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# Semidefinite relaxation aided noncoherent detection in two-way relay transmission*

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**Abstract:** A high-performance noncoherent transmission scheme is proposed in the broadcasting phase of a two-way relay transmission (TWRT), where multiple-symbol differential detection (MSDD) is performed because of its excellent detection performance with no channel estimation. Specifically, the generalized likelihood ratio test aided MSDD (GLRT-MSDD) is developed for the down-link. Furthermore, GLRT-MSDD is reformulated and a semidefinite relaxation aided MSDD (SDR-MSDD) is proposed. The reformulation of GLRT-MSDD to SDR-MSDD is desirable owing to its reduced complexity. Performance analysis and the simulations validate that the proposed SDR-MSDD provides the bit-error-rate performance close to that of GLRT-MSDD with reasonable complexity in TWRT.

**Key words:** Multiple-symbol differential detection; Generalized likelihood ratio test; Semidefinite relaxation; Two-way relay transmission

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## 1 Introduction

Relay is an effective transmission technique for coverage extension (Zeinalpour-Yazdi et al., 2010). In the transmission scheme for relay, some novel relay protocols have been proposed (Rankov and Wittneben, 2007), among which two-way relay transmission (TWRT) has attracted growing attention. Particularly, coherent transmission has been widely applied in TWRT (Cui et al., 2009; To et al., 2010). Unfortunately, a coherent detection scheme requires accurate channel state information, where the estimation of channel parameters will be a complicated task under some scenarios, such as in ultra-wideband (UWB) systems (Win and Scholtz, 1998). Therefore, a noncoherent transmission scheme has attracted growing attention since it avoids channel estimation (Simon and Alouini, 1998; Wang et al., 2016; Lv et al., 2017).

Generally speaking, the typical noncoherent techniques are transmitted reference (TR) and differential detection (DD) (Leib and Pasupathy, 1988). In TWRT-UWB, TR-based detectors can be employed in the broadcasting (BC) phase (Dong and Dong, 2010; Gao et al., 2011; Dai et al., 2014, 2015). However, there is a degradation of bit-error-rate (BER) performance for TR detectors due to the use of noise templates (Quek and Win, 2005). The system performances of TR and DD can be improved with the multiple-symbol differential detection (MSDD) technique (Wang et al., 2013). Recently, an MSDD has been introduced in a TWRT system modulated with frequency shift keying (Dang et al., 2016). An MSDD formulated as a generalized likelihood ratio test (GLRT) is

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called GLRT-MSDD (Wang et al., 2013; Dang et al., 2016). By increasing the size of the observation window, considerable performance improvements can be obtained (Wang et al., 2014). However, this GLRT-MSDD has a problem of nondeterministic polynomial time hard (NP-hard). Several algorithms have been developed to solve this problem, including the exhaustive search algorithm (Guo and Qiu, 2006), sphere decoding, and the Viterbi algorithm (Lottici and Tian, 2008). Specifically, the exhaustive search algorithm obtains optimal solutions, and its computational complexity increases exponentially with the observation window size $K$ (Guo and Qiu, 2006). The sphere decoding algorithm reduces the computational complexity of exhaustive search, but remains exponential (Lampe et al., 2005). The computational complexity of the Viterbi algorithm is polynomial with $K$, but its performance degrades significantly if the memory length is small (Lottici and Tian, 2008). Therefore, it is necessary to seek some effective algorithms with reduced computational complexity.

