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Reducing power grid cascading failure propagation by minimizing algebraic connectivity in edge addition*

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Abstract: Analyzing network robustness under various circumstances is generally regarded as a challenging problem. Robustness against failure is one of the essential properties of large-scale dynamic network systems such as power grids, transportation systems, communication systems, and computer networks. Due to the network diversity and complexity, many topological features have been proposed to capture specific system properties. For power grids, a popular process for improving a network's structural robustness is via the topology design. However, most of existing methods focus on localized network metrics, such as node connectivity and edge connectivity, which do not encompass a global perspective of cascading propagation in a power grid. In this paper, we use an informative global metric algebraic connectivity because it is sensitive to the connectedness in a broader spectrum of graphs. Our process involves decreasing the average propagation in a power grid by minimizing the increase in its algebraic connectivity. We propose a topology-based greedy strategy to optimize the robustness of the power grid. To evaluate the network robustness, we calculate the average propagation using MATCASC to simulate cascading line outages in power grids. Experimental results illustrate that our proposed method outperforms existing techniques.

Key words: Network robustness; Cascading failure; Average propagation; Algebraic connectivity; Power grid
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1 Introduction

Several important infrastructures, such as transportation systems, telecommunication systems, and electric power grids, are modeled as networks (Pizzuti et al., 2020). Such representations allow us to specify how the components are related to each other through interconnections, and allow us to study many system properties such as robustness, different partitions, and consensus problems, among other areas of interest (Tang et al., 2018).

Examining network robustness under various situations such as internal failures or external attacks is regarded as a difficult research topic, due to the diversity and complexity of networks in general. Several publications discuss how a network's topology can be used to analyze and measure its robustness (Gu et al., 2020; Wang JX et al., 2020). Various network topological features have been proposed for measuring some network properties. These include

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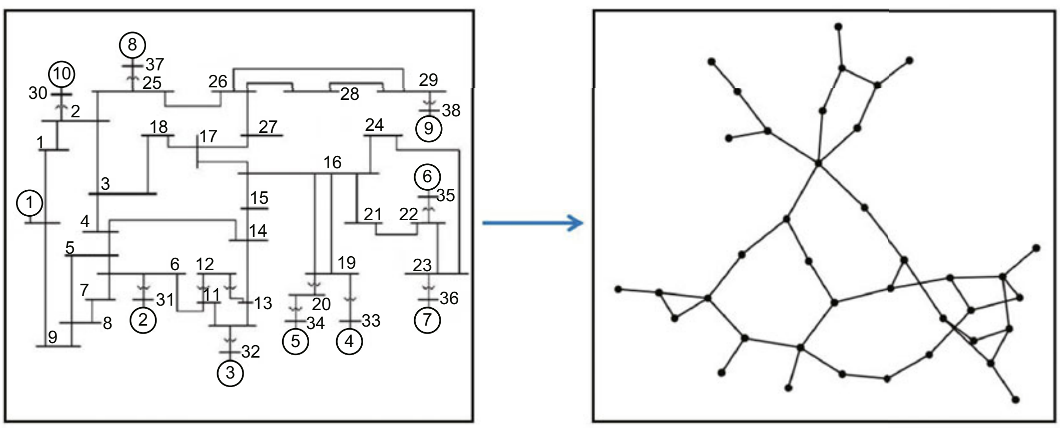
simple network metrics (such as mean shortest paths, degree centrality, and clustering coefficient) and network connectivity properties inferred from spectral graph theory. Many of these metrics have been studied in large-scale networks such as power grids (Anghel et al., 2007), which are commonly modeled as complex networks as shown in Fig. 1. Power grid robustness is typically evaluated based on abnormal events such as a cascading failure of the transmission lines, which is the consequence of the collective dynamics of a power grid (Koç et al., 2014). Because a power grid cascading failure starts from the propagation of a single local failure, the network topology can help us effectively analyze the robustness of the power grid.

However, power grid complexity goes beyond its topology. Analysis based solely on topological features may lead to inaccurate results, because it fails to capture some of the peculiarities of power networks described by Kirchoff's laws (Cuadra et al., 2015). Consequently, evaluation and validation of real cascading failure data are necessary to verify the effectiveness of topological methods. However, because cascading failures are rare events in a power grid, analyzing the system's properties directly from cascading failure data is impractical. Existing studies commonly rely on simulation tools to inspect how a power grid will behave in case of such rare events. There are several tools that simulate cascading failures and are capable of returning detailed information that is useful for understanding how cascades propagate, such as DCSIMSEP (Eppstein and Hines, 2012; Rezaei et al., 2015), OPA (Carreras et al., 2002), MATCASC (Koç et al., 2013), and COSMIC (Song et al., 2016). However, it should be noted that although these tools can give detailed simulation results, they cannot provide any insight into how the topology of a power grid affects the system's robustness.

In recent years, many studies have devoted efforts to optimizing a network's structural robustness by designing network topology (Liu W et al., 2009; Liu ZY et al., 2012; Laszka et al., 2013; Peng and Wu, 2016). Several different optimization models have been presented to generate robust networks that could withstand random failures and attacks. A lot of research has focused on localized networks, node connectivity, and edge connectivity, but these areas do not tell the whole story of cascade propagation phenomena, which encompasses the entire power grid network. There are much more variables that can be considered. For example, algebraic connectivity is considered an essential indicator of a network's resilience, and it was used by Liu W et al. (2009) to quantify the importance of network nodes and lines. Laszka et al. (2013) proposed a new metric called persistence, which was used to mitigate attacks by controlling node deployment, resulting in more robust network topologies for wireless sensor networks. Liu ZY et al. (2012) converted the optimal sensor network design problem into a multi-objective optimization problem to obtain a balanced result among the rigidity and efficiency of the connections and resilience to node disconnections. A statistical model called the branching process (Dobson et al., 2005, 2010; Qi et al., 2013; Dey et al.,



(a) (b)

Fig. 1 Electrical diagram (a) and network graph (b) of the IEEE 39 network

2016) was explored in later studies to help analyze system dynamics. It gives an accurate result of the average propagation even for a smaller number of simulations. Despite this progress, the following challenges still need to be grappled when designing robust power grid topology: (1) Due to the large number of nodes in real-world power grids, a highly scalable algorithm is required; (2) Although various methods for evaluating robustness exist, there is no widely acknowledged robustness metric and using different metrics often leads to different optimized networks.

To address these issues, a global power grid perspective, rather than a local view, should be adopted when designing a robust topology. Because the edges are sparse in real-world power grids, removing existing edges often has undesirable effects. It is natural to optimize the topology by adding edges. However, adding edges to real-world power grids is quite difficult due to resource constraints, and design planning in advance is required. We propose a topology-based power grid optimization strategy for selecting transmission lines to add to the network, which optimizes the grid's robustness. We use a more informative metric, algebraic connectivity of the power grid, because it is sensitive to connectivity in a broader spectrum of graphs (Liu W et al., 2009; Dey et al., 2016). Because graph theory does not fully address a power system's physical characteristics, power grid robustness is currently evaluated mainly by power flow simulation (Azzolin et al., 2018) and verified by cascading failure simulator based power flow equations. Average propagation is calculated to evaluate network robustness using MATCASC to simulate cascading failures. Our experimental results show that generally, the larger the algebraic connectivity is, the more severe the propagation will be. We propose a greedy algorithm to determine the optimal way to add lines to the grids so the increase in the algebraic connectivity is minimized after new connections are added. As a result, cascading failure propagation after the addition can also be expected to be low. Finally, we compare our approach with existing strategies. To evaluate our approach, we conduct case studies with real-world datasets. The results proved the usefulness and effectiveness of our proposed strategy.

The contributions of this study are as follows:

1. We optimize edge addition using a modified greedy algorithm to increase the efficiency of the process and reduce the computational time complexity.

2. We perform empirical analysis of cascading failures in several power grids based on algebraic connectivity, using average propagation as the main evaluation criterion.

2 Related works

Our work is related to the study of topological analysis of networks and algebraic connectivity, as well as cascading power grid failures. Here, we briefly review the related works in these fields of research.

