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Bipartite asynchronous impulsive tracking consensus for multi-agent systems*

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Abstract: In this study, we discuss how multi-agent systems (MASs) with a leader can achieve distributed bipartite tracking consensus using asynchronous impulsive control strategies. The proposed asynchronous impulsive control approach does not require the impulse to occur simultaneously for all agents. The communication links between neighboring nodes of MASs are antagonistic. When the leader's control input is non-zero, sufficient conditions are obtained to achieve bipartite asynchronous impulsive tracking consensus in closed-loop MASs. More extensive ranges of asynchronous impulsive effects are discussed, and the designed controller's feedback can effectively work against adverse impulsive permutation. Simple algebraic conditions for estimating the impulsive gain boundary and asynchronous impulsive interval are presented. Theoretical results are demonstrated with illustrative examples.

Key words: Bipartite tracking; Multi-agent systems; Asynchronous impulsive; Consensus
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1 Introduction

Multi-agent system (MAS) coordination has definitely been a research hotspot in control and systems over the last few decades (Guan et al., 2014; Ning et al., 2019; Zhao and Yang, 2020; Zhu YN et al., 2020; Yang ZQ et al., 2021). MASs form a complex dynamical network that includes multiple agents using a coupling protocol. Cooperation between agents enables them to cooperate with each other to complete complex tasks, such as synchronization (Yang XS et al., 2018; Wang P et al., 2021) and flight formation (Jadbabaie et al., 2003). A basic MAS control problem is to design a distributed control protocol to make multiple autonomous agents reach an agreement, which would have potential applications in many fields, such as cooperative robots, unmanned aerial vehicles, spacecraft coordination, autonomous flight formation, and wireless sensor networks (Zuo et al., 2018; Yang D et al., 2019; Gao et al., 2020).

In recent years, there have been many reports about leader and leaderless consensus control of MASs. The problem of observer design for second-order MAS consensus tracking was discussed by Yu et al. (2017). Finite (fixed)-time consensus problems for second-order MASs with reduced state information were discussed by Hong et al. (2019). Du et al. (2020) studied distributed fixed-time consensus

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for nonlinear heterogeneous MASs. Output feedback based consensus control for nonlinear MASs was investigated by Li K et al. (2020). One of the basic paradigms for the multi-agent consensus tracking problem is first to consider communication links between cooperative (positive) node communications. As already known, in the communications among multiple agents, cooperative and antagonistic interactions may exist simultaneously. In this case, bipartite consensus was introduced and has been widely studied in recent years (Altafini, 2013; Jiang Y et al., 2017; Meng, 2017; Liu F et al., 2018; Wen et al., 2018; Zhang LZ and Yang, 2020; Pan et al., 2021). Altafini (2013) proved that a certain form of protocol can be achieved in the presence of antagonistic interactions. Pan et al. (2021) discussed bipartite consensus of matrix-valued weighted directed networks. Wen et al. (2018) solved the bipartite consensus problem of MASs with non-zero control inputs. Based on the preset-time approach, bipartite consensus tracking for second-order MASs was discussed by Ning et al. (2020).

It is well known that many biological phenomena, such as threshold, optimal control model in economics, and frequency modulation systems, have impulsive effects (Lakshmikantham et al., 1989; Li YY, 2017; Ji et al., 2020). In practice, symbol flipping may occur due to communication errors when a consensus tracking impulsive controller is implemented for MASs (Li XD et al., 2019b). In addition, each agent usually possesses limited energy resources. It is impractical to transfer information constantly. Therefore, it is significant to develop impulsive control technologies (Yang T and Chua, 1997; Li XD et al., 2019b; Han et al., 2020; Jiang BX et al., 2020; Hu et al., 2021). In recent years, impulsive control protocols have been widely used (Lu JQ et al., 2012; Zhang H et al., 2017; Tan et al., 2019; Wang YQ et al., 2019; Li XD et al., 2020; Yang XS et al., 2020). For example, Yang T and Chua (1997) studied several theorems about the stability of impulsive control systems. Zhang H et al. (2017) provided an effective consensus protocol to solve the problems caused by the impulsive controlled random switching structure. Based on event-triggered impulsive control, Lyapunov stability of impulsive systems was studied by Li XD et al. (2020). The pinning synchronization of coupled Lur'e networks under mixed impulses was discussed by Wang YQ et al. (2019). It is noted that the aforementioned works on impulsive control of MASs were carried out under the synchronous setting; that is, the impulsive jumps of all the agent states occur with respect to the same time sequence.

Due to the limitation of current hardware, asynchronous impulsive control is widely used in practical applications. Therefore, to overcome the limitations of large-scale clock synchronization, some researchers are committed to researching asynchronous impulsive schemes (Jiang FC et al., 2018; Zhu W et al., 2020). Asynchronous consensus for second-order MAS networks with sampled data communication was investigated, in which each agent has its own time clock (Liu ZW et al., 2019). The asynchronous MAS consensus problem with fixed and switched communication topologies was studied by Zhu W et al. (2020). However, these works on consensus considered only MASs with cooperative communication links, which is a special communication link situation. Moreover, extensive ranges of impulsive effects are not considered. The estimation of impulsive gain and the maximum impulsive interval of asynchronous impulsive time sequences is not solved. So far, the problem of how to make MASs achieve bipartite tracking consensus under asynchronous impulsive control has not been solved.

In this work, we study mainly the bipartite tracking consensus problem of MASs by designing an asynchronous impulsive control protocol. The main challenges are modeling each agent of the asynchronous impulses, designing control gain of the asynchronous impulses, and estimating the maximum asynchronous impulsive control interval. Based on the definition of an average asynchronous impulsive interval and bipartite consensus, and combined with the relevant conclusions of graph theory, bipartite tracking consensus of MASs under different impulsive effects is obtained. First, a distributed controller with both feedback and asynchronous impulsive effects is considered. Second, the impulsive gain and maximum asynchronous impulsive interval are estimated according to the positive or negative communication links. Furthermore, the more extensive asynchronous impulsive effect is discussed, so that the asynchronous impulsive can play both positive and negative roles in the final tracking consensus.

In this paper, I_n denotes an $n \times n$ identity matrix. All the eigenvalues of matrix Q are real.