In this study, a framework of MSDD based on semidefinite relaxation (SDR-MSDD) is designed and the system performance of SDR-MSDD is analyzed in TWRT. We focus on how to transform GLRT-MSDD into SDR-MSDD. First, GLRT-MSDD is reformulated into constrained quadratic programming (Helmberg and Rendl, 1998). When the constraint of the feasible solutions is relaxed, SDR can be applied to the convex optimization problem. Furthermore, a mathematical framework for SDR-MSDD can be derived in TWRT. As a beneficial result, the computational complexity of the relaxed problem is greatly reduced. Specifically, the proposed SDR-MSDD approximately entails a complexity of $O(K^3)$ (Zhou and Ma, 2012). However, the computational complexity of GLRT-MSDD is $O(2^K)$. Therefore, the proposed SDR-MSDD is more competitive than GLRT-MSDD in terms of computational complexity. Hence, it is particularly attractive when applying the SDR-MSDD scheme to a noncoherent TWRT. Note that the proposed schemes can be extended and applied in many kinds of noncoherent transmission scenarios. In this study, the proposed noncoherent transmission schemes are implemented in the UWB system. In TWRT-UWB, the information bits are transmitted with nanosecond pulses, and the received signal consists of a number of dense multipath components (Win and Scholtz, 2000). These lead to a great challenge to the traditional coherent receivers. Thus, noncoherent transmission schemes used in TWRT-UWB are inevitable. In what follows, two kinds of detection schemes, GLRT-MSDD and SDR-MSDD, will be derived in the context of TWRT-UWB. It is noted that network coding is implemented using the exclusive-or (XOR) operation after the multiple access phase. The relay transmits network-coded information to the sources, and each source node conceives information from the other source by taking the encoded information detected and their own self-information. In addition, for simplicity, numerical simulations are carried out in the BC phase to prove the BER performance of the proposed MSDDs. As far as we know, there has been little study on the noncoherent detection of network coding based TWRT-UWB. The major contributions of this study are summarized as follows:

1. In TWRT, a GLRT-MSDD scheme is conceived, where exhaustive search mechanism is employed. In contrast to the existing noncoherent transmission schemes (Dai et al., 2015), the proposed GLRT-MSDD obtains an excellent detection performance by avoiding the noise template.

2. To reduce the complexity of GLRT-MSDD and make it practical, the Boolean constraint in GLRT is relaxed, and the SDR-MSDD scheme is derived correspondingly in the context of noncoherent TWRT-UWB. This proposed SDR-MSDD is attractive, because relying on this detector, a near-optimal performance can be obtained with polynomial time complexity in TWRT.

3. Using an approximate discrete-time model, the relationship between SDR-MSDD and GLRT-MSDD is revealed, and the system performance of SDR-MSDD has been analyzed in TWRT. In addition, the computational complexity of the proposed schemes has been analyzed to reflect the effectiveness of these MSDDs.

Notations: $\boldsymbol{I}_H$ represents the $H \times H$ identity matrix; $(\cdot)^{\mathrm{T}}$ and $\mathrm{Tr}(\cdot)$ represent the transpose and the trace of a matrix, respectively; $X_{i,i}$ represents the diagonal elements of matrix $\boldsymbol{X}$; $\mathrm{diag}(\boldsymbol{x})$ denotes a diagonal matrix with $\boldsymbol{x}$ on its main diagonal; $\lambda_{\boldsymbol{X},\max}$ is the largest eigenvalue of matrix $\boldsymbol{X}$; $\boldsymbol{X} \succeq \mathbf{0}$ represents the positive semidefinite (PSD) matrix constraint; $E[\cdot]$ and $\mathrm{Var}[\cdot]$ denote the expectation and the variance of a random variable, respectively;

$*$ is the convolution operation; $p\{\cdot\}$ represents the probability of an event occurring.

## 2 System description

As shown in Fig. 1, a three-node network is considered, and the down-link of a TWRT-UWB is investigated. During the BC phase, the relay node $R$ sends information to the user nodes, $U_1$ and $U_2$. The information bits are transmitted in blocks. Each block contains information bits as $c_k \in \{0,1\}$, $k = 1,2,\ldots,K$. Then, the information symbols $d_k \in \{-1,1\}$ are obtained by binary phase shift keying (BPSK) modulation. Finally, differential modulation is implemented and the transmitted symbol is given by $b_k = b_{k-1}d_k$, where $b_k \in \{-1,1\}$ and the referenced symbol is $b_0 = 1$. Then, at relay $R$, the transmitted signal in each block is

$$s(t) = \sum_{k=0}^{K} b_k \omega_{\rm s}(t - kT_{\rm s}), \tag{1}$$

where $\omega_{\rm s}(\cdot)$ is the symbol waveform with symbol duration $T_{\rm s}$ and $\omega_{\rm s}(t) = \sum_{j=0}^{N_{\rm f}-1} \omega(t - jT_{\rm f})$. $\omega(\cdot)$ represents the monocycle pulse. $T_{\rm s} = N_{\rm f}T_{\rm f}$, where $T_{\rm f}$ denotes the frame duration. The channel impulse response is given by Win and Scholtz (2000):

