





































We define an equivalent function  $Y(p) = \max_{\mathbf{p}} G(\mathbf{p}) - \gamma P(\mathbf{p})$  with *Theorem 3*. We define two solutions as  $y_1$  and  $y_2$ , and ascertain the function values for them to be  $Y(y_1)$  and  $Y(y_2)$ . Then we define the derivative of function  $\frac{\partial Y}{\partial p}$  as  $G'$ , and there are thus two situations in terms of two variables. If  $y_1 \geq y_2$ ,  $y_1 - y_2 \geq 0$ , we just focus on whether the  $Y'$  is positive or negative. This is according to the expression of  $Y'$  as

$$Y' = R' - \gamma^*, \quad (\text{A1})$$

where  $\gamma^*$  is the assumed optimal solution. According to the proof in Appendix B, we can see that if  $y_1 \geq y_2$ ,  $Y(y_1) \leq Y(y_2)$ , and  $\gamma > y_2 \geq y_1$ , so the derivative of function  $Y'(y_2) \leq 0$  because the minimum value has not been achieved. Thus, the first order condition of quasi-convex function expression

$$\nabla Y(x) * (y - x) \leq 0, \quad (\text{A2})$$

holds where  $x = y_1, y = y_2$ . Similarly, if  $y_1 < y_2$ , similar proofs can be obtained but are not mentioned due to space constraints for the paper.

## Appendix B: Proof of problem equivalence

We prove the problem equivalence with two steps. First, we prove the sufficient condition and

define the objective as  $\gamma^* = \frac{G(\mathbf{p}^*)}{P(\mathbf{p}^*)}$ , where  $\mathbf{p}^*$  is the optimal power allocation policy. Then, it is clearly evident that

$$\gamma^* = \frac{G(\mathbf{p}^*)}{P(\mathbf{p}^*)} \geq \frac{G(\mathbf{p})}{P(\mathbf{p})}, \quad (\text{B1})$$

and based on this, we can drive the following formulas:

$$G(\mathbf{p}) - \gamma^* P(\mathbf{p}) \leq 0, \quad (\text{B2})$$

$$G(\mathbf{p}^*) - \gamma^* P(\mathbf{p}^*) = 0. \quad (\text{B3})$$

Therefore,  $\max_{\mathbf{p}} G(\mathbf{p}) - \gamma^* P(\mathbf{p}) = 0$ , and the sufficient condition is proved.

Secondly, the necessary condition should be proved. Let us suppose that  $\tilde{\mathbf{p}}$  is the optimal policy and  $G(\tilde{\mathbf{p}}) - \gamma^* P(\tilde{\mathbf{p}}) = 0$  is established. We thus have

$$G(\mathbf{p}) - \gamma^* P(\mathbf{p}) \leq G(\tilde{\mathbf{p}}) - \gamma^* P(\tilde{\mathbf{p}}) = 0. \quad (\text{B4})$$

The above inequality can be derived as

$$\frac{G(\mathbf{p})}{P(\mathbf{p})} \leq \gamma^* \quad \text{and} \quad \frac{G(\tilde{\mathbf{p}})}{P(\tilde{\mathbf{p}})} = \gamma^*. \quad (\text{B5})$$

Therefore, *Theorem 2* is proved.