2.1 Topological analysis of network and algebraic connectivity

The study of applied graph theory, also known as complex network theory (Correa-Henao and Yusta-Loyo, 2015), is characterized by its theoretical representation of a system as a network topology. This facilitates study of the impacts of topology changes on the robustness of the system with topological measurements. Examples of these measurements include the degree of connections (Holme et al., 2002; Correa-Henao et al., 2013), betweenness (Guan et al., 2011; Marsden, 2015), and centrality and mean shortest paths (Dey et al., 2016). According to Bigdeli et al. (2009), the betweenness, degree, and clustering coefficient are all defined on a single node. To evaluate the robustness of the entire network, these metrics need to be calculated on every node, which may result in an inefficient computation process. The above metrics explicitly use the graph topology to quantify connectivity. In addition to these metrics, there exists another group of indicators, called spectrum-based measurements, which are derived from the adjacency matrix and Laplacian matrix of a network (van Mieghem, 2010; Ellens et al., 2011). Spectrum-based measurements have been shown to be associated with the inherent interconnectedness, partition ability, propagation range, and convergence rates of dynamic network processes, and thus have been widely used to quantify network robustness.

Major progress in spectral analysis was presented by Fiedler (1973), who introduced a metric of graph connectedness called algebraic connectivity, which is the second smallest eigenvalue of the Laplacian matrix of a graph. He showed that the greater the algebraic connectivity, the more difficult it is to cut the graph into smaller components. Existing studies indicate that high algebraic connectivity results in robust networks, and attempt to maximize the algebraic connectivity by adding edges (lines) to the grid (Jamakovic and Uhlig, 2007; Sydney et al., 2013). Many strategies have been proposed to optimize a network's algebraic connectivity, including edge addition (Jiang et al., 2011; Wei P et al., 2014; Ji et al., 2016), edge deletion (Wei P et al., 2014), and edge rewiring (Sydney et al., 2013). Sydney et al. (2013) studied edge rewiring on three kinds of complex networks and compared edge rewiring to edge addition. Wei P et al. (2014) introduced the flight route addition/deletion problem and compared three different methods to analyze and optimize the algebraic connectivity of air transportation networks. In recent years, some new efforts were also witnessed in network optimization in industrial sectors, such as air transportation and satellite networks (Zheng et al., 2017), and also in businesses where sensor networks are used (Liu ZY et al., 2012; Laszka et al., 2013). Due to limiting factors such as computational complexity, most researchers employ the algebraic connectivity of a network to quantify the importance of a node or an edge in a localized view rather than in a global view.

2.2 Cascading failure and network topology

Extensive literature exists on cascading failures. Large-scale cascading failures are usually the results of propagation from a single local failure into the whole network (Koç et al., 2014). The existing works can be divided into two basic categories: the evolutionary approach and the holistic approach (Li and Xue, 2019). The evolutionary approach focuses mainly on the causes and consequences of cascading failures (Chen J et al., 2005; Bompard et al., 2016). Flow analysis (Rei et al., 2000) and Markov chains (Wang ZF et al., 2012) were adopted in such studies. The holistic approach focuses on the topology and operation of power grids. This approach typically evaluates vulnerabilities to locate weak points in power systems (Chen Q and Mili, 2013; Wang YZ and Baldick, 2014). Furthermore, with the development of system engineering, cascading failures can be analyzed based on complex network theory (Saleh et al., 2018), which is used to model a power grid's ability to handle cascading outages from a macroscopic perspective (Carreras et al., 2004).

Most of the existing models for cascading failures in power grids are basic topological models. Hardly any attention has been paid to quantifying network robustness against cascading failure in terms of network's topological properties. Some researchers studied both the topological features and effect of flow dynamics on network robustness in cascading failures. Koç et al. (2014) proposed a topological metric that uses effective graph resistance to relate power grid robustness to cascading failures. Dey et al. (2016) proposed a novel approach to investigate the relationship between the average propagation of failures and the topological variations occurring in the grid. Pahwa et al. (2012) studied how topological changes can affect the robustness of the network against attacks and failures. However, due to power grid complexity, there are still insufficient network optimization techniques to design typologies that are robust against cascading failures. As most power grids are usually sparsely connected and normal changes to the topology can involve the addition of new power lines, we consider to optimize the robustness of a power grid against cascading failures with edge additions.

3 Empirical analysis of cascading failures in power grids

In this section, we present empirical analysis of the relationship between cascading failures in a power grid and its topology. To examine the effects that topology has on a cascading system, it is necessary to develop a cascade model as a complex network, where the generation, transmission, and load buses are modelled as nodes while the transmission lines are represented as edges in accordance to the circuit laws. We then explain how to use these models to run simulations of cascading failures on power grids. This is followed by a detailed explanation of average propagation.

Some existing research has already analyzed power grid networks by attacking critical power lines in the topology. The results often simulate the worst-case scenarios by causing critical power lines to fail (Pizzuti et al., 2020). In the real world, cascading failures can be caused by several factors, not just the failure of critical power lines. This means that the results from the existing research reflect only one kind of cascading failures (caused by critical line failures). In this paper, we applied a random attack method to obtain a better overall view of the impact of the cascade. The results based on random attacks on power lines are more acceptable than the results from attacking critical lines. A comparison of different attack strategies and their effects on the robustness levels of the tested networks was proposed in Koç et al. (2013).

3.1 Cascading failure simulation

A cascading failure occurs when the failure of one part of an interconnected system results in the failure of more parts, and eventually the whole system. The concept is comparable to a set of falling dominoes. In a power grid, each line has a relay that protects it from permanent damage due to events such as excessive current. To avoid permanently damaging the line, an overcurrent relay notifies a circuit breaker to trip a line when the line's current exceeds its capacity limit and this violation lasts long enough. As stated in Koç et al. (2014), we assume that the capacity C_l of a transmission line l is proportional to its initial power flow $L_l(0)$ (when there is no failure in the network):

$$C_l = \alpha_l L_l(0), \tag{1}$$

where α_l is the tolerance level of line l. Traditionally, power grid researchers have focused on the robustness of a grid for a specific grid tolerance level of the grid. The study of effects of using different tolerance levels is important in identifying the robustness and vulnerability of real-life networks; Wang JW and Rong (2009) showed that tolerance could be used negatively to cause damage by spreading rumors or positively to control epidemics.

For simplicity, we assume a deterministic model for the line tripping mechanism, where the circuit breaker for a line trips at the moment the flow of the line exceeds its capacity. When isolated islands are created by the failure, the cascading failure continues in each island in which generators or loads are separately shed, to attain a supply-demand balance. The cascade of failures continues until no more components are overloaded.

Following the notation of Dobson et al. (2006), we assume that the cascading failure is started by $\epsilon_0 > 0$ initial failures in stage 0 and continues to produce further failures $\epsilon_1, \epsilon_2, ...$ in stages 1, 2, ..., respectively. The simulation is repeated K times with different initial failures to produce K independent instances of cascading failures.

We define $Y_n^{(k)}$ as the total number of failures including stage n in the k^{th} simulation as

$$Y_n^{(k)} = \epsilon_0^{(k)} + \epsilon_1^{(k)} + \epsilon_2^{(k)} + \ldots + \epsilon_n^{(k)}. \tag{2}$$

We apply MATCASC, a MATLAB-based cascading failure analysis tool from Koç et al. (2013), to solve the power flow equations and analyze cascading line outages in power grids.

3.2 Average propagation

To deduce the growth of outages after they are initiated, it is necessary to have an estimator to predict the blackout severity. We use average propagation (AP), which was proposed by Dobson et al. (2006) to evaluate the scale of the cascading failure. For cascading data obtained from K simulations using the methods described in Section 3.1, AP is expressed as

$$\text{AP} = \frac{\sum_{k=1}^{K}\left(\epsilon_1^{(k)} + \epsilon_2^{(k)} + \ldots + \epsilon_{s(k)}^{(k)}\right)}{\sum_{k=1}^{K}\left(\epsilon_0^{(k)} + \epsilon_1^{(k)} + \ldots + \epsilon_{s(k)-1}^{(k)}\right)}. \tag{3}$$

With a triggering event leading to more outages, the chances that the entire system will collapse are rare. Rather, small independent islands will form, indicating that a few lines are still intact. Therefore, in the above equation, $s(k)$ depends on the saturation S (Dobson et al., 2006):

$$s(k, S) = \max\left\{n | Y_{n-1}^{(k)} < S \text{ and } \epsilon_{n-1}^{(k)} > 0\right\}. \tag{4}$$

We can see that the cascading failure simulation stops when the number of failures in any stage is zero or the total number of failures becomes at least S.