$$h_q(t) = \sum_{i=1}^{I} \alpha_{i,q}\delta(t - \tau_{i,q}), \tag{2}$$

where $q$ represents the $q^{\rm th}$ user in the network, $I$ denotes the total number of propagation paths, and $\alpha_{i,q}$ and $\tau_{i,q}$ represent the gain and the delay parameters in the $i^{\rm th}$ propagation path from the relay to the $q^{\rm th}$ user, respectively. Correspondingly, the signal received at the $q^{\rm th}$ user node is described as

$$\begin{aligned} r_q(t) &= s(t) * h_q(t) + n(t) \\ &= \sum_{k=0}^{K} b_k \sum_{j=0}^{N_{\rm f}-1} g_q(t - kT_{\rm s} - jT_{\rm f}) + n(t), \end{aligned} \tag{3}$$

where $g_q(t) = \omega(t) * h_q(t)$ is the overall channel response, $n(t)$ stands for the additive white Gaussian noise (AWGN), whose mean and two-sided power spectral density are zero and $N_0/2$, respectively. Based on this model, several noncoherent transmission schemes have been proposed in the point-to-point communication system (Lottici and Tian, 2008; Zhou and Ma, 2012). It is necessary to extend and develop some MSDD schemes in TWRT, where an excellent detection performance can be achieved with low complexity. Note that at the user nodes, the different receivers adopt the same detection structure. Thus, the strategy derived in the following sections is applicable to both user nodes.

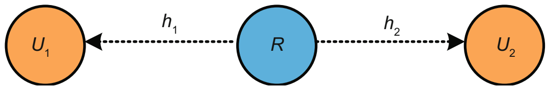


**Fig. 1 Broadcasting phase of a three-node system model**

## 3 GLRT-MSDD and SDR-MSDD in TWRT

In this section, we consider the noncoherent transmission schemes in the BC phase of TWRT-UWB. First, the GLRT-MSDD mathematical model is briefly introduced, and then by relaxing the Boolean constrained variables of GLRT-MSDD, an SDR-MSDD scheme is conceived in TWRT-UWB.

### 3.1 GLRT-MSDD in TWRT

In the down-link of the noncoherent TWRT-UWB, the receivers at the user nodes aim to recover $K$ consecutive information symbols $\boldsymbol{b}$, where $\boldsymbol{b} \triangleq [b_0, b_1, \ldots, b_K]^{\rm T}$. Specifically, the GLRT-based decision strategy can be formulated as the following optimization problem (Lottici and Tian, 2008):

$$\hat{\boldsymbol{b}} = \arg\max_{\tilde{\boldsymbol{b}}}\{\varGamma[r_q(t)|\tilde{\boldsymbol{b}}]\}, \tag{4}$$

$$\varGamma[r_q(t)|\tilde{\boldsymbol{b}}] = \sum_{u=0}^{K-1}\sum_{v=u+1}^{K} \tilde{b}_u\tilde{b}_v\rho_{u,v}, \tag{5}$$

$$\rho_{u,v} = \int_0^{T_{\rm I}} z_u(t)z_v(t){\rm d}t, \tag{6}$$

$$z_u(t) = \frac{1}{N_{\rm f}}\sum_{j=0}^{N_{\rm f}-1} r_q(t + uT_{\rm s} + jT_{\rm f}), \tag{7}$$

where $T_{\rm I} \le T_{\rm s}$, $\hat{\boldsymbol{b}}$ and $\tilde{\boldsymbol{b}}$ represent the estimated and candidate symbol vectors, respectively. Written in matrix form, Eq. (4) can be further reformulated in a more compact manner as

$$\begin{aligned} \max \quad & J_{\rm GLRT}(\tilde{\boldsymbol{b}}) = \tilde{\boldsymbol{b}}^{\rm T}\boldsymbol{G}\tilde{\boldsymbol{b}} \\ {\rm s.t.} \quad & \tilde{b}_k \in \{-1,1\}, \quad k = 1,2,\ldots,K, \\ & \tilde{b}_0 = 1, \end{aligned} \tag{8}$$

where $\boldsymbol{G}$ is a $(K+1)\times(K+1)$ matrix, and $G_{u,v}$ corresponds to the correlation function $\rho_{u-1,v-1}$. It can be seen that the optimization of Eq. (8) is an NP-hard problem. Hence, the proposed GLRT-MSDD in TWRT imposes high computational complexity, which increases exponentially with the observation window size $K$. To address this issue, we propose SDR-MSDD, where the computational complexity is polynomial with $K$.

