Frontiers of Information Technology & Electronic Engineering
www.jzus.zju.edu.cn; engineering.cae.cn; www.springerlink.com
ISSN 2095-9184 (print); ISSN 2095-9230 (online)
E-mail: jzus@zju.edu.cn

Correspondence:

Discrete fractional watermark technique*

Zai-rong WANG[1], Babak SHIRI[‡2], Dumitru BALEANU[3,4]

[1]*Data Recovery Key Laboratory of Sichuan Province, School of Computer Science, Neijiang Normal University, Neijiang 641100, China*

[2]*Data Recovery Key Laboratory of Sichuan Province, College of Mathematics and Information Science, Neijiang Normal University, Neijiang 641100, China*

[3]*Department of Mathematics, Cankaya University, Ankara 06530, Turkey*

[4]*Institute of Space Sciences, Măgurele-Bucharest 011254, Romania*

E-mail: wangzr@njtc.edu.cn; shire_babak@yahoo.com; dumitru@cankaya.edu.tr

Received Mar. 29, 2020; Revision accepted May 5, 2020; Crosschecked May 20, 2020

Abstract: The fractional logistic map holds rich dynamics and is adopted to generate chaotic series. A watermark image is then encrypted and inserted into the original images. Since the encryption image takes the fractional order within (0, 1], it increases the key space and becomes difficult to attack. This study provides a robust watermark method in the protection of the copyright of hardware, images, and other electronic files.

Key words: Discrete fractional calculus; Image encryption; Watermark
https://doi.org/10.1631/FITEE.2000133 **CLC number:** TP391.4

1 Introduction

Watermark is often used to protect the copyright of images and chips against clone products. Watermark image generation and identification are the key steps in security. Chaos encryption is a popular method since chaos has randomness and sensitivity. Chaos is very suitable for the design of encryption schemes. Up to now many chaos-based watermark methods have been developed (Li et al., 2016; Gao and Gao, 2019). Fractional calculus has a long history of about 300 years and has been adopted in many applications to tackle engineering problems (Podlubny, 1999). The motivation for introducing fractional derivatives into signal processing is clear. In fact, in classical methods, only a single signal is considered. The fractional derivatives hold memory effects or non-locality. These features can guarantee better results in applications such as image encryption (Liu et al., 2017, 2018), image enhancement (Pu et al., 2008), image retrieval (Ghose et al., 2020), image denoising (Ma et al., 2020), and image registration (Han, 2020).

However, fractional derivatives have some problems in real-world applications. On one hand, nonlocality increases the computational cost and leads to poor efficiency. Sometimes both the performance and computation speed should be considered, such as applying fractional calculus to big data processing. There should be a balance between these two aspects. On the other hand, it is challenging to do standard discretization of fractional derivatives because the numerical errors can accumulate quickly and thus it is hard to guarantee the accuracy of numerical discretization. This implies that not all of the memory effects are fully used. This causes loss of important information and introduces noise. Alternative methods or theories need to be considered

‡ Corresponding author

* Project supported by the Sichuan Science and Technology Support Program, China (No. 2018JY0120)

ORCID: Zai-rong WANG, https://orcid.org/0000-0002-4493-3246; Babak SHIRI, https://orcid.org/0000-0003-2249-282X

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to introduce the chaotic maps.

In discrete fractional calculus (DFC), finite sum is used for fractional operators (Atici and Eloe, 2009; Abdeljawad, 2011; Anastassiou, 2011; Bastos et al., 2011). The structure of this definition is suitable for signal and image processing. Discrete memory effects were successfully introduced in standard logistic maps and rich dynamics was obtained. It can be concluded that DFC provides an exact discrete-time method without loss of memory. Various fractional discrete-time chaotic systems and results have been reported.

2 Discrete fractional calculus

We review some results in fractional calculus and fractional differences. We start by introducing some definitions (Podlubny, 1999; Atici and Eloe, 2009; Abdeljawad, 2011; Anastassiou, 2011; Bastos et al., 2011).