AP can be divided into the following cases:

(1) AP < 1: In this case, the cascade tends to die after a certain number of stages. (2) AP > 1: Here, the cascade either ceases after some stages with certain probability or proceeds until the system is saturated.

3.3 Impact of tolerance level and saturation

We use simulations to evaluate the impact of tolerance level and saturation. We set the initial failure to be random attacks, where four randomly chosen transmission lines (edges) are removed.

Fig. 2a shows the impact of the tolerance level on AP for the IEEE 39 network. Data is obtained with $[\alpha_{\min} = 1,\ \alpha_{\max} = 4]$ subdivided with $\Delta\alpha = 0.1$. The impact of the tolerance level in Fig. 2a suggests that AP decreases as α increases. The larger the value of α, the slower the decrease in AP. Therefore, network robustness is positively correlated with α as measured by AP. However, in reality, due to economic considerations, the safety margins are limited and will not be very high. Fig. 2b shows the effect of saturation in a cascading failure in terms of AP. We found that the increase in saturation causes a decrease in AP. It can be seen from the figure that when the ratio of saturation to the total number of edges reaches 0.6, AP decreases much more slowly as the saturation continues to increase.

We design a line capacity tolerance and saturation level to control the scale of failure (Anghel et al., 2007; Moussawi et al., 2017; Spiewak et al., 2018). In our numerical evaluations in Sections 3.4 and 5, we set the saturation S to 60% of the total number of transmission lines in the system and the tolerance level α to 1.3 for each line.

3.4 Correlation between average propagation and algebraic connectivity

The relationship between algebraic connectivity and robustness is somewhat counter-intuitive. Dey et al. (2016) reported that after the removal of a line, the system tends to be less connected, thus increasing the network sparsity. Links tend to act independently of one another, which effectively reduces failure propagation. A system that has low algebraic connectivity tends to be less connected and there is a decrease in AP. Intuitively, a densely connected network will result in more severe propagation. As new connections typically characterize a power grid's evolution, cascading failure is an increasing concern for the power grid as it grows in size. From the summary given above, our understanding is that in a sparsely connected network, if network connectivity is increased, it may become more prone to failure propagation. Thus, we need to find an optimal method to increase the network robustness by edge addition, with minimum increase in the failure propagation rate.

We conducted simulations on the IEEE 39 network and IEEE 118 network to test and identify the impact of algebraic connectivity on propagation. As explained in Section 3.2, several simulations were performed to estimate AP using Eq. (3). In each simulation, several random edges were added to the grid. After the addition of edges, the connectivity of the system is increased. Edges tend to act independently of one another, which effectively increases the failure propagation range. We again used random attacks to generate initial failures. The relationship between the algebraic connectivity and AP is shown in Fig. 3. The correlation in Table 1 shows that when the network becomes more connected (that is, has higher algebraic connectivity), AP of failures is also increased. From these results, we can see an obvious positive correlation between algebraic connectivity and propagation spread. Because the cascading spread is also affected by some factors that are not captured by topological metrics, some discrepancies exist between algebraic connectivity and AP, but the general tendency of correlation is obvious.

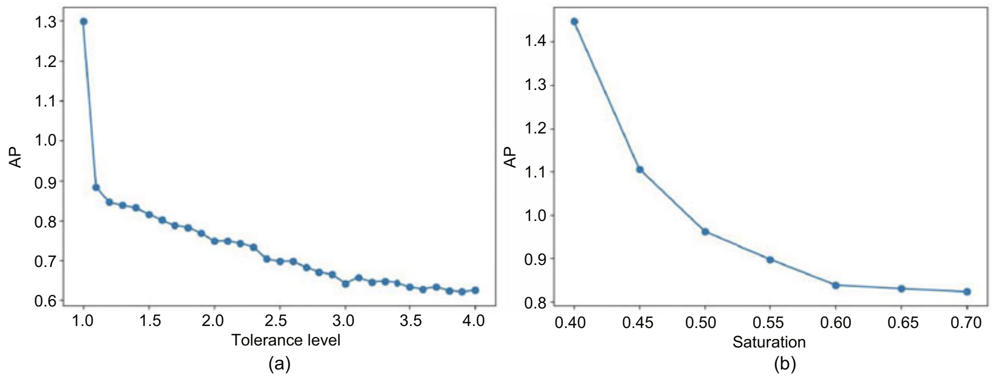


Fig. 2 Impact of the tolerance level on the average propagation (AP) for the IEEE 39 network (a) and impact of the saturation S on AP for the IEEE 39 network (b), where M is the ratio of S to the total number of edges

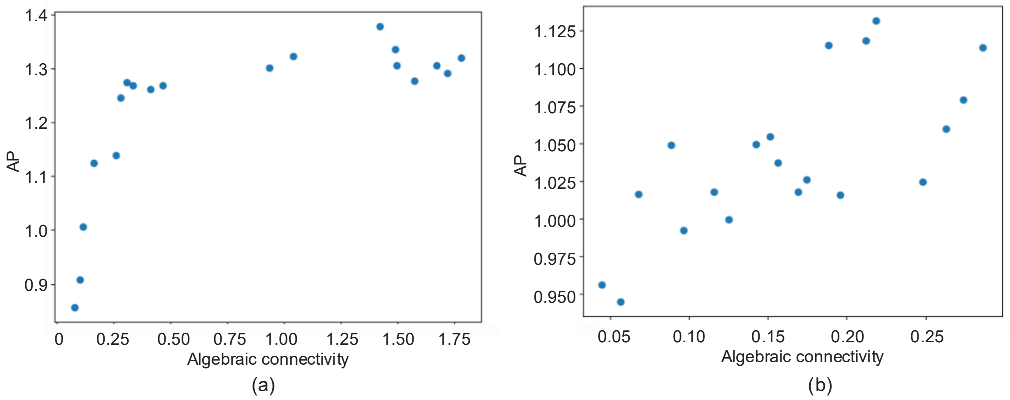


Fig. 3 Relationship between the algebraic connectivity and the average propagation (AP) in 20 simulations for the IEEE 39 network (a) and the IEEE 118 network (b)

Table 1 Correlation between the algebraic connectivity and average propagation for the IEEE 39 network and the IEEE 118 network obtained in 20 simulations

Dataset	Correlation coefficient
IEEE 39	0.686 350
IEEE 118	0.713 378

4 Cascading failure reduction via algebraic connectivity

In real life, there are few chances to construct a totally new power grid network, either in a local area or for a whole country. Instead, experts and engineers are tasked with maintaining or finding ways to improve the robustness of an existing network. The actions of these engineers and experts are restricted by a budget (e.g., adding a fixed number of edges), having to follow government regulations, and other constrictions. These issues and concerns are the major motivations of this work.

4.1 Problem formulation

In this study, we aim to optimize the algebraic connectivity of power grid networks to improve robustness by minimizing AP. Consider the electrical network's physical structure. Electric power is transferred from the generation buses to distribution substations through the transmission buses, interconnected by transmission lines. The graph of a power grid network can be described by $G(V, E_0)$, where node set V represents the generators, substations, and transformers in the power grid, and edge set E_0 contains the power transmission lines. Let n denote the size of V and m denote the size of E_0. The second smallest eigenvalue of the Laplacian matrix of a graph is called its algebraic connectivity $\lambda_2(\boldsymbol{L})$, and the corresponding normalized eigenvector is called the Fiedler vector (Fiedler, 1973). According to Ghosh and Boyd (2006), the Laplacian matrix $\boldsymbol{L}$ can be represented by the dot product summation of edge vectors. For an edge e connecting two nodes i and j, we define the edge vector $\boldsymbol{h}_e \in \mathbb{R}^n$ as $h_e(i) = 1$, $h_e(j) = -1$, and all other entries equal to 0. Then the Laplacian matrix $\boldsymbol{L}$ of G is an $n \times n$ matrix:

$$\boldsymbol{L} = \sum_{e=1}^{m} \boldsymbol{h}_e \boldsymbol{h}_e^{\mathrm{T}}. \tag{5}$$

The objective of this study is to reduce propagation of a cascading failure by minimizing the increase in algebraic connectivity after a fixed number of edge additions.