### 3.2 SDR-MSDD in TWRT

In this subsection, a near-optimal noncoherent detection scheme will be developed in TWRT. In particular, the difficult discrete optimization of Eq. (8) is equivalent to the following optimization problem:

$$\begin{aligned} \max \quad & J_{\mathrm{GLRT}}(\boldsymbol{X}) = \mathrm{Tr}(\boldsymbol{XG}) \\ \mathrm{s.t.} \quad & \boldsymbol{X} = \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}, \\ & X_{v,v} = 1. \end{aligned} \tag{9}$$

Note that $\tilde{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{G}\tilde{\boldsymbol{b}} = \mathrm{Tr}(\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{G})$. The constraint $\boldsymbol{X} = \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}$ indicates that $\boldsymbol{X}$ is a PSD matrix with $\mathrm{rank}(\boldsymbol{X}) = 1$. Because of the nonconvex constraint of $\mathrm{rank}(\boldsymbol{X}) = 1$, Eq. (9) is difficult to solve. When this constraint is dropped, Eq. (9) is relaxed to

$$\begin{aligned} \max \quad & J_{\mathrm{SDR}}(\boldsymbol{X}) = \mathrm{Tr}(\boldsymbol{XG}) \\ \mathrm{s.t.} \quad & \boldsymbol{X} \succeq \boldsymbol{0}, \\ & X_{v,v} = 1. \end{aligned} \tag{10}$$

Since Eq. (10) is convex, it can be solved with a general class of optimization techniques. Using the sign of the principal eigenvector of $\boldsymbol{X}_{\mathrm{opt}}$, we have

$$(\boldsymbol{G} - \lambda_{\boldsymbol{G},\max}\boldsymbol{I})\boldsymbol{w} = \boldsymbol{0}, \tag{11}$$

where $\boldsymbol{w} = [w_0, w_1, \ldots, w_K]^{\mathrm{T}}$ is the eigenvector corresponding to the largest eigenvalue $\lambda_{\boldsymbol{G},\max}$ of the optimal solution of Eq. (10). Then, the estimated information symbols at the user nodes can be obtained as

$$\hat{\boldsymbol{b}}_k = \mathrm{sgn}(w_k/w_1), \quad k = 0, 1, \ldots, K. \tag{12}$$

The proposed SDR-MSDD converts the original NP-hard problem into a convex optimization problem with the solvable complexity of polynomial time (Rendi et al., 1995). For clarity, the whole SDR-MSDD scheme in TWRT is summarized in the following:

Step 1: Set $\boldsymbol{G}$ in Eq. (8), and then solve the optimization problem $\boldsymbol{X}_{\mathrm{opt}} = \mathrm{argmax}\{\mathrm{Tr}(\boldsymbol{XG})\}$.

Step 2: If $\boldsymbol{X}_{\mathrm{opt}}$ is of rank one, let $\boldsymbol{X}_{\mathrm{opt}} = \boldsymbol{x}_{\mathrm{opt}}\boldsymbol{x}_{\mathrm{opt}}^{\mathrm{T}}$, and then $\boldsymbol{x}_{\mathrm{opt}}$ is the solution to Eq. (9); else, go to step 3.

Step 3: Factorize $\boldsymbol{X}_{\mathrm{opt}} = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}$, and solve the estimated symbols as Eq. (12).

**Remark 1** GLRT-MSDD and SDR-MSDD are considered for a two-way relay system, where XOR network coding is employed at the relay after completing the communication of the multiple-access phase. In the broadcasting phase, the network encoded information is transmitted to the users, and one of the users obtains the message from another user using the received signal and the self-information through the XOR operation again. For simplicity, the MSDD schemes are discussed during the BC phase in the study. In fact, the proposed two MSDD algorithms are also suitable for the multiple-access phase under two scenarios: (1) Three time phases are needed to communicate, where the two sources transmit signals during the first and second phases, respectively, and then the receiver at the relay obtains the network encoded symbol and broadcasts it to the sources during the third time phase. (2) The communication is completed within two time phases; the two sources simultaneously transmit signals to the relay in the first phase, and then XOR is operated at the relay and the network encoded information is broadcasted in the second phase.

