(0)\lim_{z\to 1}\frac{N'_{\mathrm{AR}}(z)}{D'_{\mathrm{AR}}(z)}, \tag{24}$$

$$B_{\mathrm{r}}(1)=A(0)\lim_{z\to 1}\frac{N'_{\mathrm{BR}}(z)}{D'_{\mathrm{BR}}(z)}, \tag{25}$$

where $N'_{\mathrm{AS}}(1)$, $D'_{\mathrm{AS}}(1)$, $N'_{\mathrm{AR}}(1)$, $D'_{\mathrm{AR}}(1)$, $N'_{\mathrm{BS}}(1)$, $D'_{\mathrm{BS}}(1)$, $N'_{\mathrm{BR}}(1)$, $D'_{\mathrm{BR}}(1)$, $X_1(1)$, $X_2(1)$, $X'_1(1)$, and $X'_2(1)$ are given by Eqs. (A28)–(A39) in the Appendix.

Since the probability summation of the entire states has to be one, Eqs. (22)–(25) are given by

$$A_{\mathrm{s}}(1)+B_{\mathrm{s}}(1)+r\left(A_{\mathrm{r}}(1)+B_{\mathrm{r}}(1)\right)=1. \tag{26}$$

Then we can obtain

$$A(0)=\left(\frac{N'_{\mathrm{AS}}(1)}{D'_{\mathrm{AS}}(1)}+\frac{N'_{\mathrm{BS}}(1)}{D'_{\mathrm{BS}}(1)}+r\frac{N'_{\mathrm{AR}}(1)}{D'_{\mathrm{AR}}(1)}+r\frac{N'_{\mathrm{BR}}(1)}{D'_{\mathrm{BR}}(1)}\right)^{-1}. \tag{27}$$

Summing $A_{\mathrm{s}}(z)$, $A_{\mathrm{r}}(z)$, $B_{\mathrm{s}}(z)$, and $B_{\mathrm{r}}(z)$, the overall system behavior can be obtained as

$$N_{\mathrm{SW}}(z)=A_{\mathrm{s}}(z)+B_{\mathrm{s}}(z)+\sum_{m=0}^{r}\frac{A_{\mathrm{r}}(z)+B_{\mathrm{r}}(z)}{S^m(z)}. \tag{28}$$

At the last part of the right-hand side of Eq. (28) every buffer occupancy at a buffer is increasing by multiplying by $S(z)$ in Fig. 5.

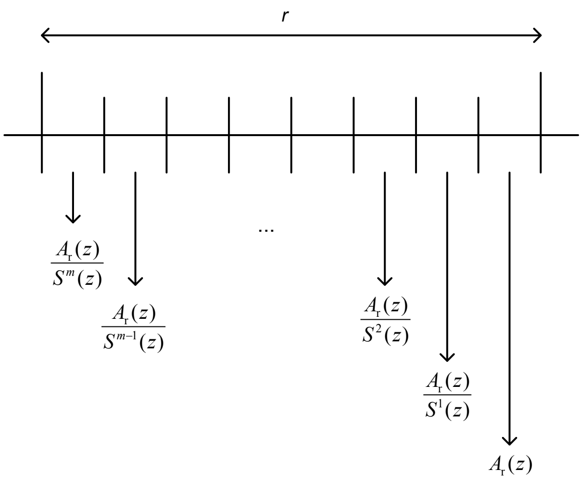

Fig. 5 Buffer behaviors preceding the round-trip delay time epoch

The average buffer occupancy can be solved by differentiating Eq. (28) and substituting z=1:

$$\begin{aligned} N'_{\mathrm{SW}}(1) &= A'_{\mathrm{s}}(1)+B'_{\mathrm{s}}(1)+r\left(A'_{\mathrm{r}}(1)+B'_{\mathrm{r}}(1)\right) \\ &\quad -\frac{r(r-1)}{2}\left(A_{\mathrm{r}}(1)+B_{\mathrm{r}}(1)\right)S'(1), \end{aligned} \tag{29}$$