Definition 1 For $\alpha > 0$, the Riemann-Liouville integral of α order for function y on $[t_0, +\infty)$ is defined as (Podlubny, 1999)

$$ {}_{t_0}I_t^{\alpha} y(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} y(s) \mathrm{d}s, \ t > t_0. \quad (1)$$

Definition 2 For $0 < \alpha < 1$ and $y(t) \in C^1([t_0, +\infty))$, the Caputo derivative of α order is defined by

$$ {}_{t_0}^{C}D_t^{\alpha} y(t) := \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-s)^{-\alpha} y'(s) \mathrm{d}s, \ t > t_0. \quad (2)$$

When considering image processing or sampled signals, we can see they hold discrete structures. We need to discretize the fractional derivatives and integral for image enhancement and denosing, respectively. As mentioned in the introduction, in this treatment, computation speed and numerical discretization accuracy should both be considered. We first turn to DFC.

Definition 3 (Fractional sum) (Atici and Eloe, 2009; Bastos et al., 2011) Given $u : \mathbb{N}_a \to \mathbb{R}$ and $\nu > 0$, the ν order sum is

$$\Delta_a^{-\nu} u(t) := \frac{1}{\Gamma(\nu)} \sum_{s=a}^{t-\nu} (t-\sigma(s))^{\nu-1} u(s), \quad (3)$$

where $\sigma(s) = s + 1$, $a \in \mathbb{R}$, and $t \in \mathbb{N}_{a+\nu}$. t^{ν} is the falling factorial functional defined by

$$t^{\nu} = \frac{\Gamma(t+1)}{\Gamma(t+1-\nu)}.$$

Definition 4 (Fractional Riemann-Liouville difference) (Atici and Eloe, 2009; Bastos et al., 2011) Given $u : \mathbb{N}_a \to \mathbb{R}$ and $\nu > 0$, the ν-order Riemann-Liouville (R-L) difference is

$$\Delta_a^{\nu} u(t) := \frac{1}{\Gamma(-\nu)} \sum_{s=a}^{t+\nu-1} (t-\sigma(s))^{-\nu-1} u(s), \quad (4)$$

where $\sigma(s) = s + 1$, $a \in \mathbb{R}$, $t \in \mathbb{N}_{a+m-\nu}$, and $m = \lfloor \nu \rfloor + 1$. Here the notation $\lfloor \cdot \rfloor$ stands for the floor function.

Definition 5 (Caputo difference) (Abdeljawad, 2011; Anastassiou, 2011) For $u(t)$ defined on $\mathbb{N}_a$ and $\nu > 0$ ($\nu \notin \mathbb{N}$), the Caputo difference is defined by

$$ {}^{C}\Delta_a^{\nu} u(t) := \Delta_a^{-(m-\nu)} \Delta^m u(t), \quad (5)$$

where $t \in \mathbb{N}_{a+m-\nu}$, $m = \lfloor \nu \rfloor + 1$, and $\Delta u(t) = u(t+1) - u(t)$. For $\nu = m$, ${}^{C}\Delta_a^{\nu} u(t) := \Delta^m u(t)$.

Definitions 3–5 are given in the finite sum concerning the finite given signals. Hence, DFC is very suitable for signal processing. We adopt the fractional logistic map (Wu and Baleanu, 2014) to generate chaotic series:

$$u(n+1) = u(0) + \frac{\mu}{\Gamma(\nu)} \sum_{j=0}^{n} \frac{\Gamma(n-j+\nu)}{\Gamma(n-j+1)} u(j)(1-u(j)). \quad (6)$$

3 New watermark technique

3.1 Algorithm

We adopt the following steps for the watermark technique. Suppose P is an original image and W is a watermark image.

Step 1: Encrypt the watermark image W using the chaotic series from Eq. (6) and obtain an encryption image W^*.