## 4 Performance analysis of SDR-MSDD in TWRT

In this section, the necessary and sufficient condition to produce a rank-one solution will be developed for SDR-MSDD in TWRT. Specifically, to clearly reveal the detection process, a discrete-time model will be derived. For the $q^{\mathrm{th}}$ user node, the received signal in Eq. (7) can be represented with a discrete-time sampling as

$$z_k(t) = b_k g_q(t) + \sigma\varpi_k(t), \tag{13}$$

where $\varpi_k(t)$ is a band-limited AWGN and $\sigma^2 = N_0/(2N_{\mathrm{f}})$. For the $k^{\mathrm{th}}$ received signal, the sampled noise is defined as $\boldsymbol{\varpi}_k$, $\boldsymbol{\varpi} = [\boldsymbol{\varpi}_0, \boldsymbol{\varpi}_1, \ldots, \boldsymbol{\varpi}_k]^{\mathrm{T}}$; the sampled channel template is defined as $\boldsymbol{g}_q = [g_{q,0}, g_{q,1}, \ldots, g_{q,M-1}]^{\mathrm{T}}$ and $\boldsymbol{G}_q$ is an $M \times M$ unitary matrix whose column vectors constitute the complementary space of $\boldsymbol{g}_q$; the vector of the sampled received signal is $\boldsymbol{z}_k = b_k\boldsymbol{g}_q + \sigma\boldsymbol{\varpi}_k$. Define $\boldsymbol{Z} = [\boldsymbol{z}_0, \boldsymbol{z}_1, \ldots, \boldsymbol{z}_k]$ and $\boldsymbol{A} = \boldsymbol{Z}^{\mathrm{T}}\boldsymbol{Z}$; $\boldsymbol{F} = \boldsymbol{G}_q^{\mathrm{T}}\boldsymbol{\varpi}$, and $\boldsymbol{f}_k^{\mathrm{T}}$ represents the $k^{\text{th}}$ row of matrix $\boldsymbol{F}$. These notations will facilitate the performance analysis of the proposed scheme. First, a lemma is given as follows:

**Lemma 1** At a high signal-to-noise ratio (SNR), the necessary and sufficient condition is $\lambda_{\boldsymbol{D},\max} \leq (K+1)/\sigma$ for the proposed SDR-MSDD to develop a rank-one solution, where

$$\begin{aligned}\boldsymbol{D} =& \boldsymbol{f}_1\boldsymbol{b}^{\mathrm{T}} + \boldsymbol{b}\boldsymbol{f}_1^{\mathrm{T}} + \sigma\sum_{m=1}^{M}\boldsymbol{f}_m\boldsymbol{f}_m^{\mathrm{T}} \\ &-\mathrm{diag}(\boldsymbol{b})\mathrm{diag}\left(\left(\boldsymbol{f}_1\boldsymbol{b}^{\mathrm{T}} + \boldsymbol{b}\boldsymbol{f}_1^{\mathrm{T}} + \sigma\sum_{m=1}^{M}\boldsymbol{f}_m\boldsymbol{f}_m^{\mathrm{T}}\right)\boldsymbol{b}\right).\end{aligned} \tag{14}$$

**Proof** According to the Karush-Kuhn-Tucker (KKT) conditions, when Eq. (10) obtains the optimal solution, the necessary and sufficient condition can be written as

$$\begin{cases}\boldsymbol{X} \succeq \boldsymbol{0}, \\ X_{v,v} = 1, \\ \boldsymbol{L} \succeq \boldsymbol{0}, \\ \boldsymbol{L} + \mathrm{diag}(\boldsymbol{l}) = -\boldsymbol{A}, \\ \boldsymbol{X}\boldsymbol{L} = \boldsymbol{0},\end{cases} \tag{15}$$

where $\boldsymbol{L}$ and $\boldsymbol{l}$ are the dual variables of Eq. (10). Based on $\boldsymbol{L} + \mathrm{diag}(\boldsymbol{l}) = -\boldsymbol{A}$, we have

$$\boldsymbol{L} = -\boldsymbol{A} - \mathrm{diag}(\boldsymbol{l}). \tag{16}$$

Due to $\boldsymbol{X} = \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}$ and $\boldsymbol{X}\boldsymbol{L} = \boldsymbol{0}$, we have

$$\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}\boldsymbol{L} = \boldsymbol{0}. \tag{17}$$

