

## MODELING AND ROBUST LQ REGULATOR DESIGNING FOR REFINING PROCESS\*

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**Abstract:** This paper presents a synthetic design procedure for a robust LQ regulator for refining process, including modeling and robust optimal system designing. The paper discusses three major topics: mathematical modeling of the process with large uncertainty, determination of a synthetic performance index for optimizing the process, and design of the robust optimal system with robust guaranteed stability. This research result is a part of preliminary results of the real refining process optimal control system implemented in the Minfeng Paper Mill, Zhejiang Province. Simulation test results showed that the proposed modeling and control algorithm are efficient and practicable.

**Key words:** Refining process, robust control, optimal control

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### INTRODUCTION

An important part in the pulping and paper making-industry is the refining process. Whether the refined pulp is good or not is crucial for achieving consistent pulp quality. What people expect and strive for in the refining process is guaranteed refined pulp quality, safety in production, and minimal energy consumption. Refined pulp quality and energy consumption are key factors in the refining process. Experience showed that a stable refining process results in stable refined-pulp quality (Pi et al., 1993). The existence of large uncertainties and the interacting effects of various kinds of factors in the refining process make it very difficult to develop an accurate refining process model, where robust control can be quite useful. On the other hand, mechanism analysis of refining processes by Specific Edge Load (SEL) theory (Leider et al., 1977) and Specific Source Load (SSL) theory (Lumianinen et al., 1990) indicates that the quality indexes are relevant to the state and input variables respectively and also to the cross-product of these variables. For example, the mass flow of pulp (dry) through the refiner ( $M$ ) should be kept stable to guarantee the refined pulp quality relates directly to the product of outlet pulp con-

sistency and inlet pulp flow. Moreover, the reactive power of disc refiners is composed of basic power consumption (constant) and power for heating pulp ( $P_h$ ) which relates to the product of the temperature differential and the inlet pulp flow. Thus, to achieve efficient control, we should consider the interacting effects of the state and input variables in such a specific process. However, there is no mention in relevant literature, of a general procedure for building a robust modeling-based control system to deal with both the uncertainties in the refining process and the special performance requirement. A synthetic design method for use in refining process control seems worthy of attention.

In this respect, standard linear quadratic (LQ) optimal regulators have advantages because they ensure the property of asymptotic stability and considerably large guaranteed stability margins (Safonov et al., 1977 and Lehtomaki et al., 1981), i. e., the guaranteed GM (gain margin) of  $(1/2, \infty)$  and PM (phase margin) of 60 degrees. However, LQ optimal regulators for the performance index with cross-product terms of states and inputs (LQRCPT) do not guarantee the stability margins of the standard LQ optimal regulators. Furthermore, both standard regulators and LQRCPT are not robust to parameter un-

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certainty. In fact, "blindly" designing a LQ regulator for some nominal systems does not guarantee the stability of the closed-loop system, even if the actual system is guaranteed to be open-loop stable (Douglas et al., 1994).

This paper focuses on both parameter uncertainty in the system matrix and the cross-product term in the performance index for LQ optimal regulator. To design a robust LQ optimal regulator for the performance index with cross-product terms of states and inputs (RLQRCPT) with guaranteed stability margins, we developed a modified robust return difference equality for the RLQRCPT problem. Then, sufficient conditions for RLQRCPT are presented. The proposed results are based upon robust algebraic matrix inequality and positive-real condition for some transfer matrices. Moreover, conditions to guarantee RLQRCPT with GM of  $(1/2, \infty)$  and PM of 60 degrees are derived. Our results also showed that the standard robust LQ regulator problem is a special case of the RLQRCPT problem. Finally, a robust modeling-based optimal control system guaranteeing that the refining process is robust and optimal was designed using the

proposed methods.

## PROCESS MODELING

Little was known about the refining process just a few years ago. Although some modern refining theories based on researches on mechanism analysis of refining process have been proposed, there is no uniform theory to explain the process's mechanism clearly. Among existing theories, SEL theory and SSL theory seem to be more effective in practical use. However, most models built by using SEL theory are based on only a single variable, which cannot clearly describe the whole refining process. Due to the existence of large uncertainties and the lack of on-line measuring technique in the refining process, it is very difficult or impossible to develop accurate models of the refining process. Hence, in this work, we tried to design an uncertain multi-variable model explaining the process more suitably for the three serial-connected 450 disc refiners used in the Minfeng Paper Mill, Zhejiang Province. The system is shown in Fig. 1.