All possible edges that can be added are given in a pre-determined set P. We denote the edges chosen to be added as a set ΔE. Thus, the edge addition problem can be formulated as

$$\begin{aligned} &\min \lambda_2 \left(G\left(V, E_0 + \Delta E\right)\right) \\ \text{s.t.} \quad &|\Delta E| = k,\ \Delta E \subseteq P,\ P \cap E_0 = \varnothing. \end{aligned} \tag{6}$$

4.2 Minimizing algebraic connectivity using greedy edge addition

Ghosh and Boyd (2006) presented a greedy local heuristic, where they added k edges to the grid based on the Fiedler vector. In a sparsely connected network, adding k edges all at once may not produce an optimal result. By extending their method, we present an algorithm called modified greedy edge addition (MGEA) to reduce the propagation range by adding k edges using a selection criterion and minimizing the increase in algebraic connectivity.

According to Mohar et al. (1991), the algebraic connectivity can be computed by

$$\lambda_2(\boldsymbol{L}(x)) = \min\left\{\frac{\boldsymbol{y}^{\mathrm{T}}\boldsymbol{L}(x)\boldsymbol{y}}{\boldsymbol{y}^{\mathrm{T}}\boldsymbol{y}} \mid \boldsymbol{y} \neq \boldsymbol{0},\ \boldsymbol{1}^{\mathrm{T}}\boldsymbol{y} = \boldsymbol{0}\right\}, \tag{7}$$

where $\boldsymbol{y}$ is an $n \times 1$ non-zero vector and it is orthogonal to the all-one vector $\boldsymbol{1}$. Furthermore, Eq. (7) can be transformed into

$$\lambda_2(\boldsymbol{L}(x)) = \min\left\{\frac{\boldsymbol{y}^{\mathrm{T}}\boldsymbol{L}(x)\boldsymbol{y}}{\|\boldsymbol{y}\|^2} \mid \boldsymbol{y} \neq \boldsymbol{0},\ \boldsymbol{1}^{\mathrm{T}}\boldsymbol{y} = \boldsymbol{0}\right\}, \tag{8}$$

where we can replace vector $\boldsymbol{y}$ with the normalized vector $\boldsymbol{v} = \boldsymbol{y}/\|\boldsymbol{y}\|$ and we have Eq. (9):

$$\lambda_2(\boldsymbol{L}(x)) = \min\left\{\boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}(x)\boldsymbol{v} \mid \|\boldsymbol{v}\| = \boldsymbol{1},\ \boldsymbol{1}^{\mathrm{T}}\boldsymbol{v} = \boldsymbol{0}\right\}. \tag{9}$$

The normalized vector $\boldsymbol{v}$ in Eq. (9) is the Fiedler vector, because

$$\lambda_2(\boldsymbol{L}(x))\boldsymbol{v} = \boldsymbol{L}(x)\boldsymbol{v}. \tag{10}$$

Multiplying $\boldsymbol{v}^{\mathrm{T}}$ to the left of both sides of Eq. (10) yields

$$\boldsymbol{v}^{\mathrm{T}}\lambda_2(\boldsymbol{L}(x))\boldsymbol{v} = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}(x)\boldsymbol{v}. \tag{11}$$

Because vector $\boldsymbol{v}$ is normalized,

$$\boldsymbol{v}^{\mathrm{T}}\lambda_2(\boldsymbol{L}(x))\boldsymbol{v} = \lambda_2(\boldsymbol{L}(x))\left(\boldsymbol{v}^{\mathrm{T}}\boldsymbol{v}\right) = \lambda_2(\boldsymbol{L}(x)). \tag{12}$$

Therefore, if $\boldsymbol{v}$ is a Fiedler vector, the minimum in Eq. (9) can be achieved:

$$\lambda_2(\boldsymbol{L}(x)) = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}(x)\boldsymbol{v}. \tag{13}$$

Based on Eq. (5), the Laplacian matrix after edge addition is

$$\boldsymbol{L}_{\text{new}} = \boldsymbol{L}_0 + \sum_{l=1}^{|P|}\boldsymbol{h}_e\boldsymbol{h}_e^{\mathrm{T}} = \boldsymbol{L}_0 + \boldsymbol{L}', \tag{14}$$

where $\boldsymbol{L}' = \sum_{l=1}^{|P|}\boldsymbol{h}_e\boldsymbol{h}_e^{\mathrm{T}}$. If $\boldsymbol{v}$ is a Fiedler vector, we can obtain the algebraic connectivity as

$$\begin{aligned}\lambda_2(\boldsymbol{L}_{\text{new}}) &= \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}_{\text{new}}\boldsymbol{v} = \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}_0\boldsymbol{v} + \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}'\boldsymbol{v} \\ &\geq \lambda_2(\boldsymbol{L}_0) + \lambda_2\left(\boldsymbol{L}'\right).\end{aligned} \tag{15}$$

So, minimizing $\lambda_2(\boldsymbol{L}_{\text{new}})$ can be relaxed to minimizing $\lambda_2(\boldsymbol{L}')$. Note that $\Delta E \subseteq P$, and minimizing $\lambda_2(\boldsymbol{G}')$ is equivalent to selecting edges from P that have the smallest impact on $\lambda_2(\boldsymbol{L}_P)$. If G is a large graph with many edges, adding one edge has only an insignificant impact on its second eigenvector. According to Eq. (13), we have

$$\begin{aligned}\lambda_2(\boldsymbol{L}_{\text{new}}) &= \boldsymbol{v}^{\mathrm{T}}_{(\boldsymbol{L}_{\text{new}})}\boldsymbol{L}_{\text{new}}\boldsymbol{v}_{(\boldsymbol{L}_{\text{new}})} \approx \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}_{\text{new}}\boldsymbol{v} \\ &= \boldsymbol{v}^{\mathrm{T}}\boldsymbol{L}_0\boldsymbol{v} + \boldsymbol{v}^{\mathrm{T}}\left(\boldsymbol{h}_e\boldsymbol{h}_e^{\mathrm{T}}\right)\boldsymbol{v} \\ &= \lambda_2(\boldsymbol{L}_0) + (v_i - v_j)^2.\end{aligned} \tag{16}$$

Because $(v_i - v_j)^2 \geq 0$, we can see that adding an edge can never decrease the algebraic connectivity. Fig. 4 shows 200 random instances of edge addition for the IEEE 118 network and the IEEE 2383 network. From the result, we can see that there is a strong positive correlation between the value of $(v_i - v_j)^2$ of an edge and the increase in the algebraic connectivity (λ_2) after edge addition. Therefore, we believe that it is appropriate to apply our method to increase the robustness of the system, because it

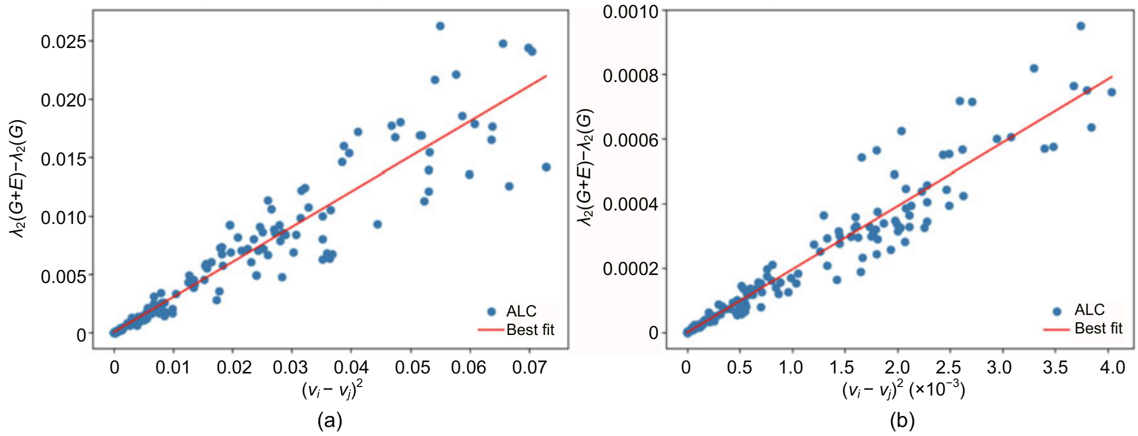


Fig. 4 Relationship between the value of $(v_i - v_j)^2$ and the increase in algebraic connectivity after edge addition for the IEEE 118 network (a) and the IEEE 2383 network (b)

leads to small algebraic connectivity after edge addition, thus reducing the failure propagation range.