By replacing $\boldsymbol{L}$ with Eq. (16), Eq. (17) can be rewritten as

$$\tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}}[-\boldsymbol{A} - \mathrm{diag}(\boldsymbol{l})] = \boldsymbol{0}. \tag{18}$$

Multiplying on the left with $\boldsymbol{b}^{\mathrm{T}}$ and taking the transpose, we have

$$(-\boldsymbol{A})\tilde{\boldsymbol{b}} = \mathrm{diag}(\boldsymbol{l})\tilde{\boldsymbol{b}}, \tag{19}$$

$$\boldsymbol{l} = -\mathrm{diag}(\boldsymbol{b})^{-1}\boldsymbol{A}\boldsymbol{b}. \tag{20}$$

Then Eq. (16) can be rewritten as

$$\boldsymbol{L} = -\boldsymbol{A} + \mathrm{diag}(\boldsymbol{b})^{-1}\mathrm{diag}(\boldsymbol{A}\boldsymbol{b}). \tag{21}$$

Expanding the matrix (Jalden et al., 2003), the right part of Eq. (21) can be equivalent to

$$\begin{aligned}\boldsymbol{L} =& -\boldsymbol{A} + \mathrm{diag}(\boldsymbol{b})^{-1}\mathrm{diag}(\boldsymbol{A}\boldsymbol{b}) \\ =&(K+1)\boldsymbol{I} - \tilde{\boldsymbol{b}}\tilde{\boldsymbol{b}}^{\mathrm{T}} - \sigma\boldsymbol{D}.\end{aligned} \tag{22}$$

This implies that $\boldsymbol{L}$ is a PSD matrix, if and only if $\lambda_{\boldsymbol{D},\max} \leq (K+1)/\sigma$. In other words, SDR-MSDD in TWRT has a rank-one solution if and only if $\boldsymbol{L}$ is a PSD matrix. With the increase of SNR, it can be concluded that

$$\lim_{\sigma\to 0} p\left\{\lambda_{\boldsymbol{D},\max} \leq \frac{K+1}{\sigma}\right\} = 1. \tag{23}$$

Thus, Lemma 1 is proved.

**Remark 2** At a high SNR, the condition is satisfied in Lemma 1. On the other hand, when SNR decreases, the noise increases gradually and the largest eigenvalue of $\sigma\boldsymbol{D}/(K+1)$ is usually large. Therefore, there may be an error with a high probability.

## 5 Complexity analysis of noncoherent detection in TWRT

Some noncoherent detection strategies have been developed to reduce the complexity of exhaustive search in point-to-point systems (Lottici and Tian, 2008; Wang et al., 2013). In this study, GLRT-MSDD is proposed in TWRT. To reduce the complexity of GLRT-MSDD, the SDR-based approximation scheme with polynomial complexity is proposed. Specifically, the exhaustive search scheme for GLRT-MSDD entails a computational complexity of $O(2^K)$ (Ma et al., 2002). However, a computational complexity of $O(K^3) - O(K^{3.5})$ is used with the proposed SDR-MSDD in TWRT, where the complexity comes mainly from solving the eigenvalues and eigenvectors of matrix $\boldsymbol{G}$ in Eq. (10) (Rendi et al., 1995; Mao et al., 2007). Hence, we conclude that the proposed SDR-MSDD obtains a lower complexity than GLRT-MSDD in TWRT.

## 6 Simulation results and discussion

In this section, Monte-Carlo simulations are carried out to validate the effectiveness of the proposed noncoherent transmission schemes in TWRT, where the proposed GLRT-MSDD and SDR-MSDD detectors are characterized in the BC phase of UWB.

The channel is generated as prescribed in the IEEE 802.15.3a CM2 model (Win and Scholtz, 2000). The transmitted monocycle waveform $\omega(t)$ is the second derivative of the Gaussian function with duration $T_\omega = 0.5$ ns. The frame duration is set to $T_\mathrm{f} = 100$ ns, and the integration interval is $T_\mathrm{I} = 20$ ns with energy normalization.