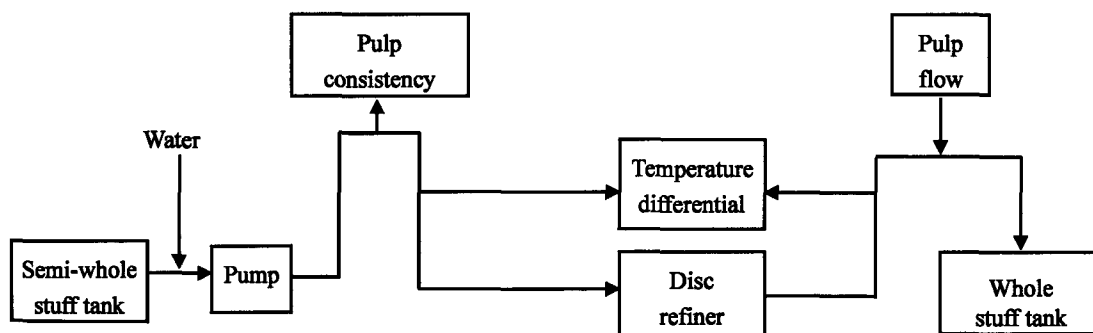


Fig. 1 Refining process control system

Design of an appropriate control system requires an understanding of the dominant dynamic features and coupling between the manipulative inputs and outputs. In the refining process, the pulp fiber is beat physically so that it displays the specific property ready for paper making. Our work focuses on the development of a practical control model, which includes uncertain parameters and reveals the essential dynamics of the refining process. The model is developed by using the basic principle of SEL theory, synthetical analyses and reasonable mechanical assumption for the refining process. Analysis and

in-situ tests indicated that good quality refined pulp can be guaranteed by beating the raw pulp to make the fiber cells distribute as plainly as possible.

Although various factors affect the pulp quality, the degree of beating and the wet weight of the refined-pulp which directly resulted from the refining process are two key quality indexes in the refining process. On-line measurements of these two indexes for low consistency pulp are not available now. Mechanical analysis of the pulping process showed that these two indexes are related directly to inlet pulp flow, pulp con-

sistency, water flow and rotational velocity of the refiner discs. Hence, these measurable indexes are usually used in practical refining processes to evaluate the refined-pulp quality. What is important here is to select proper states and control variables according to the requirements for stabilizing refined-pulp quality and decreasing energy consumption. And also, the selected states and control variables should be applicable to robust optimal control algorithms.

For the three serial-connected  $\phi 450\text{mm}$  disc refiners, a corresponding model including uncertainty is given below:

$$\dot{x}(t) = (A_0 + \Delta A)x(t) + Bu(t) \quad (1)$$

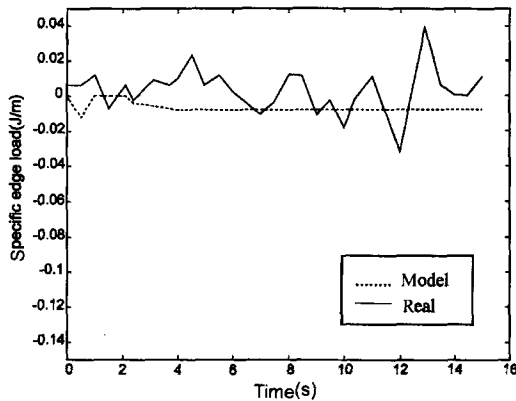
where

$$x = (x_1, x_2, x_3)^T, u = (u_1, u_2, u_3)^T$$

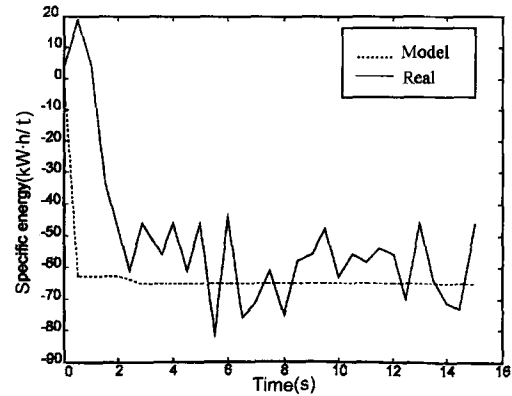
$$A_0 = \begin{pmatrix} -1.25 & 0 & 0 \\ 0 & -1.56 & 2.21 \\ 0 & 0 & -6.20 \end{pmatrix},$$

$$B = \begin{pmatrix} -0.07 & 0.07 & 0 \\ 0 & -1.57 & 0 \\ 0 & 0 & 3.10 \end{pmatrix},$$

$$\Delta A = \begin{pmatrix} k_{11} & k_{12} & 0 \\ 0 & k_{22} & k_{23} \\ 0 & 0 & 0 \end{pmatrix}$$



(a)



(b)

Fig. 2 Specific edge load (a) and specific energy increments (b) as inlet pulp flow varies by 11%.

ty, i.e., to make specific edge load and specific source load stable. In order to reach this objective, stabilizing is necessary. Here

$$M = c_1 x_1 u_2 \quad (2)$$

where  $c_1$  is pulp density (constant). Moreover,

The uncertain parameters  $|k_{11}| \leq 4.5$ ,  $|k_{12}| \leq 3.2$ ,  $|k_{22}| \leq 6.24$ ,  $|k_{23}| \leq 8.84$ ,  $x_1, x_2, x_3$  are outlet pulp consistency (%), temperature differential ( $^{\circ}\text{C}$ ) and rotational velocity (rev/s) of refiner discs, respectively,  $u_1, u_2, u_3$  are inlet water flow rates ( $\text{m}^3/\text{s}$ ), inlet pulp flow rates ( $\text{m}^3/\text{s}$ ), and total power ( $W$ ) of refiner, respectively.