In summary, the MGEA algorithm picks one edge from the candidate set with the minimum $(v_i - v_j)^2$ at each iteration, where v_i and v_j are the i^{th} and j^{th} items of the Fiedler vector $\boldsymbol{v}$ of the current Laplacian matrix $\boldsymbol{L}$, respectively. The complete algorithm is listed in Algorithm 1.

Algorithm 1 Modified greedy edge addition

Given graph $G(V, E_0)$ and candidate edge set P
Let $E = E_0$
for 1 to k **do**
  Calculate $\lambda_2(G(V, E))$ and its Fiedler vector $\boldsymbol{v}$
  $e_{ij} = \arg\min_{e_{ij} \in P} (v_i - v_j)^2$
  $E = E + e_{ij}$
  $P = P - e_{ij}$
end for
Output $G(V, E)$

5 Experiments and results

Determining the optimal location of a new edge is challenging. The added edges should increase the power grid's robustness by minimizing failure propagation. In this section, we review the results from the experiments that we conducted to evaluate the greedy edge addition algorithm's ability to improve the robustness of power grid networks. Our study is intended to examine the following question: Does MGEA outperform the other four baseline strategies?

5.1 Experimental protocol

5.1.1 Datasets

Four datasets, IEEE 39, IEEE 57, IEEE 118, and IEEE 2383, are used in our experiments. Some properties of the datasets are presented in Table 2. We will evaluate the performance of our algorithm in terms of (1) minimizing the algebraic connectivity increase and (2) reducing the average propagation rate. Considering the issue of protecting the grid against random failures or targeted attacks, in our experiments, cascading failures are triggered by random attacks, where four randomly chosen transmission lines (edges) are removed. For the IEEE 2383 dataset, which has a total of 2896 edges, we remove 10 lines instead, to compensate for the large number of edges compared with the other three datasets. Because different initial attacks can lead to different results, we performed 200 iterations when calculating AP (i.e., $K = 200$ in Eq. (3)).

Table 2 Numbers of nodes and edges in each dataset

Dataset	Number of nodes	Number of edges
IEEE 39	39	46
IEEE 57	57	80
IEEE 118	118	186
IEEE 2383	2383	2896

The number of lines removed randomly in each cycle was determined by experiments. Based on $N-k$ contingency analysis (Wei XG et al., 2019), we narrowed down the initial powerline removal number to be between 2 and 20. We found that removing 4 edges in a small network and 10 edges in an extensive network satisfied the needs for computational efficiency, while preserving the experimental results' trends. In real-world networks, most changes are restricted by limitations and budgets, such as limits on the number of powerlines that can be added and the need to comply with government regulations, among other restrictions. Based on real-world power grid topologies, our simulations return the top 20 edges that can be added to the original network, which can help simplify a human engineer's decision making to improve the grid's robustness. How those edges are chosen in each strategy is explained below.

5.1.2 Baseline strategies

Here, we compare MGEA with the baseline strategies. Table 3 summarizes the strategies and their corresponding computational complexities.

1. Random (RD)

The random addition strategy simply chooses an

Table 3 Summary of the strategies and their computational complexities

Strategy	Edge addition criterion	Complexity
RD	Random selection	$O(kM)$
DP	$\arg\min_{i,j} (d_i d_j)$	$O(kN^2)$
BT	$\arg\min_{i,j} ((C_B(i)+1)(C_B(j)+1))$	$O(kN^3)$
ER	$\arg\max_{i,j} (R_{ij})$	$O(kN^3)$
MGEA	$\arg\min_{e_{ij} \in P} (v_i - v_j)^2$	$O(kN^3)$

k is the number of edges to be added; N is the number of nodes; M is the number of edges in the initial grid. RD: random; DP: degree product; BT: betweenness; ER: effective resistance; MGEA: modified greedy edge addition edge from the candidate set at random. It is often selected as a reference to be compared with other edge addition strategies.

2. Degree product (DP) (Marsden, 2015)

The degree of a node is the simplest centrality metric that reflects a node's importance in its locality (Marsden, 2015). For an undirected network, the degree of a node is equal to the number of edges connected to it. The edge in the candidate set with the lowest degree product is added.

3. Betweenness (BT) (Guan et al., 2011; Jiang et al., 2011)

Betweenness is one of the most important metrics to evaluate the routing strategy performance of a network (Jiang et al., 2011). We define the betweenness product of an edge (i, j) in the candidate set as $(C_B(i) + 1)(C_B(j) + 1)$, where $C_B(v)$ is the betweenness centrality of node v as defined in Guan et al. (2011). Note that one is added to the betweenness before calculating the product because the betweenness for some nodes can be 0. The edge in the candidate set with the lowest betweenness product is added.

4. Effective resistance (ER) (van Mieghem, 2010; Koç et al., 2014)

According to Ohm's law, the effective resistance R_{ij} is the potential difference between nodes i and j when a unit current is injected at node i and withdrawn at node j (van Mieghem, 2010). The edge in the candidate set with the highest effective resistance is added.

Note that each algorithm is repeated multiple times, adding one edge each time, until the target number of edge additions is reached. For all the experiments performed in this study, the candidate set is chosen to be the set of all possible edge additions that do not lead to self-loops or parallel edges.

5.2 Performance comparison

Tables 4–7 present the values of AC and AP for each algorithm on the four datasets we used. Overall, we can see that the random, degree product, and effective resistance algorithms are far inferior to the MGEA algorithm proposed in this study. The performance of the betweenness algorithm is similarly to that of our algorithm when the first few edges are added. However, after more edges are added, our algorithm surpasses the betweenness algorithm substantially.

5.2.1 Algebraic connectivity

As shown in Figs. 5a, 6a, 7a, and 8a, the results suggest that the MGEA algorithm performs better than the other baseline methods in all four datasets in terms of minimizing the increase in AC. Fig. 5a shows that the results of the betweenness algorithm are relatively close to those of our algorithm initially.

However, after 10 edges are added, the distinction becomes clear, with our algorithm having the best results. Therefore, for the IEEE 39 network, we can say that the performance of the MGEA algorithm is the best among the five we tested. In addition, we find that the topology of the IEEE 39 network and some grid-related parameters are relatively close to those of the IEEE 118 network, so it is reasonable for the IEEE 39 to be similar to the IEEE 118 in experimental results.

For the IEEE 2383 network, Fig. 8a shows that the performance of the MGEA algorithm is also better than those of the baseline algorithms. We can conclude that each of the five algorithms tested adds different lines to the network, and each added line influences the topology's overall strength in a different way. The MGEA algorithm appears to be the most successful one among the five tested in improving the overall robustness of the whole network.