The effectiveness of the proposed transmission schemes is demonstrated in the BC phase of TWRT. In Fig. 2, we compare the BER performance between SDR-MSDD and GLRT-MSDD with different observation window size $K$. When $K = 1$, MSDD is equivalent to the differential detection scheme in TWRT. As $K$ increases, the proposed transmission schemes can offer detection gain and further improve system performance. It can be seen that SDR-MSDD yields a BER performance identical to that of the GLRT-MSDD benchmarker when $K = 2, 4, 7$. This indicates that the SDR-based detector achieves accurate approximation to the optimal GLRT-based detector in TWRT.

In Fig. 3, we compare the average computational complexity of different detection schemes in TWRT. It can be seen that the computational complexities of GLRT-MSDD and SDR-MSDD detectors are similar when the observation window size $K$ is less than nine. By comparison, GLRT-MSDD imposes significantly higher computational complexity than SDR-MSDD when $K$ is large. This is consistent with the complexity analysis. For example, the complexity of GLRT-MSDD at SNR = 6 dB and $K = 13$ is about three times that of SDR-MSDD in TWRT-UWB. Specifically, the GLRT-MSDD benchmarker requires the computational complexity of $O(2^K)$ and SDR-MSDD requires polynomial computational complexity in terms of $K$. This implies that, although GLRT-MSDD is competitive in a single-antenna TWRT-UWB, it might be difficult to use the detector in the multi-antenna TWRT-UWB or massive multiple-input multiple-output (MIMO) systems, where the SDR-MSDD detector might be more promising.

## 7 Conclusions

Two kinds of noncoherent transmission schemes are proposed in TWRT. In particular, the proposed SDR-MSDD detector bypasses exhaustive search decisions and directly calculates the eigenvalue and eigenvector of the matrix. Based on this contribution, an NP-hard problem of GLRT-MSDD has been transformed into a convex optimization problem. On the one hand, the performance analysis showed that the SDR-MSDD scheme can obtain the solution of rank one with a high probability; in other words, the solution of SDR-MSDD is very close to that of GLRT-MSDD. On the other hand, complexity analysis showed that for a large observation window size $K$, the computational complexity of SDR-MSDD is significantly lower than that of GLRT-MSDD. Monte-Carlo simulations confirmed that in the down-link of TWRT-UWB, SDR-MSDD provides almost the same BER performance as GLRT-MSDD regardless of the window size, but with a lower complexity.

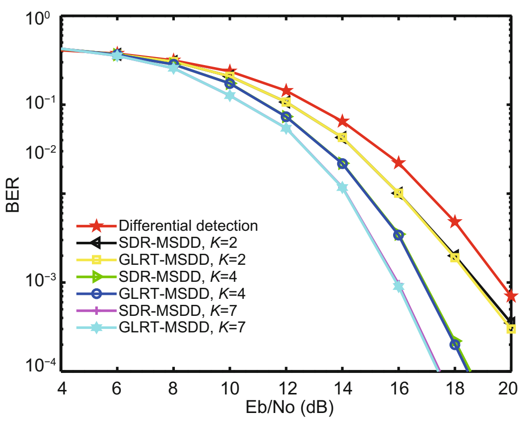


**Fig. 2 BER performance comparison between SDR-MSDD and GLRT-MSDD in TWRT with different observation window size *K***

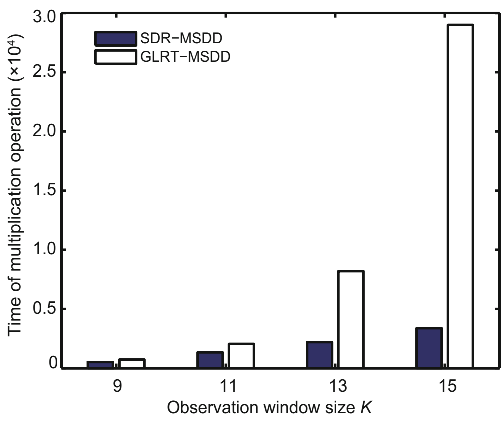


**Fig. 3 Complexity comparisons between SDR-MSDD and GLRT-MSDD in TWRT-UWB with different observation window size *K***

## Compliance with ethics guidelines

Chan-fei WANG, Ji-ai HE, Wei-fang WANG, and Ya-mei XU declare that they have no conflict of interest.

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