where $A'_{\mathrm{s}}(1)$, $B'_{\mathrm{s}}(1)$, $A'_{\mathrm{r}}(1)$, and $B'_{\mathrm{r}}(1)$ are given as

$$A_s'(1)=\left(\frac{N_{AS}''(1)}{2D_{AS}'(1)}-\frac{N_{AS}'(1)D_{AS}''(1)}{2(D_{AS}'(1))^2}\right)A(0), \quad (30)$$

$$B_s'(1)=\left(\frac{N_{BS}''(1)}{2D_{BS}'(1)}-\frac{N_{BS}'(1)D_{BS}''(1)}{2(D_{BS}'(1))^2}\right)A(0), \quad (31)$$

$$A_r'(1)=\left(\frac{N_{AR}''(1)}{2D_{AR}'(1)}-\frac{N_{AR}'(1)D_{AR}''(1)}{2(D_{AR}'(1))^2}\right)A(0), \quad (32)$$

$$B_r'(1)=\left(\frac{N_{BR}''(1)}{2D_{BR}'(1)}-\frac{N_{BR}'(1)D_{BR}''(1)}{2(D_{BR}'(1))^2}\right)A(0), \quad (33)$$

where $N_{AS}''(1)$, $D_{AS}''(1)$, $N_{AR}''(1)$, $D_{AR}''(1)$, $N_{BS}''(1)$, $D_{BS}''(1)$, $N_{BR}''(1)$, $D_{BR}''(1)$, $X_1''(1)$, and $X_2''(1)$ are given by Eqs. (A40)–(A49) in the Appendix.

Then, from Little's law, the average waiting time is given by

$$W_{SW}=\frac{N_{SW}'(1)}{\lambda}. \quad (34)$$

5 Numerical results

The analysis and computer simulation of the performance of packet data multiplexers were conducted. C++ and Java languages for Windows 7 were used. For the Markovian interruption signal, any parameter value can be used for α_s and β_s. However, an integrated system in which the voice signal had a higher priority than the data signal was considered.

In this study, the default parameter values were chosen based on Jung (2013): the length of an average available period $\overline{A}=1.8$ s; the length of an average blocked period $\overline{B}=1.3$ s; the slot time T=5 ms; the retransmission interval time r=4 time slots.

From Eqs. (6) and (7), some parameters in Fig. 4 were calculated as follows (Ghaderi and Boutaba, 2006): $1/(1-\alpha)$=1800/5=360 slots, $1/(1-\beta)$=1300/5=260 slots, $\alpha_s=1-T/\overline{A}$, and $\beta_s=1-T/\overline{B}$.

From Eq. (6) α_s, α_r, β_s, and β_r can be found as follows: α_s=1−1/360=0.99722, β_s=1−1/260=0.99615, α_r=1−r/360=0.98889, and β_r=1−r/260=0.98462.

Simulations have been done under the following conditions to partially verify the validity of the above analysis:

1. The buffer length is infinite.

2. The number of arrivals in a time slot is given by round-trip delay time r, where the random number RN should satisfy the following:

(1) The range of RN is 0<RN<1.

(2) If CDF[r–1]≤RN≤CDF[r], then r packets are arriving.

3. We have run the simulations for 10^6 time slots.

Using simulation data we partially verify our analysis results.

Figs. 6 and 7 illustrate the average buffer occupancy and average waiting time of the system, respectively. They were obtained from Eqs. (29) and (34), respectively, with the following parameter values: T=5 ms, $\overline{A}=1.8$ s, $\overline{B}=1.3$ s. From these parameters, α_s=0.9972 and β_s=0.9962 were found. As expected, the figures illustrated that the buffer occupancy and waiting time after some point increased abruptly as traffic intensity λ increased. For example, the curve with p=0.5 had an abrupt trend after the point at which traffic load λ was 0.06, and had no stationary probability behavior when λ was over 0.08. Also, when $p=10^{-6}$, the analytically calculated buffer occupancy and waiting time were the same as the simulation data. So, we can say that the system stability is dependent on the error probability and whether the system has Markovian interruptions.