$\lambda_{\min}(Q)$ and $\lambda_{\max}(Q)$ represent the minimum and maximum eigenvalues, respectively. 1_n is an n-dimensional column vector with elements 1, and $\mathrm{sign}(\cdot)$ denotes the sign function. "$\otimes$" represents the Kronecker product. Let S be the vector norm set on C^n. For any $\varrho \in S$, matrix measure is denoted by μ_ϱ. Denote $\tau(\Lambda) = \max_p\{\mathrm{Re}(\lambda_p(\Lambda))\}$. $\rho_{\max}(A)$ is the maximum singular value of matrix A and $\sigma(A) = \max_{1\le p\le n}|\lambda_p(A)|$ is the spectral radius.

2 Preliminaries

Let $\mathcal{G}^{\mathrm{c}} = (\mathcal{V}, \mathcal{E}, \mathcal{C})$ represent a directed signed graph with node set $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$, where $\mathcal{C} = (c_{pq})_{N\times N}$ denotes a signed weighted adjacency matrix and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ denotes a set of link edges. A link edge $\mathcal{E}_{pq} \in \mathcal{E}$ is denoted by (v_p, v_q). c_{pq} could be positive or negative (i.e., $c_{pq} \neq 0$) if v_p can send information to agent v_q, that is, $\mathcal{E}_{pq} \in \mathcal{E}$. For $p = 1, 2, \cdots, N$, assume $c_{pp} = 0$. The neighbor set of agent p is defined as $\mathcal{N} = \{q|c_{pq} \neq 0\}$. A directed path from node v_p to node v_q is a subgraph of $\mathcal{G}^{\mathrm{c}}$, and the path is marked as (v_p, v_{p_1}), (v_{p_1}, v_{p_2}), $\cdots$, (v_{p_k}, v_q) with distinct agents v_{p_l}, $l = 1, 2, \cdots, k$. An undirected graph $\mathcal{G}^{\mathrm{c}}$ is called a connected graph if there is a path between any pair of different nodes v_p and v_q in $\mathcal{G}^{\mathrm{c}}$, $p, q = 1, 2, \cdots, N$. If a directed graph $\mathcal{G}^{\mathrm{c}}$ contains a root node r and r has paths to all other nodes in $\mathcal{G}^{\mathrm{c}}$, then the directed graph $\mathcal{G}^{\mathrm{c}}$ is believed to contain a directed spanning tree. Denote the Laplacian matrix by $\mathcal{L} = (l_{pq})_{N\times N}$ with $l_{pp} = \sum_{q=1,q\neq p}^{N}|l_{pq}|$, $l_{pq} = -c_{pq}, p \neq q$.

Definition 1 (Altafini, 2013) A signed graph $\mathcal{G}^{\mathrm{c}}$ is called structurally balanced if the node set $\mathcal{V}$ of $\mathcal{G}^{\mathrm{c}}$ can be partitioned into two subsets $\mathcal{V}_1$, $\mathcal{V}_2$ satisfying $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$ and $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, such that $c_{pq} \geq 0, v_p, v_q \in \mathcal{V}_i$ $(i \in \{1, 2\})$ and $c_{pq} \leq 0, v_p \in v_i$, $v_q \in \mathcal{V}_{3-i}$ $(i \in \{1, 2\})$. If not, the signed graph $\mathcal{G}^{\mathrm{c}}$ is said to be structurally unbalanced.

The algebraic definition of structural balance of a connected signed digraph $\mathcal{G}^{\mathrm{c}}$ is as follows:

Definition 2 (Altafini, 2013) A signed graph $\mathcal{G}^{\mathrm{c}}$ of order N is structurally balanced if and only if $\exists M \in \mathcal{M}$, $\mathcal{M} = \{M = \mathrm{diag}(m_1, m_2, \cdots, m_N)|m_p \in \{\pm 1\},\ p = 1, 2, \cdots, N\}$, such that matrix $\mathcal{MCM}$ contains all nonnegative entries and provides a partition $\mathcal{V}_1 = \{p|m_p > 0\}$ and $\mathcal{V}_2 = \{p|m_p < 0\}$.

In this study, we consider a network of $N + 1$ linear agents with a single leader and N followers. The dynamic of each agent is described by

$$\dot{x}_p(t) = Ax_p(t) + Bu_p(t),\ p = 0, 1, \cdots, N, \tag{1}$$

where $x_p(t) \in \mathbb{R}^n$ is the state of agent p, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $u_p(t) \in \mathbb{R}^m$ is the input of agent p. The agent labeled 0 is the leader, while the remaining agents represent the followers. For MAS (1), let $\tilde{\mathcal{C}} = (c_{pq})_{(N+1)\times(N+1)}$ $(p, q = 0, 1, \cdots, N)$ be an augmented graph that describes the communication topology between the leader and followers. If the leader is a neighbor of node k $(k = 1, 2, \cdots, N)$, then $c_{k0} > 0$; otherwise, $c_{k0} = 0$. Denote $\mathcal{C}$ and $\mathcal{L}$ as the adjacency matrix and Laplacian matrix of MAS (1), respectively.

Assumption 1 The graph composed of a leader and N followers has a directed spanning tree, and the leader is the root node. Furthermore, it is assumed that the subgraph describing the followers' information communication topology is structurally balanced and connected.

Remark 1 For MAS (1), the link edge $\mathcal{E}_{pq} \in \mathcal{E}$ is a directed link. When $c_{pq} > 0$, it indicates that the information exchange relationship between node p and node q is a cooperative positive connection, and that the coupling term can be represented by $c_{pq}(x_p(t) - x_q(t))$. When $c_{pq} < 0$, it means that there is a competitive negative connection between node p and node q, and that the coupling term can be expressed as $-c_{pq}(x_p(t) + x_q(t))$. Obviously, if there is $l \in \{1, 2, \cdots, N\}$ such that $c_{pl} < 0$, then Laplacian matrix $\mathcal{L}$ is not a row-sum-zero matrix.

Let $K = \mathrm{diag}(c_{10}, c_{20}, \cdots, c_{N0})$. It follows from Definitions 1 and 2 and Assumption 1 that all diagonal elements of $\mathcal{MCM}$ are nonnegative and that $\mathcal{MLM}$ is a row-sum-zero matrix. Denote

$$\mathcal{L}_M = M\mathcal{L}M + K,\ \ \mathcal{L}_K = \mathcal{L} + K. \tag{2}$$

According to the above analysis, matrix $\mathcal{L}_M$ is positive definite.