5.2.2 Average propagation

We can see from the results that among all the strategies tested, the MGEA algorithm yields the most robust network configuration against cascading failures, as measured by AP. However, as shown in Fig. 6b, the results of the DP algorithm are comparable to those of our algorithm except for the last few edges, where the performance of the DP algorithm deteriorates. Electric power is transmitted from the generation buses to the load buses through intermediate (transmission) components, which deliver electric power from generators to consumers. We believe that this may be because the DP algorithm adds only edges from one of the two nodes in the initial grid with very small degrees to the other nodes. When these edges fail, there are more neighboring lines around the edges that can carry the fault. The load makes the edges of the surrounding nodes less prone to overloading and failure, so AP of the cascading failure is relatively small. Another possible reason is that after studying the IEEE 57 network, we find

Table 4 Algebraic connectivity (AC) and average propagation (AP) after edge addition for the IEEE 39 network

Number of edge additions	AC					AP				
	RD	DP	BT	ER	MGEA	RD	DP	BT	ER	MGEA
1	0.083 98	0.076 97	0.076 74	0.084 05	0.076 19	0.874 29	0.855 45	0.845 00	0.930 96	0.843 80
2	0.093 00	0.098 75	0.076 81	0.118 71	0.076 19	0.903 71	0.862 64	0.858 08	1.077 30	0.865 88
3	0.101 93	0.099 41	0.077 32	0.143 87	0.076 19	0.960 63	0.973 10	0.871 07	1.078 00	0.889 59
4	0.116 36	0.139 96	0.077 32	0.175 90	0.076 19	0.983 47	0.999 60	0.876 01	1.069 09	0.883 40
5	0.127 36	0.146 97	0.077 33	0.193 20	0.076 19	1.008 18	1.114 74	0.934 75	1.100 74	0.869 73
6	0.139 99	0.147 40	0.077 33	0.260 56	0.076 19	1.060 62	1.162 07	0.974 36	1.134 75	0.915 13
7	0.152 70	0.152 52	0.093 14	0.285 77	0.076 19	1.071 86	1.121 51	1.040 72	1.239 76	0.900 49
8	0.165 09	0.186 58	0.093 55	0.285 92	0.076 19	1.102 32	1.120 20	1.055 04	1.227 78	0.898 92
9	0.172 93	0.186 76	0.093 91	0.292 41	0.076 19	1.114 72	1.125 20	1.045 13	1.190 48	0.923 62
10	0.186 80	0.190 49	0.164 15	0.365 48	0.076 20	1.127 05	1.139 60	1.091 73	1.201 44	0.909 70
11	0.200 67	0.196 08	0.181 20	0.419 35	0.076 20	1.156 52	1.156 38	1.145 35	1.216 05	0.954 74
12	0.215 05	0.208 59	0.181 20	0.434 91	0.076 21	1.171 30	1.161 17	1.111 81	1.189 11	0.931 86
13	0.233 23	0.208 72	0.196 28	0.436 77	0.076 22	1.181 88	1.141 94	1.185 71	1.196 45	0.979 22
14	0.241 98	0.211 75	0.198 90	0.445 81	0.076 24	1.189 58	1.163 15	1.193 38	1.235 52	1.011 24
15	0.257 36	0.223 24	0.251 38	0.492 99	0.076 26	1.195 07	1.202 19	1.192 98	1.221 09	1.006 66
16	0.282 45	0.224 51	0.252 15	0.604 67	0.076 27	1.207 48	1.188 25	1.278 35	1.277 34	0.989 51
17	0.301 41	0.228 42	0.288 23	0.633 26	0.076 28	1.217 31	1.133 14	1.249 77	1.277 48	0.985 32
18	0.316 32	0.228 56	0.288 29	0.655 32	0.076 30	1.236 10	1.174 24	1.293 51	1.285 93	1.008 81
19	0.338 53	0.229 02	0.288 32	0.657 43	0.076 32	1.237 18	1.197 06	1.323 50	1.313 81	0.995 63
20	0.351 44	0.229 15	0.289 10	0.665 88	0.076 33	1.242 83	1.176 28	1.336 80	1.318 20	1.004 42

RD: random; DP: degree product; BT: betweenness; ER: effective resistance; MGEA: modified greedy edge addition

Table 5 Algebraic connectivity (AC) and average propagation (AP) after edge addition for the IEEE 57 network

Number of edge additions	AC					AP				
	RD	DP	BT	ER	MGEA	RD	DP	BT	ER	MGEA
1	0.096 56	0.138 47	0.132 08	0.125 25	0.088 22	1.018 10	1.049 97	1.054 77	0.997 76	1.000 51
2	0.107 54	0.144 37	0.132 16	0.137 62	0.088 22	1.034 72	1.029 92	1.012 95	1.039 94	1.008 50
3	0.117 31	0.146 46	0.132 48	0.137 68	0.088 22	1.038 02	0.997 47	1.062 02	1.067 64	0.997 52
4	0.130 61	0.148 16	0.132 54	0.159 49	0.088 22	1.057 19	1.021 93	1.092 66	1.113 42	0.980 45
5	0.136 67	0.155 04	0.132 54	0.176 65	0.088 22	1.055 37	1.024 24	1.067 77	1.092 82	1.035 05
6	0.146 08	0.162 72	0.132 77	0.184 82	0.088 22	1.052 87	1.030 19	1.071 50	1.064 04	1.030 71
7	0.157 55	0.166 07	0.132 77	0.184 85	0.088 22	1.082 25	1.047 28	1.083 49	1.106 35	1.032 46
8	0.165 74	0.168 77	0.139 54	0.190 56	0.088 22	1.080 64	1.051 51	1.085 56	1.111 34	1.033 50
9	0.179 31	0.175 19	0.139 66	0.191 05	0.088 22	1.100 48	1.055 27	1.104 72	1.157 93	1.070 71
10	0.187 10	0.175 21	0.139 73	0.200 69	0.088 22	1.098 43	1.060 99	1.112 50	1.190 68	1.037 74
11	0.194 03	0.179 46	0.140 06	0.233 49	0.088 22	1.104 36	1.060 84	1.110 96	1.108 24	1.049 14
12	0.201 38	0.179 50	0.140 33	0.238 82	0.088 23	1.118 21	1.054 86	1.161 04	1.139 20	1.082 74
13	0.210 38	0.179 67	0.140 50	0.239 51	0.088 23	1.124 98	1.040 54	1.148 70	1.144 77	1.068 20
14	0.219 07	0.193 72	0.200 49	0.243 15	0.088 23	1.140 04	1.065 74	1.161 46	1.217 22	1.101 45
15	0.226 00	0.194 45	0.219 15	0.243 42	0.088 23	1.141 23	1.067 82	1.166 67	1.226 36	1.067 73
16	0.234 35	0.200 12	0.221 48	0.243 42	0.088 23	1.141 31	1.087 53	1.192 23	1.216 10	1.078 52
17	0.247 80	0.200 20	0.226 39	0.300 37	0.088 23	1.153 67	1.088 38	1.160 65	1.130 81	1.079 78
18	0.260 54	0.200 61	0.246 87	0.315 73	0.088 23	1.153 99	1.141 63	1.178 28	1.141 68	1.068 30
19	0.269 11	0.201 32	0.279 12	0.316 89	0.088 23	1.161 85	1.136 63	1.180 43	1.184 11	1.071 54
20	0.280 26	0.202 35	0.286 12	0.340 30	0.088 23	1.161 00	1.111 29	1.159 84	1.207 59	1.052 68

RD: random; DP: degree product; BT: betweenness; ER: effective resistance; MGEA: modified greedy edge addition

that its topology is slightly different from those of other networks. The generators are concentrated in one area, and the generator degrees are very low. When a new line is added to a generator, AP of the cascading failure may be smaller if the new line is added in a network with a different topology. For the IEEE 2383 network, the difference among the algorithms is not very obvious when adding the first

Table 6 Algebraic connectivity (AC) and average propagation (AP) after edge addition for the IEEE 118 network