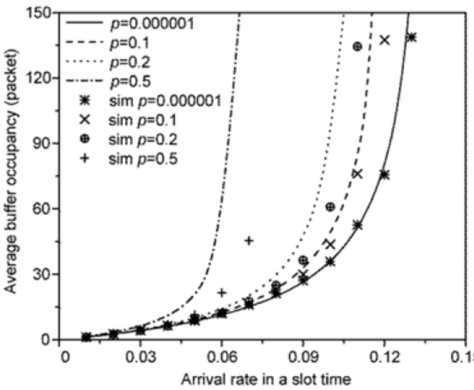

Fig. 6 Average buffer occupancy in the SW protocol as a function of the arrival rate with the packet error probability as a parameter (α_s=0.9972 and β_s=0.9962)

Fig. 7 shows the average waiting time in the queue when round-trip delay r was four slots, with the Markovian interruptions. The waiting time had a nonzero value even when there was very low traffic load. This is explained as follows: if a packet arrives during state B, an arriving packet should wait until state B ends. This average waiting time when there is very low traffic load is as given by Eq. (7).

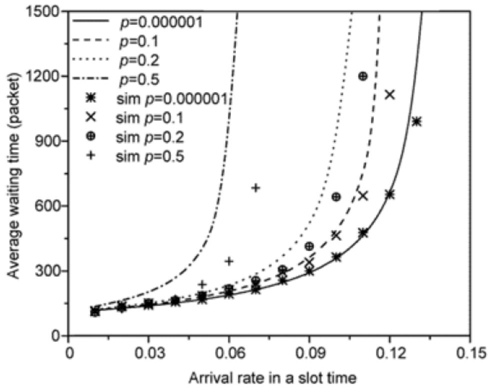

Fig. 7 Average waiting time in the SW protocol as a function of the arrival rate with the packet error probability as a parameter (α_s=0.9972 and β_s=0.9962)

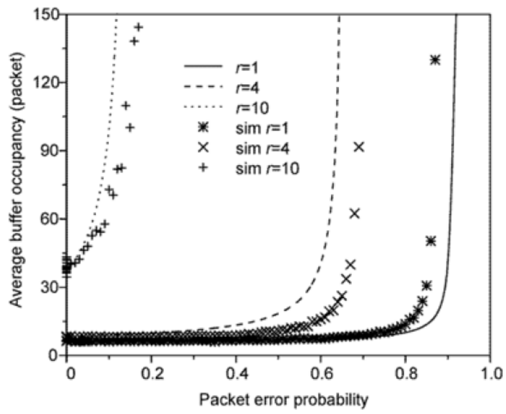

Fig. 8 Average buffer occupancy in the SW protocol as a function of the packet error probability with the round-trip delay as a parameter (λ=0.05, α_s=0.9972, and β_s=0.9962)

Figs. 8 and 9 show the average buffer occupancy and waiting time of the data queue as a function of packet error probability p. In these figures, we analyze the SW system when the number of arrivals per slot time was λ=0.05. When r=1 simulation results were larger than analysis ones under the same conditions. This is because a packet is discarded before the previous round-trip delay from receiving the latest correspondent ACK signal in analysis, but in the real system it is discarded after receiving the latest correspondent ACK signal. Difference of the sojourn time of a packet in the data queue between the real system and analysis induces these results.

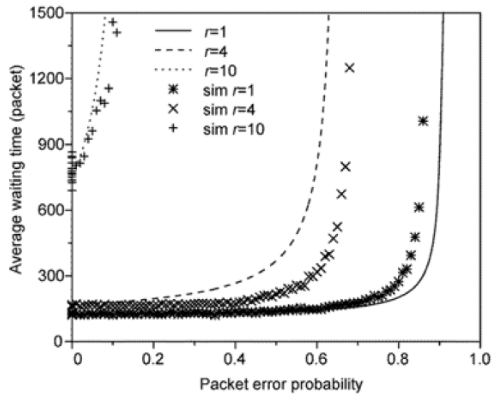

Fig. 9 Average waiting time in the SW protocol as a function of the packet error probability with the round-trip delay as a parameter (λ=0.05, α_s=0.9972, and β_s=0.9962)