Assume that asynchronous impulsive time instants $T_p = \{t_1^p, t_2^p, \cdots\}$ $(p = 1, 2, \cdots, N)$ have no finite accumulation points. The impulsive effects for agent p are proposed as follows:

$$\begin{cases} u_p(t) = d_1 H\Big(\sum_{q=1}^{N} |c_{pq}|(x_p(t) - \mathrm{sign}(c_{pq})x_q(t)) \\ \qquad + c_{p0}(x_p(t) - m_p x_0(t))\Big),\ t \in [t_k^p, t_{k+1}^p), \\ x_p(t) = x_p(t^-) - \mu g_p(t^-),\ t = t_k^p, \end{cases} \tag{3}$$

where $m_p \in \{\pm 1\}$, the scalar d_1 and the control parameter $H \in \mathbb{R}^{m\times n}$ are to be designed later, μ represents the impulsive control gain and $\mu > 0$, $g_p(t^-) = \sum_{q=1}^{N} l_{pq}x_q(t^-) - c_{p0}(x_p(t) - m_p x_0(t))$, $x(t)$ is right-hand continuous, and $x_p(t_k^{p^-})$ and $g_p(t_k^{p^-})$ are the left limits of $x(t)$ and $g(t)$ at $t = t_k^p$ respectively.

For simplicity, merge all impulsive time instants $\{t_k^p\}$ as $T^\star = \{t_k\} = \bigcup_{p=1}^{N}\{t_k^p\}$ in chronological order. In this way, $T^\star$ represents the set of impulsive time instants of all agents and satisfies $t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$. Then, asynchronous impulsive control protocol (3) is reorganized as follows:

$$\begin{cases} u_p(t) = d_1 H\Big(\sum_{q=1}^{N} |c_{pq}|(x_p(t) - \mathrm{sign}(c_{pq})x_q(t)) \\ \qquad + c_{p0}(x_p(t) - m_p x_0(t))\Big),\ t \in [t_k, t_{k+1}), \\ x_p(t) = x_p(t^-) - \mu s_p(t) g_p(t^-),\ t = t_k, \end{cases} \tag{4}$$

where $s_p(t) = 0$ if t_k does not belong to set $T^\star$ and $s_p(t) = 1$ if t_k belongs to $T^\star$.

Definition 3 MAS (1) is called to achieve bipartite asynchronous impulsive tracking consensus under the designed control protocol (4), and the impulsive sequence $T^\star$ is independent if the following condition holds:

$$\lim_{t\to\infty} \|x_p(t) - m_p x_0(t)\| = 0. \tag{5}$$

Lemma 1 (Zahreddine, 2003) For any matrix E, the following equation holds:

$$\inf_{\varrho\in S}\{\mu_\varrho(E)\} = \max_{1\le p\le n}\{\mathrm{Re}(\lambda_p(E))\}.$$

Definition 4 (Lu JQ et al., 2010) In time interval (t_0, t), the impulsive sequence $T^\star$ with chronological order is considered. The number of impulsive times in sequence $T^\star$ is represented by $k(t_0, t)$. If there exist $\mathcal{T}$ ($\mathcal{T} > 0$) and ϖ ($\varpi > 0$) such that

$$\frac{t - t_0}{\mathcal{T}} - \varpi \le k(t_0, t) \le \frac{t - t_0}{\mathcal{T}} + \varpi, \quad t \ge t_0,$$

then in sequence $T^\star$, the average impulsive interval is less than $\mathcal{T}$.

3 Theoretical results

In this section, we consider bipartite tracking consensus for MAS (1) defined over a signed communication graph under different impulsive effects.

For $t \in [t_k, t_{k+1})$, considering control protocol (4), the dynamics of MAS (1) and the leader node is described by

$$\begin{aligned} \dot{x}(t) = {} & (I_N \otimes A)x(t) + d_1(\mathcal{L} \otimes BH)x(t) \\ & - d_1(\mathcal{L} \otimes BH)(MI_N \otimes x_0(t)) \\ & + d_1(K \otimes BH)x(t) \\ & - d_1(K \otimes BH)(MI_N \otimes x_0(t)), \end{aligned} \tag{6}$$

with the fact $d_1(\mathcal{L} \otimes BH)MI_N \otimes x_0(t) = 0$.

$$\begin{aligned} MI_N \otimes \dot{x}_0(t) &= MI_N \otimes (Ax_0(t) + Bu_0(t)) \\ &= (I_N \otimes A)(MI_N \otimes x_0(t)) \\ &\quad + MI_N \otimes Bu_0(t). \end{aligned} \tag{7}$$

Meanwhile, let $p = 1$. From Definition 1, we have $|c_{12}|m_1 = c_{12}m_2$: if node $1 \in \mathcal{V}_1$, node $2 \in \mathcal{V}_1$, then $c_{12} > 0$, $m_1 > 0$, $m_2 > 0$, and it is easy to obtain $|c_{12}|m_1 = c_{12}m_2$; if node $1 \in \mathcal{V}_2$, node $2 \in \mathcal{V}_2$, then $c_{12} > 0$, $m_1 < 0$, $m_2 < 0$, we also obtain $|c_{12}|m_1 = c_{12}m_2$; if node $1 \in \mathcal{V}_1$, node $2 \in \mathcal{V}_2$, then $c_{12} < 0$, $m_1 > 0$, $m_2 < 0$, $|c_{12}|m_1 = c_{12}m_2$ is still valid; if node $1 \in \mathcal{V}_2$, node $2 \in \mathcal{V}_1$, then $c_{12} < 0$, $m_1 < 0$, $m_2 > 0$, $|c_{12}|m_1 = c_{12}m_2$ still holds. Let $p = 1$, $q = 1, 2, \cdots, N$, that is

$$\begin{aligned} & l_{11}m_1 + l_{12}m_2 + \cdots + l_{1N}m_N \\ = & (|c_{12}| + |c_{13}| + \cdots + |c_{1N}|)m_1 - c_{12}m_2 \\ & - c_{13}m_3 - \cdots - c_{1N}m_N. \end{aligned}$$

Let $p = 1, 2, \cdots, N$. From the above analysis, we can obtain

$$\mathcal{L}MI_N \otimes x_0(t) = 0.$$

Let $e_p^\star(t) = x_p(t) - m_p x_0(t)$ $(p = 1, 2, \cdots, N)$ and $e^\star(t) = \big((e_1^\star(t))^{\mathrm{T}}, (e_2^\star(t))^{\mathrm{T}}, \cdots, (e_N^\star(t))^{\mathrm{T}}\big)^{\mathrm{T}}$. Rewrite control protocol (4) at $t = t_k$ in chronological order in the matrix form as

$$e_p^\star(t_k) = \Big(I_N - \mu S(t_k)(\mathcal{L} + K)\Big)e^*(t_k^-).$$

From Definition 2 and Remark 1, one has $m_p m_q c_{pq} \ge 0$ for $p, q = 1, 2, \cdots, N$ and $m_p \in \{\pm 1\}$, and then it is easy to obtain $c_{pq}m_p = |c_{pq}|m_q$ and