Step 2: Add watermark image W^* to the original image P to obtain watermark result R.

Step 3: Extract W^* from R to decrypt and compare the decrypted image W^* with the original one W.

We can see that the main computational cost comes from step 1. Hence, to address this problem, software and a patent were designed for long-term calculation (Wu et al., 2014). The length of the chaotic time series can be obtained as long as 10 000 in 10 s. Then image encryption is considered and several schemes have been provided in Bai et al. (2018) and Wu et al. (2019).

Considering step 2, we use the method in Wu et al. (2019). Suppose that $\widetilde{g}_{i,j}$, $g_{i,j}$, and $c_{i,j}$ are pixel values of R with size 256×256, P with size 256×256, and W^* with size 64×64. $p_{(i-1)\times4+k,(j-1)\times4+l}$ is the probability density function with respect to $(i,\ j)$, defined by

$$p_{(i-1)\times4+k,(j-1)\times4+l}=\frac{g_{(i-1)\times4+k,(j-1)\times4+l}}{\sum\limits_{k=1}^{4}\sum\limits_{l=1}^{4}g_{(i-1)\times4+k,(j-1)\times4+l}},$$

where $k,l=1,2,3,4$.

The pixel values of R can be obtained as

$$\begin{aligned}\widetilde{g}_{(i-1)\times4+k,(j-1)\times4+l}=&\,g_{(i-1)\times4+k,(j-1)\times4+l}\\&+p_{(i-1)\times4+k,(j-1)\times4+l}\times c_{i,j},\end{aligned}$$

where $k,l=1,2,3,4$.

Finally, to extract the watermark image, we have

$$c_{i,j}=\frac{\widetilde{g}_{(i-1)\times4+1,(j-1)\times4+1}-g_{(i-1)\times4+1,(j-1)\times4+1}}{p_{(i-1)\times4+1,(j-1)\times4+1}},$$

by which we can decrypt the image with the known keys to obtain W^*.

3.2 Experiment

We choose the images given in Figs. 1a and 1c as W and P, respectively. In the fractional logistic map (6), set $u(0)=0.3$, $\mu=2.8$, and $\nu=0.9$. This leads to the fractional chaotic series. Following steps 1–3 in Section 3.1, we obtain the watermark results in Fig. 1. In comparison with the existing methods based on chaotic maps, for example, the logistic map

$$u(n+1)=\mu u(n)(1-u(n)), \tag{7}$$

here we have an additional parameter, the fractional order ν, which is used as the key in the watermark technique. It is clear that this increases the computational cost for key space in an attack. Also, our technique can better protect privacy and copyright.

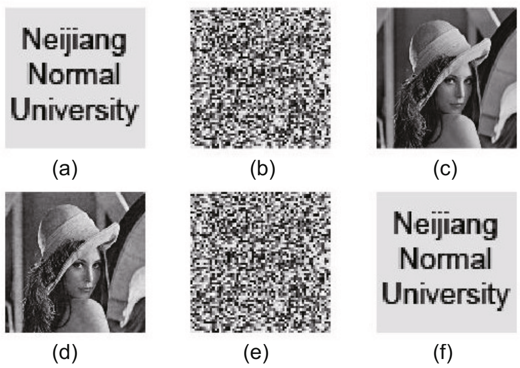

Fig. 1 Discrete fractional watermark technique: (a) image W; (b) encryption of W; (c) Lena; (d) watermark result; (e) extraction of image W^*; (f) decrypted image

Contributors

Zai-rong WANG and Dumitru BALEANU designed the research. Zai-rong WANG introduced the algorithm. Babak SHIRI implemented the method and drafted the manuscript. Zai-rong WANG, Babak SHIRI, and Dumitru BALEANU revised and finalized the manuscript.

Compliance with ethics guidelines

Zai-rong WANG, Babak SHIRI, and Dumitru BALEANU declare that they have no conflict of interest.

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