6 Conclusions

In this paper, a packet voice/data multiplexer with a fully reliable SW ARQ scheme has been analyzed. When the voice signal is inactive (i.e., the output link has been "available" for the data packet), the data packet is in transmission. The buffer behavior of the SW ARQ protocol is of particular interest in this study, and we found that, depending on the activity of the voice signal (inactive/active), the system is modeled into a two-state (A and B) Markov chain. Once this is established, the joint probability of buffer occupancy and system state is formulated. Combining the system states, the z-transform of buffer occupancy has been found, from which the average buffer occupancy and the corresponding average waiting time have been obtained. From the analytical results, the slot time has to be carefully picked out under the given packet error probability to prevent over buffer occupancy and waiting time. The closed-form solution for buffer occupancy is important when queueing theory is applied to data communications. The solution for buffer occupancy and queueing delay can be achieved. Eventually, the number of voice interruptions that can be accepted for the quality of service (QoS) when values among buffer occupancy AND/OR delay are specified can be calculated. Inversely, buffer occupancy AND/OR delays can be calculated for a specific number of voice signals. This allows for faster and more efficient data communication systems.

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Compliance with ethics guidelines

Dashdondov KHONGORZUL, Yong-Ki KIM, and Mi-Hye KIM declare that they have no conflict of interest.

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Appendix: Derivation of buffer occupancy for SW ARQ protocol

The polynomials $A_s(z)$, $A_r(z)$, $B_s(z)$, and $B_r(z)$ are calculated here, and $A_s(1)$, $A_r(1)$, $B_s(1)$, and $B_r(1)$ will be found. Finally, their derivative values such as $A_s'(1)$, $A_r'(1)$, $B_s'(1)$, and $B_r'(1)$ will be derived.

From the state transition diagram of Fig. 4, the z-transform of buffer occupancy $A_s(z)$, $A_r(z)$, $B_s(z)$, and $B_r(z)$ can be given by

$$\begin{aligned} A_s(z) &= \alpha_s\left(A_s(0)+A_r(0)\right)S(z) \\ &\quad +(1-\beta_s)\left(B_s(z)+B_r(z)\right)S(z), \end{aligned} \tag{A1}$$

$$\begin{aligned} A_r(z) &= (qz^{-1}+p)\alpha_r\left(A_r(z)-A_r(0)\right) \\ &\quad +(qz^{-1}+p)\alpha_r\left(A_s(z)-A_s(0)\right)R(z), \end{aligned} \tag{A2}$$

$$\begin{aligned} B_s(z) &= (1-\alpha_s)\left(A_s(0)+A_r(0)\right)S(z)R(z) \\ &\quad +\beta_s\left(B_s(z)+B_r(z)\right)S(z), \end{aligned} \tag{A3}$$

$$\begin{aligned} B_r(z) &= (qz^{-1}+p)(1-\alpha_r)\left(A_r(z)-A_r(0)\right)R(z) \\ &\quad +(qz^{-1}+p)(1-\alpha_r)\left(A_s(z)-A_s(0)\right)R(z). \end{aligned} \tag{A4}$$

Solving Eq. (A2) with respect to $A_r(z)$, we can derive the following equation:

$$\begin{aligned} A_r(z) &= \frac{(qz^1+p)\alpha_r R(z)}{1-(qz^1+p)\alpha_r R(z)}\left(A_s(z)-(A_s(0)+A_r(0))\right) \\ &= \frac{X_1(z)}{1-X_1(z)}\left(A_s(z)-A(0)\right), \end{aligned} \tag{A5}$$

where $X_1(z)=a_r(qz^{-1}+p)R(z)$, $A(0)=A_s(0)+A_r(0)$.