$|c_{pq}|m_p = c_{pq}\mathrm{sign}(c_{pq})m_p = |c_{pq}|m_q\mathrm{sign}(c_{pq})$. Combined with Eqs. (4), (6), and (7), the evolution dynamics of $e^\star(t)$ with impulsive effect is described by

$$\begin{cases} \dot{e}^\star(t) = \Big(I_N \otimes A + (d_1\mathcal{L}_K \otimes BH)\Big)e^\star(t), \\ \qquad t \in [t_k, t_{k+1}), \\ e_p^\star(t_k) = \Big(I_N - \mu S(t_k)\mathcal{L}_K\Big)e^\star(t_k^-), \quad t = t_k, \end{cases} \tag{8}$$

where $\mathcal{L}_K$ is defined in Eq. (2). $S(t_k) = \mathrm{diag}(s_1(t_k), s_2(t_k), \cdots, s_N(t_k))$. Let $e^*(t) = (M \otimes I_n)e^\star(t)$.

From Definition 2, one has $MM = I_N$ and $M\mathrm{sign}(s) = \mathrm{sign}(Ms)$. Recall that $M\mathcal{L}M$ is a row-sum-zero matrix. Combined with Eq. (8), we have

$$\begin{cases} \dot{e}^*(t) = \Big(I_N \otimes A + (d_1 M\mathcal{L}_K M \otimes BH)\Big)e^*(t), \\ \qquad t \in [t_k, t_{k+1}), \\ e^*(t_k) = (M \otimes I_n)(I_N - \mu S(t_k)(\mathcal{L} \\ \qquad + K)(M \otimes I_n)e^*(t_k^-), \ t = t_k, \end{cases} \tag{9}$$

and rewriting Eq. (9), we obtain

$$\begin{cases} \dot{e}^*(t) = \Big(I_N \otimes A + (d_1\mathcal{L}_M \otimes BH)\Big)e^*(t), \\ \qquad t \in [t_k, t_{k+1}), \\ e^*(t_k) = \Big(I_N - \mu S(t_k)(\mathcal{L}_M \otimes I_n)\Big)e^*(t_k^-), \ t = t_k. \end{cases} \tag{10}$$

3.1 Bipartite positive impulse consensus analysis

Through the above analysis, the following theorems are established. First, we consider the bipartite asynchronous impulsive tracking consensus problem of MAS (1) under positive impulse effects. Let $\Theta = (I_N \otimes I_n) - \mu S(t_k)(\mathcal{L}_M \otimes I_n)$.

Theorem 1 If Assumption 1 holds, there exists a constant η such that $0 < \eta < 1$ and $\frac{\ln \eta}{\mathcal{T}} + \omega < 0$. For the impulsive sequence $T^\star = \{t_k\} = \bigcup_{p=1}^N \{t_k^p\}$, suppose that the average impulsive interval is less than $\mathcal{T}$ ($\mathcal{T} > 0$), and

$$\begin{pmatrix} s\eta(I_N \otimes Q) & \Theta^{\mathrm{T}}(I_N \otimes Q) \\ (I_N \otimes Q)\Theta & (I_N \otimes Q) \end{pmatrix} > 0. \tag{11}$$

Then MAS (1) achieves bipartite asynchronous impulsive tracking consensus, where $\omega = \tau(\varLambda)\lambda_{\min}(Q^{-1})$.

Proof Construct a Lyapunov function as

$$V(t) = (e^*(t))^{\mathrm{T}}(I_N \otimes Q)e^*(t), \tag{12}$$

where Q is a positive-definite matrix. For $\forall t \in (t_k, t_{k+1})$, taking time derivative of $V(t)$ along the solutions of Eq. (10) yields

$$\begin{aligned} \dot{V}(t) &= 2(e^*(t))^{\mathrm{T}}(I_N \otimes QA + d_1\mathcal{L}_M \otimes QBH)e^*(t) \\ &= (e^*(t))^{\mathrm{T}}(I_N \otimes (QA + A^{\mathrm{T}}Q) \\ &\quad + 2(d_1\mathcal{L}_M \otimes QBH))e^*(t). \end{aligned}$$

Let $\varLambda = (QA + A^{\mathrm{T}}Q) + 2(d_1\mathcal{L}_M \otimes QBH)$. According to Lemma 1, one may obtain

$$\dot{V}(t) \le \tau(\varLambda)\lambda_{\min}(Q^{-1})V(t), \tag{13}$$

where $\tau(\varLambda) = \max_p\{\mathrm{Re}(\lambda_p(\varLambda))\}$. From inequality (13), it is easy to obtain

$$V(t) \le \mathrm{e}^{\omega(t-t_k)}V(t_k), \quad t \in (t_k, t_{k+1}), \tag{14}$$

where $\omega = \tau(\varLambda)\lambda_{\min}(Q^{-1})$.

When $t = t_k$, from Eq. (10), one has

$$\begin{aligned} e^*(t_k) &= \Big(I_N - \mu S(t_k)(\mathcal{L}_M \otimes I_n)\Big)e^*(t_k^-) \\ &= \Theta e^*(t_k^-). \end{aligned}$$

According to Schur complement and by combining inequality (11), one has

$$\eta I_N \otimes Q - \Theta^{\mathrm{T}}Q\Theta > 0.$$

Then,

$$\begin{aligned} V(t_k^+) &= (e^*(t))^{\mathrm{T}}(I_N \otimes Q)e^*(t_k) \\ &= (e^*(t_k^-))^{\mathrm{T}}\Theta^{\mathrm{T}}(I_N \otimes Q)\Theta e^*(t_k^-) \\ &\le \eta V(t_k^-). \end{aligned} \tag{15}$$

Combining inequalities (14) and (15) yields

$$\begin{aligned} V(t) &\le \mathrm{e}^{\omega(t-t_k)}V(t_k) \le \eta\mathrm{e}^{\omega(t-t_k)}V(t_k^-) \\ &\le \cdots \le \eta^k\mathrm{e}^{\omega(t-t_0)}V(t_0^-). \end{aligned}$$

Based on Definition 4, one has $k(t_0, t) \ge \frac{t-t_0}{\mathcal{T}} - \varpi$, since $0 < \eta \le 1$. Then

$$\begin{aligned} V(t) &\le \eta^k\mathrm{e}^{\omega(t-t_0)}V(t_0^-) \\ &\le \eta^{\frac{t-t_0}{\mathcal{T}} - \varpi}\mathrm{e}^{\omega(t-t_0)}V(t_0^-) \\ &= \eta^{-\varpi}\mathrm{e}^{\left(\frac{\ln\eta}{\mathcal{T}} + \omega\right)(t-t_0)}V(t_0^-). \end{aligned}$$

By the condition of Theorem 1, $\frac{\ln\eta}{\mathcal{T}} + \omega < 0$, as $t \to \infty$, $e^*(t)$ in Eq. (10) converges to zero exponentially. This completes the proof.