Number of edge additions	AC					AP				
	RD	DP	BT	ER	MGEA	RD	DP	BT	ER	MGEA
1	0.033 45	0.028 63	0.027 13	0.038 78	0.027 13	1.095 33	1.075 81	1.075 23	1.105 65	1.062 03
2	0.038 68	0.028 72	0.027 20	0.057 91	0.027 13	1.105 80	1.115 57	1.062 82	1.135 24	1.081 94
3	0.042 73	0.033 77	0.027 20	0.066 69	0.027 13	1.116 14	1.110 93	1.051 47	1.152 52	1.086 59
4	0.048 88	0.033 77	0.027 34	0.068 73	0.027 13	1.119 75	1.136 60	1.076 88	1.147 70	1.072 18
5	0.053 87	0.033 79	0.027 41	0.070 77	0.027 13	1.127 17	1.136 16	1.060 06	1.160 86	1.078 83
6	0.059 50	0.034 22	0.027 42	0.079 96	0.027 13	1.140 93	1.146 34	1.081 83	1.153 88	1.077 86
7	0.064 68	0.034 63	0.027 59	0.081 41	0.027 13	1.145 65	1.129 91	1.077 30	1.189 66	1.080 40
8	0.068 76	0.034 63	0.027 67	0.094 37	0.027 13	1.155 17	1.164 04	1.082 68	1.268 91	1.087 17
9	0.074 13	0.034 67	0.031 99	0.110 77	0.027 13	1.159 80	1.139 39	1.115 19	1.186 10	1.088 06
10	0.079 75	0.034 68	0.050 21	0.123 10	0.027 13	1.164 26	1.143 91	1.169 77	1.187 68	1.072 09
11	0.083 53	0.034 68	0.052 54	0.123 53	0.027 13	1.167 63	1.143 52	1.160 16	1.170 65	1.068 36
12	0.089 21	0.034 80	0.053 36	0.131 43	0.027 13	1.173 83	1.126 14	1.149 49	1.196 23	1.070 45
13	0.094 40	0.034 97	0.068 77	0.144 91	0.027 13	1.181 08	1.141 76	1.178 52	1.240 81	1.071 17
14	0.100 94	0.035 04	0.070 55	0.159 65	0.027 13	1.190 13	1.145 69	1.187 43	1.243 83	1.073 74
15	0.105 83	0.035 06	0.072 16	0.169 98	0.027 13	1.189 57	1.130 73	1.190 05	1.219 32	1.083 09
16	0.112 17	0.035 32	0.072 50	0.180 02	0.027 13	1.196 31	1.165 72	1.180 75	1.219 51	1.082 55
17	0.119 48	0.035 32	0.073 54	0.200 06	0.027 13	1.197 06	1.155 79	1.197 06	1.218 62	1.068 93
18	0.122 72	0.035 32	0.074 51	0.201 23	0.027 13	1.201 83	1.156 61	1.216 87	1.249 14	1.076 93
19	0.129 01	0.035 32	0.074 69	0.209 17	0.027 13	1.208 50	1.133 41	1.195 38	1.249 61	1.063 03
20	0.134 90	0.035 41	0.087 77	0.209 58	0.027 13	1.207 70	1.128 35	1.229 91	1.225 49	1.062 65

RD: random; DP: degree product; BT: betweenness; ER: effective resistance; MGEA: modified greedy edge addition

Table 7 Algebraic connectivity (AC) and average propagation (AP) after edge addition for the IEEE 2383 network

Number of edge additions	AC					AP				
	RD	DP	BT	ER	MGEA	RD	DP	BT	ER	MGEA
1	0.003 43	0.003 54	0.003 22	0.003 25	0.003 23	1.001 05	1.000 30	0.999 78	1.003 46	0.999 24
2	0.003 64	0.003 74	0.003 23	0.003 27	0.003 23	1.001 73	0.996 77	0.999 11	1.000 21	0.999 82
3	0.003 80	0.003 74	0.003 23	0.003 30	0.003 23	1.002 47	0.996 69	0.999 83	1.001 24	0.999 66
4	0.004 00	0.004 24	0.003 26	0.003 41	0.003 23	1.002 27	1.000 20	1.002 94	0.999 40	0.999 55
5	0.004 14	0.004 55	0.003 31	0.003 41	0.003 23	1.002 54	1.001 97	1.006 13	0.999 57	1.001 68
6	0.004 32	0.004 69	0.003 32	0.003 51	0.003 23	1.001 92	1.001 51	1.005 55	1.002 05	1.001 74
7	0.004 52	0.005 04	0.003 33	0.003 51	0.003 23	1.002 00	1.002 53	1.006 98	1.002 14	1.000 75
8	0.004 68	0.005 32	0.003 78	0.003 58	0.003 23	1.002 28	1.005 41	1.008 57	1.004 30	1.000 37
9	0.004 83	0.005 47	0.004 14	0.003 60	0.003 23	1.002 60	1.005 44	1.005 33	1.004 97	1.001 74
10	0.004 96	0.005 48	0.004 16	0.003 66	0.003 23	1.002 79	1.006 49	1.005 56	1.003 24	1.001 12
11	0.005 22	0.005 56	0.004 16	0.003 68	0.003 23	1.003 36	1.006 18	1.005 15	1.002 69	1.002 69
12	0.005 41	0.005 68	0.004 16	0.003 69	0.003 23	1.003 47	1.004 37	1.007 64	1.004 92	1.001 39
13	0.005 57	0.005 73	0.004 16	0.003 72	0.003 23	1.002 93	1.005 16	1.005 91	1.003 81	1.001 30
14	0.005 71	0.005 74	0.004 16	0.003 87	0.003 23	1.003 13	1.003 58	1.006 39	1.006 22	1.001 63
15	0.005 90	0.005 74	0.004 16	0.003 87	0.003 23	1.003 45	1.004 66	1.007 32	1.004 63	1.002 01
16	0.006 05	0.005 78	0.004 16	0.003 89	0.003 23	1.003 96	1.002 88	1.005 29	1.008 15	1.001 92
17	0.006 14	0.005 76	0.004 16	0.003 95	0.003 23	1.004 30	1.004 82	1.006 84	1.008 38	1.002 24
18	0.006 30	0.005 77	0.004 17	0.003 96	0.003 23	1.003 52	1.005 56	1.005 68	1.007 17	1.001 59
19	0.006 42	0.005 77	0.004 17	0.004 12	0.003 23	1.003 86	1.006 86	1.005 24	1.004 29	1.001 45
20	0.006 57	0.005 79	0.004 17	0.004 14	0.003 23	1.003 80	1.007 65	1.006 42	1.005 54	1.000 43

RD: random; DP: degree product; BT: betweenness; ER: effective resistance; MGEA: modified greedy edge addition

10 edges, due to the large number of existing edges. Also, because the electrical characteristics of power grids are not captured in the algorithms, the tendency of the results is not uniform. However, after the addition of a significant number of edges, the performance of the MGEA algorithm shows superiority against the other algorithms tested.

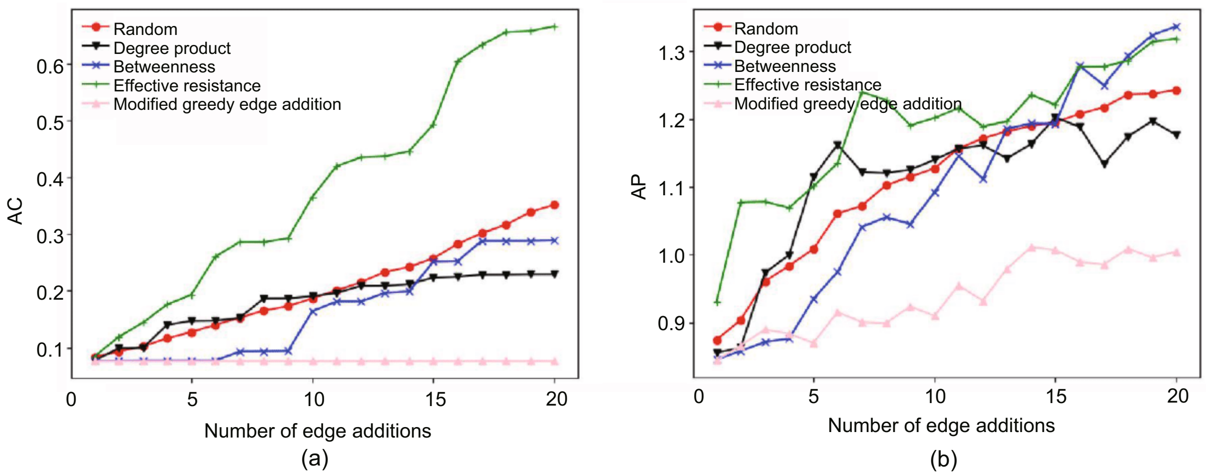


Fig. 5 Algebraic connectivity (AC) (a) and average propagation (AP) (b) after edge addition for the IEEE 39 network