Substituting $A_r(z)$ in Eq. (A5) into Eq. (A4), we can derive the following equation:

$$B_r(z) = (qz^{-1}+p)(1-\alpha_r)R(z)\left(A_s(z)-\frac{A(0)}{1-X_1(z)}\right). \tag{A6}$$

From Eq. (A3), we have

$$B_s(z)\left(1-\beta_s S(z)\right) = S(z)\left((1-\alpha_s)A(0)+\beta_s B_r(z)\right). \tag{A7}$$

Likewise, substituting Eq. (A6) into Eq. (A7), we can derive the following equation:

$$\begin{aligned} B_s(z) = \frac{S(z)}{1-\beta_s S(z)}\Bigg(& A(0)(1-\alpha_s) \\ & -(qz^{-1}+p)(1-\alpha_r)\beta_s R(z)\frac{A(0)}{1-X_1(z)} \\ & +(qz^{-1}+p)(1-\alpha_r)\beta_s R(z)\frac{A_s(z)}{1-X_1(z)}\Bigg). \end{aligned} \tag{A8}$$

Next, substituting Eqs. (A6) and (A8) into Eq. (A1), we have $A_s(z)$ as follows:

$$A_s(z)=A(0)\left(\alpha_s S(z)+\frac{(1-\alpha_s)(1-\beta_s)S^2(z)}{1-\beta_s S(z)}\right.$$
$$\left.-\frac{X_2(z)}{1-X_1(z)}-\frac{\beta_s S(z)X_2(z)}{(1-\beta_s S(z))(1-X_1(z))}\right)$$
$$+\left(\frac{X_2(z)}{1-X_1(z)}+\frac{X_2(z)\beta_s S(z)}{(1-X_1(z))(1-\beta_s S(z))}\right)A_s(z), \tag{A9}$$

where $X_2(z)$ is given as follows:

$$X_2(z)=(1-\alpha_r)(1-\beta_s)(qz^{-1}+p)S(z)R(z). \tag{A10}$$

Solving Eq. (A9) for $A_s(z)$, we have

$$A_s(z)=\frac{(\alpha_s S(z)+(1-\alpha_s-\beta_s)S^2(z))(1-X_1(z))}{(1-\beta_s S(z))(1-X_1(z))-X_2(z)}A(0)$$
$$-\frac{X_2(z)}{(1-\beta_s S(z))(1-X_1(z))-X_2(z)}A(0)$$
$$=\frac{N_{AS}(z)}{D_{AS}(z)}A(0). \tag{A11}$$

Similarly, $B_s(z)$, $A_r(z)$, and $B_r(z)$ will be given as

$$B_s(z)=\frac{S(z)A(0)}{1-\beta_s S(z)}\left((1-\alpha_s)\right.$$
$$\left.-\frac{(qz^1+p)(1-\alpha_r)\beta_s(N_{AS}(z)-D_{AS}(z))}{(1-X_1(z))D_{AS}(z)}\right)$$
$$=\frac{N_{BS}(z)}{D_{BS}(z)}A(0), \tag{A12}$$

$$A_r(z)=\frac{X_1(z)}{1-X_1(z)}\left(\frac{N_{AS}(z)}{D_{AS}(z)}-1\right)A(0)=\frac{N_{AR}(z)}{D_{AR}(z)}A(0), \tag{A13}$$

$$B_r(z)=\frac{(1-\alpha_r)(qz^{-1}+p)R(z)}{(1-X_1(z))D_{AS}(z)}(N_{AS}(z)-D_{AS}(z))A(0)$$
$$=\frac{N_{BR}(z)}{D_{BR}(z)}A(0), \tag{A14}$$

where $N_{AS}(z)$, $D_{AS}(z)$, $N_{AR}(z)$, $D_{AR}(z)$, $N_{BS}(z)$, $D_{BS}(z)$, $N_{BR}(z)$, and $D_{BR}(z)$ are given by

$$N_{AS}(z)=(\alpha_s S(z)+(1-\alpha_s-\beta_s)S^2(z))\cdot(1-X_1(z))-X_2(z), \tag{A15}$$