Remark 2 The effects of a control matrix and interconnection among nodes are described by the asynchronous impulsive control gain η, which satisfies inequality (11). Inequality $\frac{\ln\eta}{\mathcal{T}}+\omega<0$ relates to the average asynchronous impulsive interval and asynchronous impulsive control gain η in stabilizing system (10). It can be seen from inequality (11) that control gain η depends on coupling strength d_1 and control matrix K. The problem is to find the number of asynchronous impulsive times $k(t_0,t)$ and to estimate the average asynchronous impulsive interval, coupling strength d_1, and control matrix K to stabilize system (10).

Because the dimension of the matrix in inequality (11) is $2N\times 2n$, it is a challenge to calculate matrix inequality (11) when the number of nodes is large. To decouple the matrix inequality (11) into a lower-dimensional one, we consider the following conclusions. Let the chronological order impulsive sequence be $\{t_k\}$ with $\theta_{\max}=\sup\{t_{k+1}-t_k\}$ and $\theta_{\min}=\inf\{t_{k+1}-t_k\}$ $(k=1,2,\cdots)$, and suppose $0<\theta_{\min}\le\theta_{\max}<\infty$.

Corollary 1 If Assumption 1 holds, there exists a constant η_1 such that $0<\eta_1<1$ and $\frac{\ln\eta_1}{\theta_{\max}}+\omega_1<0$, for the impulsive sequence $T^\star=\{t_k\}=\bigcup_{p=1}^{N}\{t_k^p\}$. Suppose that the average impulsive interval is less than $\mathcal{T}$ $(\mathcal{T}>0)$, and

$$\rho^2_{\max}(\varTheta)\le\eta_1. \tag{16}$$

So, MAS (1) achieves bipartite asynchronous impulsive tracking consensus, where $\omega_1=\tau(\varLambda)\lambda_{\min}(Q^{-1})$.

Proof Select a Lyapunov function as $V_1(t)=\frac{1}{2}(e^*(t))^{\mathrm{T}}(I_N\otimes Q)e^*(t)$. For $\forall t\in(t_k,t_{k+1})$, taking time derivative of $V_1(t)$ yields

$$\dot{V}_1(t)=(e^*(t))^{\mathrm{T}}(I_N\otimes QA+d_1\mathcal{L}_M\otimes QBH)e^*(t).$$

According to Lemma 1, it is easy to obtain

$$V_1(t)\le \mathrm{e}^{\omega_1(t-t_k)}V_1(t_k),\ t\in(t_k,t_{k+1}), \tag{17}$$

where $\omega_1=\tau(\varLambda)\lambda_{\min}(Q^{-1})$.

When $t=t_k$, from Eq. (10), one has

$$\begin{aligned}V_1(t_k^+)&=\frac{1}{2}(e^*(t))^{\mathrm{T}}(I_N\otimes Q)e^*(t)\\&=\frac{1}{2}(e^*(t_k^-))^{\mathrm{T}}\varTheta^{\mathrm{T}}(I_N\otimes Q)\varTheta e^*(t_k^-)\\&\le\frac{1}{2}\rho^2_{\max}(\varTheta)(e^*(t_k^-))^{\mathrm{T}}(I_N\otimes Q)e^*(t_k^-)\\&\le\frac{1}{2}\eta_1(e^*(t_k^-))^{\mathrm{T}}(I_N\otimes Q)e^*(t_k^-)=\eta_1V_1(t_k^-).\end{aligned} \tag{18}$$

Combining inequalities (17) and (18) yields

$$\begin{aligned}V_1(t)&\le \mathrm{e}^{\omega_1(t-t_k)}V(t_k)\le\eta_1\mathrm{e}^{\omega_1(t-t_k)}V(t_k^-)\\&\le\cdots\le\eta_1^k\mathrm{e}^{\omega_1(t-t_0)}V(t_0^-).\end{aligned}$$

Based on Definition 4, one has $k(t_0,t)\ge\frac{t-t_0}{\mathcal{T}}-\varpi$, because $0<\eta_1<1$. Then

$$\begin{aligned}V(t)&\le\eta_1^k\mathrm{e}^{\omega_1(t-t_0)}V(t_0^-)\\&\le\eta_1^{\frac{t-t_0}{\theta_{\max}}-\varpi}\mathrm{e}^{\omega_1(t-t_0)}V(t_0^-)\\&=\eta_1^{-\varpi}\mathrm{e}^{\left(\frac{\ln\eta_1}{\theta_{\max}}+\omega_1\right)(t-t_0)}V(t_0^-).\end{aligned}$$

By the condition of Corollary 1, $\frac{\ln\eta_1}{\theta_{\max}}+\omega_1<0$, as $t\to\infty$, $e^*(t)$ in Eq. (10) converges to zero exponentially. This completes the proof.

Remark 3 If leaderless consensus of MASs is considered, where all the communication links of graph $\mathcal{G}^{\mathrm{c}}$ are cooperative (positive), the proof method of Theorem 2 is still valid for linear MASs described in Zhu W et al. (2020).

Corollary 2 If Assumption 1 holds, there exists a constant η_1 such that $0<\eta_1<1$ and $\frac{\ln\eta_1}{\theta_{\max}}+\omega_1<0$, for the impulsive sequence $T^\star=\{t_k\}=\bigcup_{p=1}^{N}\{t_k^p\}$. Suppose that the average impulsive interval is less than $\mathcal{T}$ $(\mathcal{T}>0)$, and

$$0<\mu<\frac{2}{\lambda_{\max}(\mathcal{L}_M)}, \tag{19}$$

$$0<\theta_{\max}<-\frac{2\ln(I_N-\mu S(t_k)\mathcal{L}_M)}{\omega_1}. \tag{20}$$

Then MAS (1) achieves bipartite tracking consensus, where $\omega_1=\tau(\varLambda)\lambda_{\min}(Q^{-1})$ and $\eta_1=\sigma^2(I_N-\mu S(t_k)\mathcal{L}_M)$.