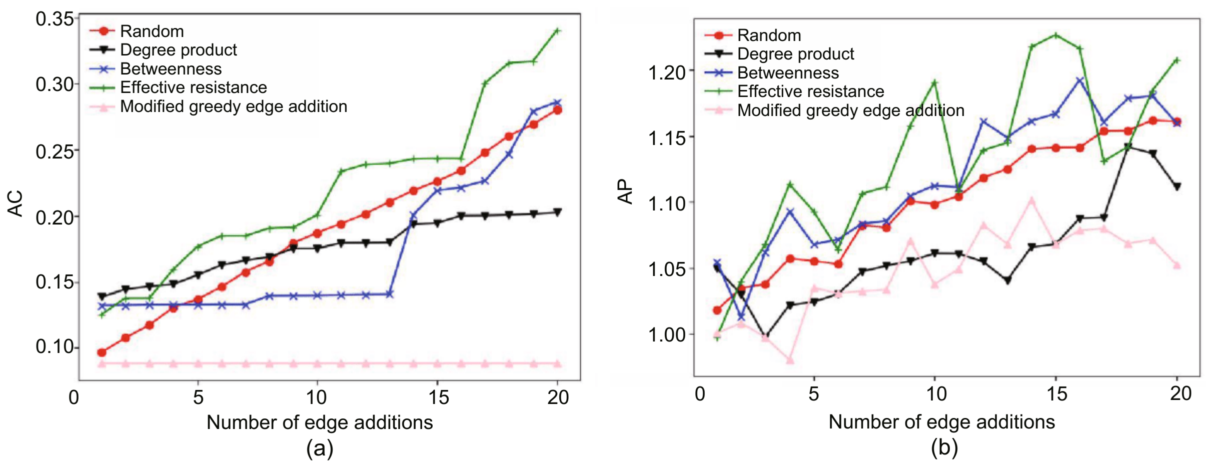


Fig. 6 Algebraic connectivity (AC) (a) and average propagation (AP) (b) after edge addition for the IEEE 57 network

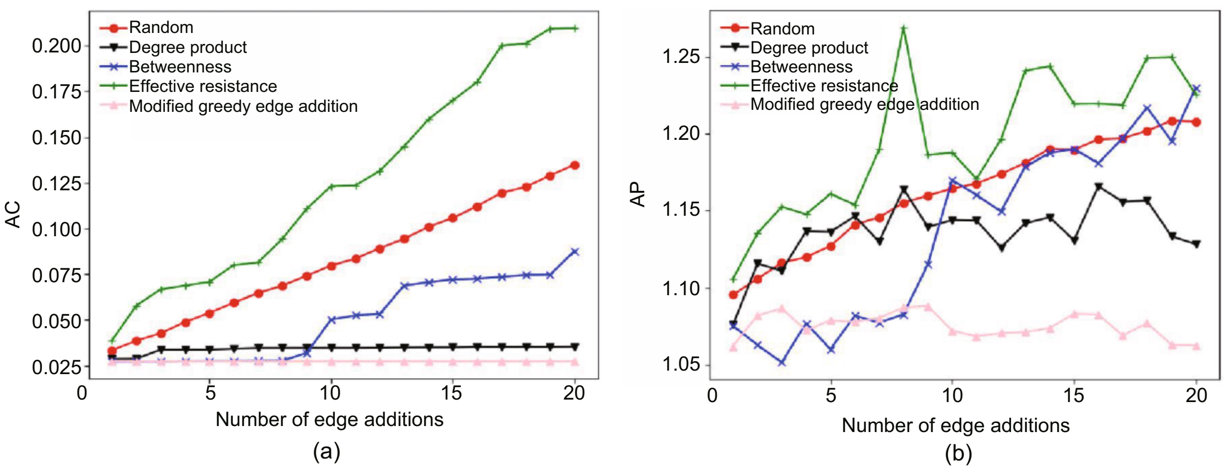


Fig. 7 Algebraic connectivity (AC) (a) and average propagation (AP) (b) after edge addition for the IEEE 118 network

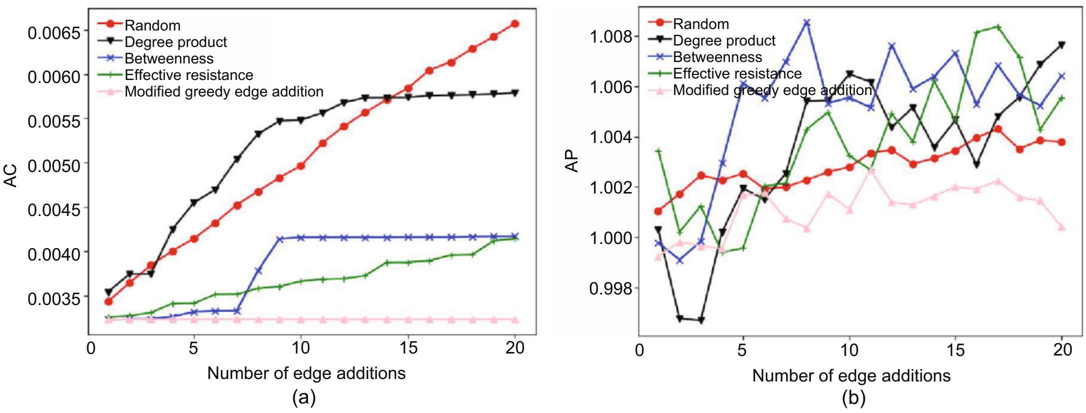


Fig. 8 Algebraic connectivity (AC) (a) and average propagation (AP) (b) after edge addition for the IEEE 2383 dataset

6 Conclusions

In this study, we studied topology-based design optimization strategies for selecting transmission lines to add a power grid's topology to a power grid, to increase its robustness in terms of average propagation. Experimental results on four power grid datasets showed that our proposed MGEA algorithm outperforms all the compared algorithms. We expect our algorithm to help experts better design new power grid topologies and maintain power grid systems in the future.

At the same time, it is noted that algebraic connectivity is a topological measurement that is widely used to assess network characteristics. Although a topological approach is appropriate to evaluate the power distribution grid, purely topological metrics fail to capture some inherent electrical characteristics of power grids. In our work, the network modeled does not fully distinguish different types of buses in the system because buses in a power grid can be categorized into different categories such as generation, transmission, or load buses. We will leave that to the future work to incorporate more electrical properties in a power grid model.

Furthermore, power grids of the same structure can also display different robustness in practice. Among existing studies, the robustness of a power grid network can be related to various factors, such as consumers' accessibility to generators (Zhang and Tse, 2015), tolerance factors (Liu J et al., 2019), and electrical properties such as resistance and power flow (Koç et al., 2014). Discovering such relationships is an important direction for future research.

Contributors

Supaporn LONAPALAWONG designed the study and addressed the problems. Supaporn LONAPALAWONG and Jiangzhe YAN processed the data and designed the algorithms. Jiayu LI and Deshi YE helped deduce the mathematical models and algorithms. Supaporn LONAPALAWONG drafted the manuscript. Yong TANG and Yanhao HUANG helped with the technical information. Wei CHEN supported with the resources. Can WANG helped organize the manuscript. Supaporn LONAPALAWONG, Jiangzhe YAN, and Can WANG revised and finalized the paper.

Compliance with ethics guidelines

Supaporn LONAPALAWONG, Jiangzhe YAN, Jiayu LI, Deshi YE, Wei CHEN, Yong TANG, Yanhao HUANG, and Can WANG declare that they have no conflict of interest.

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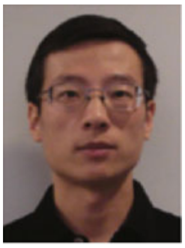

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