$$D_{AS}(z)=(1-\beta_s S(z))(1-X_1(z))-X_2(z), \tag{A16}$$

$$N_{AR}(z)=X_1(z)(N_{AS}(z)-D_{AS}(z)), \tag{A17}$$

$$D_{AR}(z)=(1-X_1(z))D_{AS}(z), \tag{A18}$$

$$N_{BS}(z)=(1-\alpha_s)(1-X_1(z))S(z)D_{AS}(z)+(1-\alpha_r)\beta_s(qz^{-1}+p)S(z)R(z)\cdot(N_{AS}(z)-D_{AS}(z)), \tag{A19}$$

$$D_{BS}(z)=(1-\beta_s S(z))(1-X_1(z))D_{AS}(z), \tag{A20}$$

$$N_{BR}(z)=(1-\alpha_r)(qz^{-1}+p)R(z)(N_{AS}(z)-D_{AS}(z)), \tag{A21}$$

$$D_{BR}(z)=(1-X_1(z))D_{AS}(z). \tag{A22}$$

Since the probability summation of the entire states has to be one, Eqs. (A11)–(A14) are given by

$$A_s(1)+B_s(1)+r(A_r(1)+B_r(1))=1. \tag{A23}$$

First $A_s(1)$ can be given by $A_s(1)=\lim_{z\to 1}A_s(z)=A(0)\lim_{z\to 1}\frac{N_{AS}(z)}{D_{AS}(z)}$. Since numerator $N_{AS}(z)$, $N_{AR}(z)$, $N_{BS}(z)$, or $N_{BR}(z)$, and denominator $D_{AS}(z)$, $D_{AR}(z)$, $D_{BS}(z)$, or $D_{BR}(z)$ go to 0 when z goes to 1 infinitesimally, the L'Hopital theorem can be used in Eqs. (A11)–(A14) as

$$A_s(1)=A(0)\lim_{z\to 1}\frac{N'_{AS}(z)}{D'_{AS}(z)}, \tag{A24}$$

$$B_s(1)=A(0)\lim_{z\to 1}\frac{N'_{BS}(z)}{D'_{BS}(z)}, \tag{A25}$$

$$A_r(1)=A(0)\lim_{z\to 1}\frac{N'_{AR}(z)}{D'_{AR}(z)}, \tag{A26}$$

$$B_r(1)=A(0)\lim_{z\to 1}\frac{N'_{BR}(z)}{D'_{BR}(z)}. \tag{A27}$$

Using the L'Hopital theorem, the steady probabilities that the system in states A_s, A_r, B_s, and B_r are as given in Eqs. (A24)–(A27). In Eqs. (A15)–(A22) differentiating these polynomials and substituting z=1, we have

$$N'_{AS}(1)=(1-X_1(1))S'(1)(2-\alpha_s-2\beta_s)-X'_2(1)-(1-\beta_s)X'_1(1), \tag{A28}$$

$$D'_{AS}(1)=-(1-\beta_s)X'_1(1)-\beta_s S'(1)\big(1-X_1(1)\big)-X'_2(1), \tag{A29}$$

$$N'_{AR}(1)=X_1(1)\big(N'_{AS}(1)-D'_{AS}(1)\big), \tag{A30}$$

$$D'_{AR}(1)=\big(1-X_1(1)\big)D'_{AS}(1), \tag{A31}$$

$$\begin{aligned}N'_{BS}(1)=&(1-\alpha_s)\big(1-X_1(1)\big)D'_{AS}(1)\\&+(1-\alpha_r)\beta_s\big(N'_{AS}(1)-D'_{AS}(1)\big),\end{aligned} \tag{A32}$$