Proof Because $M\mathcal{L}M$ is a zero-row-sum matrix and $K=\mathrm{diag}(c_{10},c_{20},\cdots,c_{N0})$, $\mathcal{L}_M$ is a positive matrix. Then all eigenvalues of $\mathcal{L}_M$ are positive (Lu WL et al., 2010). The following proof is similar to the proof of Corollary 3 in He et al. (2015).

Let $0<\lambda_1(\mathcal{L}_M)=\lambda_{\min}(\mathcal{L}_M)\le\lambda_2(\mathcal{L}_M)\le\lambda_3(\mathcal{L}_M)\le\cdots\le\lambda_N(\mathcal{L}_M)=\lambda_{\max}(\mathcal{L}_M)$. From inequality (19), one has $1-\mu\lambda_{\max}(\mathcal{L}_M)>-1$. Obviously, the inequality $1-\mu_1\lambda_{\min}(\mathcal{L}_M)<1$ holds. So,

$$-1<1-\mu\lambda_p(\mathcal{L}_M)<1,\ \ p=1,2,\cdots,N.$$

Note that $\lambda_p(I_N-\mu S(t_k)\mathcal{L}_M)=1-\mu\lambda_p(S(t_k)\mathcal{L}_M)$. Thus, matrix $\sigma(I_N-\mu S(t_k)\mathcal{L}_M)<$

1. From the definition of $S(t_k)$, one has that $S(t_k)\mathcal{L}_M$ is symmetric. Then

$$\begin{aligned}&\rho^2_{\max}(I_N-\mu S(t_k)\mathcal{L}_M)\\=&\sigma(I_N-\mu S(t_k)\mathcal{L}_M)^{\mathrm{T}}\sigma(I_N-\mu S(t_k)\mathcal{L}_M)\\=&\sigma^2(I_N-\mu S(t_k)\mathcal{L}_M)<1.\end{aligned}$$

From Corollary 1, we can complete the proof of Corollary 2.

Remark 4 When the topology of MASs is determined, inequalities (19) and (20) give the methods to select asynchronous impulsive control gain and the maximum asynchronous impulsive control interval respectively.

3.2 Bipartite negative impulse consensus analysis

In this part, the value range of asynchronous impulsive effects is extended. Although it has negative effects on the final bipartite consensus of MAS (1), bipartite tracking consensus can be achieved by relying on the feedback control term in controller (3), which can effectively counteract the side effects created by disadvantageous (negative) impulsive effects.

Theorem 2 If Assumption 1 holds, (A, B) is stable, the control parameter satisfies $\eta_2>1$, and for the impulsive sequence $T^\star=\{t_k\}=\bigcup_{p=1}^{N}\{t_k^p\}$, suppose that the average impulsive interval is less than $\mathcal{T}$ ($\mathcal{T}>0$) and

$$-\lambda_{\max}(PA+A^{\mathrm{T}}P-2PBB^{\mathrm{T}}P)\lambda_{\min}(P^{-1})>0,$$

$$\omega_2-\frac{\ln\eta_2}{\mathcal{T}}>0,$$

$$\begin{pmatrix}\eta_2(I_N\otimes P) & \Theta^{\mathrm{T}}(I_N\otimes P)\\ (I_N\otimes P)\Theta & (I_N\otimes P)\end{pmatrix}>0, \tag{21}$$

then MAS (1) achieves bipartite asynchronous impulsive tracking consensus, where $\omega_2=-\lambda_{\max}(PA+A^{\mathrm{T}}P-2PBB^{\mathrm{T}}P)\lambda_{\min}(P^{-1})>0$.

Proof Select a Lyapunov function as

$$V_2(t)=(e^*(t))^{\mathrm{T}}(I_N\otimes P)e^*(t),$$

where P is a positive-definite matrix.

For $\forall t\in(t_k,t_{k+1})$, we have

$$\begin{aligned}\dot V_2(t)&=2(e^*(t))^{\mathrm{T}}(I_N\otimes PA+d_1\mathcal{L}_M\otimes PBH)e^*(t)\\&=(e^*(t))^{\mathrm{T}}(I_N\otimes(PA+A^{\mathrm{T}}P)\\&\quad+2(d_1\mathcal{L}_M\otimes PBH))e^*(t).\end{aligned} \tag{22}$$

Let $H=-B^{\mathrm{T}}P$. From the algebraic Riccati inequality of Zhou and Doyle (1998), one has $PA+A^{\mathrm{T}}P-2PBB^{\mathrm{T}}P<0$. Denote $\omega_2=-\lambda_{\max}(PA+A^{\mathrm{T}}P-2PBB^{\mathrm{T}}P)\lambda_{\min}(P^{-1})>0$. According to Eq. (22), one has

$$\dot V_2(t)\le-\omega_2V_2(t). \tag{23}$$

From inequality (23), it is easy to obtain

$$V(t)\le \mathrm{e}^{-\omega_2(t-t_k)}V(t_k),\ t\in(t_k,t_{k+1}). \tag{24}$$

When $t=t_k$, similar to the proof of Theorem 1, the following inequality can be obtained:

$$V(t)\le\eta_2^k\mathrm{e}^{-\omega_2(t-t_0)}V(t_0^-).$$

Based on Definition 4, $k(t_0,t)\le\frac{t-t_0}{\mathcal{T}}+\varpi$. Because $\eta_2>1$, then

$$\begin{aligned}V(t)&\le\eta_2^k\mathrm{e}^{-\omega_2(t-t_0)}V(t_0^-)\\&\le\eta_2^{\frac{t-t_0}{\mathcal{T}}+\varpi}\mathrm{e}^{-\omega_2(t-t_0)}V(t_0^-)\\&=\eta_2^{\varpi}\mathrm{e}^{\left(\frac{\ln\eta_2}{\mathcal{T}}-\omega_2\right)(t-t_0)}V(t_0^-),\end{aligned}$$

where the fact that $\frac{\ln\eta_2}{\mathcal{T}}-\omega_2<0$ is easy to obtain: as $t\to\infty$, system (10) converges to zero exponentially. The proof is completed.