$$D'_{BS}(1)=\big(1-\beta_s\big)\big(1-X_1(1)\big)D'_{AS}(1), \tag{A33}$$

$$N'_{BR}(1)=(1-\alpha_r)\big(N'_{AS}(1)-D'_{AS}(1)\big), \tag{A34}$$

$$D'_{BR}(1)=(1-X_1(1))D'_{AS}(1), \tag{A35}$$

where $X_1(1)$, $X_2(1)$, $X'_1(1)$, and $X'_2(1)$ are given by

$$X_1(1)=\alpha_r, \tag{A36}$$

$$X_2(1)=(1-\alpha_r)(1-\beta_s), \tag{A37}$$

$$X'_1(1)=\alpha_r(R'(1)-q), \tag{A38}$$

$$X'_2(1)=(1-\alpha_r)(1-\beta_s)\big(R'_1(1)+S'(1)-q\big). \tag{A39}$$

Differentiating Eqs. (A11)–(A14) and substituting z=1, the final calculation is as given by Eqs. (30)–(33), where $N''_{AS}(1)$, $D''_{AS}(1)$, $N''_{AR}(1)$, $D''_{AR}(1)$, $N''_{BS}(1)$, $D''_{BS}(1)$, $N''_{BR}(1)$, and $D''_{BR}(1)$ will be calculated in the following. By differentiating Eqs. (A15)–(A22) twice and substituting z=1, we have

$$\begin{aligned}N''_{AS}(1)=&\big(1-X_1(1)\big)(4-3\alpha_s-4\beta_s)S''(1)\\&-(4-2\alpha_s-4\beta_s)S'(1)X'_1(1)\\&-(1-\beta_s)X''_1(1)-X''_2(1),\end{aligned} \tag{A40}$$

$$\begin{aligned}D''_{AS}(1)=&-\beta_s\big(1-X_1(1)\big)S''(1)-X''_2(1)\\&+2\beta_s X'_1(1)S'(1)-(1-\beta_s)X''_1(1),\end{aligned} \tag{A41}$$

$$\begin{aligned}N''_{AR}(1)=&2X'_1(1)\big(N'_{AS}(1)-D'_{AS}(1)\big)\\&+X_1(1)\big(N''_{AS}(1)-D''_{AS}(1)\big),\end{aligned} \tag{A42}$$

$$D''_{AR}(1)=\big(1-X_1(1)\big)D''_{AS}(1)-2X'_1(1)D'_{AS}(1), \tag{A43}$$

$$\begin{aligned}N'_{BS}(1)=&(1-\alpha_s)\Big(\big(2S'(1)D_{AS}{}'(1)+D''_{AS}(1)\big)\\&\cdot(1-X_1(1))-2X'_1(1)D'_{AS}(1)\Big)+(1-\alpha_r)\beta_s\\&\cdot 2\big(S'(1)+R'(1)-q\big)\big(N'_{AS}(1)-D'_{AS}(1)\big)\\&+(1-\alpha_r)\beta_s\big(N''_{AS}(1)-D''_{AS}(1)\big),\end{aligned} \tag{A44}$$

$$\begin{aligned}D''_{BS}(1)=&\big(1-\beta_s\big)\big(1-X_1(1)\big)D''_{AS}(1)\\&-\big(1-\beta_s\big)2X'_1(1)D'_{AS}(1)\\&-2\beta_s\big(1-X_1(1)\big)S'(1)D'_{AS}(1),\end{aligned} \tag{A45}$$

$$\begin{aligned}N''_{BR}(1)=&2(1-\alpha_r)\big(N'_{AS}(1)-D'_{AS}(1)\big)\big(R'(1)-q\big)\\&+(1-\alpha_r)\big(N''_{AS}(1)-D''_{AS}(1)\big),\end{aligned} \tag{A46}$$

$$D''_{BR}(1)=\big(1-X_1(1)\big)D''_{AS}(1)-2X'_1(1)D'_{AS}(1), \tag{A47}$$

where $X''_1(1)$ and $X''_2(1)$ can be obtained from $X_1(1)$ and $X_2(1)$ in Eqs. (A36) and (A37), respectively.

From Eqs. (A38) and (A39), we can find $X''_1(1)$ and $X''_2(1)$ as follows:

$$X_1''(1)=\alpha_r\big(2q-2qR'(1)+R''(1)\big), \tag{A48}$$

$$\begin{aligned}X''_2(1)=&(1-\alpha_r)(1-\beta_s)\big(2q(1-R'(1)-S'(1))\\&+R''(1)+2S'(1)R'(1)+S''(1)\big),\end{aligned} \tag{A49}$$

where $R(z)$ is given in Eq. (3) and $R''(1)$ is given by $R''(1)=\lambda^2r^2$.