Remark 5 In the design of controller (4), the impulsive control input we considered does not need to occur synchronously for all nodes. However, previous works on impulsive control considered mainly that impulses are simultaneously applied to the nodes (He et al., 2015; Yang XS et al., 2020). In addition, a wider impulsive effect is considered, and therefore η_2, which was limited to 0–1 in He et al. (2015), could be greater than 1.

Remark 6 Controller (3) has two parts. When the impulsive effects are positive, the feedback control gain d_1 can be zero to save control cost. When the impulsive effects are negative and the impulse effects prevent the bipartite consensus of MASs, then Eq. (10) is unstable. The feedback control term $d_1H\Big(\sum_{q=1}^{N}|c_{pq}|(x_p(t)-\mathrm{sign}(c_{pq})x_q(t))+c_{p0}(x_p(t)-m_px_0(t))\Big)$ could work effectively to counteract the side effects caused by adverse impulsive effects. In addition, in the previous results (Lu JQ et al., 2012; Zhu W et al., 2020), the asynchronous impulsive intervals were approximately taken as $\theta_{\min}=\min\{t_{k+1}-t_k\}$ and $\theta_{\max}=\max\{t_{k+1}-t_k\}$. In this study, the definition of average impulsive interval is considered. The number of asynchronous impulsive times in the time interval (t_0, t) can be estimated by freely adjusting ϖ, $\mathcal{T}$, and time interval (t_0, t). Compared with existing results of Lu JQ et al. (2012) and Zhu W et al. (2020), the results of this study are less conservative.

Remark 7 In practice, there is usually a requirement to complete the tracking task of an MAS within finite time or a preset time (Ning et al., 2019). Existing controllers for fixed time and finite time include a sign function, and the chattering phenomenon is not overcome. Future research will consider fixed-time and finite-time bipartite consensus of first- and second-order MASs by designing a continuous controller. In addition, considering the asynchronous impulsive method, a time-based generator approach will be designed to study the fixed-time bipartite consensus of an MAS with lower initial control input cost.

4 Simulations

In our simulations, we consider MAS (1) with a leader marked 0 and five followers marked from 1 to 5 (Wen et al., 2018), and parameters are taken as

$$x_p(t) = \begin{pmatrix} x_{p1}(t) \\ x_{p2}(t) \end{pmatrix}, A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

p=0, 1, 2, 3, 4, 5.

Let $u_0(t) = \cos(x_0(t))$. The communication topology of MAS (1) is depicted in Fig. 1 at impulsive moments t_k^p.

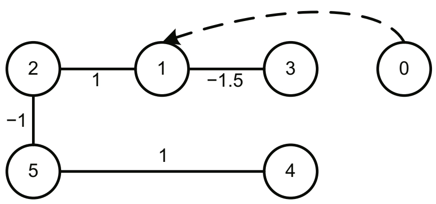

Fig. 1 Communication topology of six agents

In Fig. 1, the agent labeled "0" represents the leader, the solid lines represent the relationship between followers, and the dotted line represents the relationship between followers and the leader. From Fig. 1, it can be verified that the topology corresponding to the followers of MAS (1) is structurally balanced by dividing $\mathcal{V}$ into $\mathcal{V}_1 = \{1, 2\}$ and $\mathcal{V}_2 = \{3, 4, 5\}$. The adjacency matrix $\mathcal{C}$ and Laplacian matrix $\mathcal{L}$ are given as follows:

$$\mathcal{C} = \begin{pmatrix} 0 & 1 & -1.5 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ -1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 \end{pmatrix},$$

$$\mathcal{L} = \begin{pmatrix} 2.5 & -1 & 1.5 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 \\ 1.5 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 2 \end{pmatrix},$$

with $K = \mathrm{diag}\{2, 0, 0, 0, 0\}$. Matrices $\mathcal{L}_K$ and $\mathcal{L}_M$ can be calculated as

$$\mathcal{L}_K = \begin{pmatrix} 4.5 & -1 & 1.5 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 \\ 1.5 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 2 \end{pmatrix},$$

$$\mathcal{L}_M = \begin{pmatrix} 4.5 & -1 & -1.5 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 \\ -1.5 & 0 & 1.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 & 2 \end{pmatrix},$$

with $M = \mathrm{diag}(1, 1, -1, -1, -1)$.

Example 1 Suppose that the impulsive time sequences $T^{\star}$ of agents 0–2 are $0.2k$ and those of agents 3–5 are $0.1k$, where $k = 1, 2, \cdots$. Let $\mathcal{T} = 0.02$. We obtain $\mu = 0.3 < \frac{2}{\lambda_{\max}(\mathcal{L}_M)} = 0.3703$, $\eta = 0.92$, $\omega = 3.6186$, and $\frac{\ln \eta}{\mathcal{T}} + \omega = -0.5505 < 0$, and matrix inequality (11) holds if $Q = \begin{pmatrix} 1.4028 & 0 \\ 0 & 1.4028 \end{pmatrix}$.

For each follower, we select $d_1 = 0$ under positive impulsive effects and set $t = 0$, $x_0 = [3, 5]^{\mathrm{T}}$, $x_1 = [-1, 3]^{\mathrm{T}}$, $x_2 = [4, 6]^{\mathrm{T}}$, $x_3 = [0, 8]^{\mathrm{T}}$, $x_4 = [-3, -5]^{\mathrm{T}}$, $x_5 = [2, -5]^{\mathrm{T}}$. The states of the followers and the leader with positive impulsive effects are shown in Fig. 2. The trajectories of tracking errors (Eq. (10)) are shown in Fig. 3.

Example 2 Suppose that the impulsive time sequences $T^{\star}$ of agents 0–2 are $0.4k$ and those of agents 3–5 are $0.8k$, where $k = 1, 2, \cdots$. Let $\mathcal{T} = 0.4$. By calculating, we have $\eta_2 = 1.2 > 1$, $\omega_2 > 0.458$, and $\omega_2 - \frac{\ln \eta}{\mathcal{T}} > 0$, and matrix inequality (21) holds if $P = \begin{pmatrix} 0.5732 & -0.1251 \\ -0.1251 & 0.5732 \end{pmatrix}$. For each follower by selecting $d_1 = 6.8 > \frac{1}{\lambda_{\min}(\mathcal{L}_M)} = 6.7159$ under negative impulsive effects and setting $t = 0$, $x_0 = [3,5]^{\mathrm{T}}$, $x_1 = [-1,3]^{\mathrm{T}}$, $x_2 = [4,6]^{\mathrm{T}}$, $x_3 = [0,8]^{\mathrm{T}}$, $x_4 = [-3,-5]^{\mathrm{T}}$, $x_5 = [2,-5]^{\mathrm{T}}$, the states of the followers and the leader with negative impulsive effects are shown in Fig. 4. The trajectories of tracking errors (Eq. (10)) are shown in Fig. 5. Thus, the feedback control is contrary to the impulsive permutation, and the bipartite tracking consensus of MAS (1) can be achieved.

If we set $d_1 = 0$, it can be found that bipartite tracking consensus of MAS (1) cannot be achieved under negative impulsive effects. This implies that the term $d_1 H\Big(\sum_{q=1}^{N} |c_{pq}|(x_p(t) - \mathrm{sign}(c_{pq})x_q(t)) + c_{p0}(x_p(t) - m_p x_0(t))\Big)$ in Eq. (3) could work effectively to eliminate the adverse effects created by the negative impulsive effects.

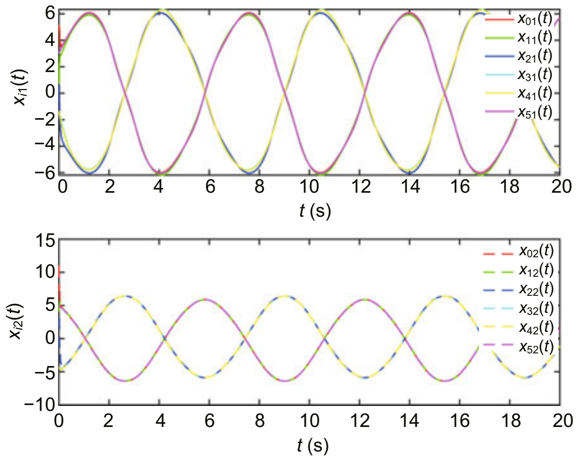

Fig. 2 States of $x_p(t)$ for $p = 0, 1, \ldots, 5$ under the positive impulsive effects in Example 1

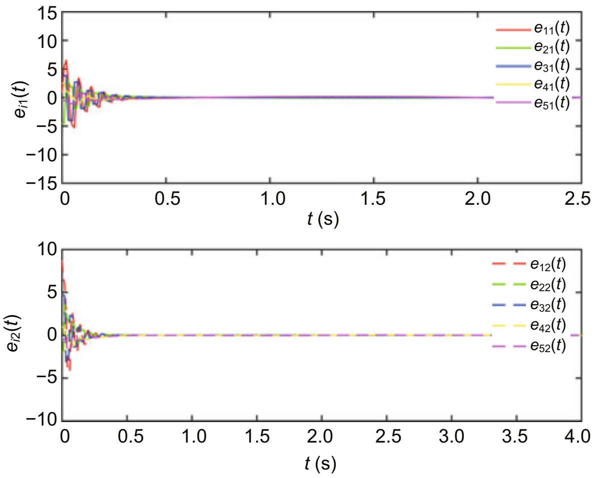

Fig. 3 Tracking errors $e_p(t)$ for $p = 1, 2, \ldots, 5$ under the positive impulsive effects in Example 1

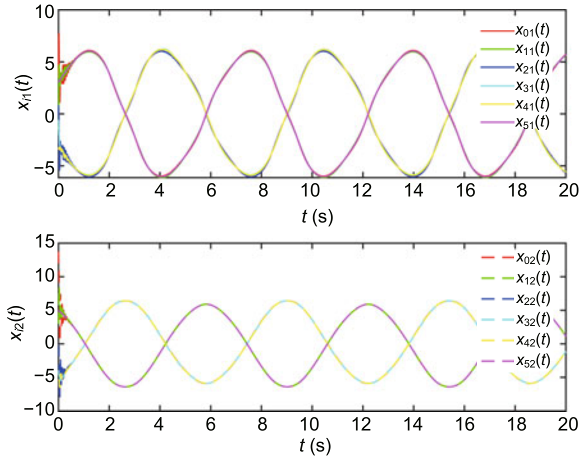

Fig. 4 States of $x_p(t)$ for $p = 0, 1, \ldots, 5$ under the negative impulsive effects in Example 2

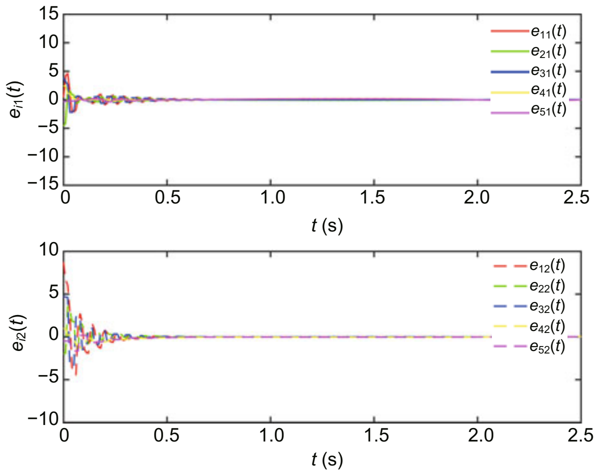

Fig. 5 Tracking errors $e_p(t)$ for $p = 1, 2, \ldots, 5$ under the negative impulsive effects in Example 2

5 Conclusions

We studied bipartite tracking consensus in linear MASs via a distributed asynchronous impulsive protocol. We considered the situation where the impulse instants of different agents are independent. Using the relative information from a neighboring agent, a distributed impulsive control protocol with positive or negative impulsive effects was designed. We showed that bipartite tracking consensus is achieved if the underlying communication topology has a balanced structure and a spanning tree. Furthermore, we determined a way to design asynchronous impulsive coupling strength and presented a basic guideline for designing a maximum asynchronous impulsive interval. Solving bipartite tracking consensus for more general systems such as switching communication topologies and doubleintegrator systems with impulsive permutation will be considered in our future studies.

Contributors

Lingzhong ZHANG and Yuanyuan LI designed the research. Lingzhong ZHANG, Jungang LOU, and Jianquan LU processed the data. Lingzhong ZHANG, Yuanyuan LI, and Jungang LOU drafted the paper. Jianquan LU helped organize the paper. Lingzhong ZHANG and Yuanyuan LI revised and finalized the paper.

Compliance with ethics guidelines

Lingzhong ZHANG, Yuanyuan LI, Jungang LOU, and Jianquan LU declare that they have no conflict of interest.

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