The model accuracy is evaluated by comparing the in-situ measured values to model output values of the specific edge load and specific energy increments of inlet pulp flow, water flow and refining power, for a certain step change, respectively. The relevant results are shown in Fig. 2 as inlet pulp flow varies by about 11%. Other results were reported by Pi and Sun (1993). Detailed comparison and analyses of experiment results showed that the model is approximately consistent with the refining process. The model errors are mainly caused by some uncertainties and parameter variations in the process.

## PERFORMANCE INDEX

The first main objective of refining process control is to guarantee stable refined-pulp quali-

the other main objective is to decrease power consumption for heating pulp ( $P_h$ ). Here

$$P_h = c_2 x_2 u_2 \quad (3)$$

where  $c_2$  is a constant related to pulp consistency and specific heat of pulp. To achieve optimal

control, we should consider the interacting effects between state variables and input variables besides considering the effects of those variables respectively. Therefore, the following quadratic cost functional with cross-product term is a good choice.

$$J = \int_0^\infty (x^T(t)Qx(t) + 2x^T(t)Nu(t) + \rho u^T(t)u(t)) dt \quad (4)$$

where  $Q - 1/\rho NN^T \geq 0, \rho > 0$ . We shall refer to (1) and (4) as robust LQ optimal regulator for performance index with cross-product term (RLQRCPT).

Suppose  $(A_0, B)$  is stabilizable and  $(D, A_0 - 1/\rho BN^T)$  is detectable, where  $D$  is defined as  $DD^T = Q - 1/\rho NN^T$ , then the optimal control for the nominal system  $A_0$  is given by (5) below (Anderson and Moore, 1989).

$$u(t) = Gx(t) = -1/\rho(B^T P + N^T)x(t) \quad (5)$$

where  $P$  is the positive solution of the algebraic Riccati equation (ARE)

$$P(A_0 - 1/\rho BN^T) + (A_0 - 1/\rho BN^T)^T P + (Q - 1/\rho NN^T) - 1/\rho PBB^T P = 0 \quad (6)$$

It is well known that the optimal control  $u(t)$  ensures the asymptotic stability of the nominal closed loop system, but does not guarantee the GM of  $(1/2, \infty)$  and the PM of 60 degrees due to the existence of the cross product terms in the performance index. Furthermore for RLQRCPT with parameter (structured) uncertainty, no general analysis and design methods are available in the literature.

We want to derive a robust optimal controller so that the refining process is robust and has guaranteed stability margins. It seems a difficult thing to do because RLQRCPT is described in time domain, but the stability margins are described in frequency domain. What is possible, however we think, is to get an expression for the return difference function in terms of the RLQRCPT design parameters. This idea will be helpful for guiding us in "robustifying" the RLQRCPT with guaranteed stability margins.

### SUFFICIENT CONDITIONS

**Theorem 1** For the RLQRCPT prob-

lem, if there exists a positive matrix  $P > 0$  which satisfies ARE (6), and the existence and asymptotic stability conditions of the optimal control for the nominal system and the cost functional (4) are satisfied, then the following equality holds

$$(I + G\Phi(-s)B)^T(I + G\Phi(s)B) = I + 1/\rho B^T \Phi^T(-s)(-P\Delta A - \Delta A^T P + Q) \cdot \Phi(s)B + E(s) \quad (7)$$

where

$$\Phi(s) = (sI - A)^{-1}, G = 1/\rho(B^T P + N^T) \\ E(s) = L(s) + L^T(-s) = 1/\rho N^T \Phi(s)B + 1/\rho B^T \Phi^T(-s)N$$

**Proof** For nominal system  $A_0$  and the cost function (4), ARE (6) can be rewritten as

$$-PA_0 - A_0^T P - Q + 1/\rho(PB + N)(PB + N)^T = 0 \quad (8)$$

Let  $A = A_0 + \Delta A$ , to account for the uncertainty, add and subtract  $PA + A^T P$  and  $sIP$ ; rearrange (8) to get

$$P(sI - A) - (sI + A^T)P + P(A - A_0) + (A^T - A_0^T)P - Q + 1/\rho(PB + N)(B^T P + N^T) = 0 \quad (9)$$

Next, postmultiply and premultiply (9) by  $(sI - A)^{-1}$  and  $B^T(-sI - A^T)^{-1}$ , respectively. Then in view of (6), we obtain

$$P(A_0 - A) + (A_0^T - A^T)P + Q = P(sI - A) + (-sI - A^T)P + 1/\rho(PB + N)(PB + N)^T \quad (10)$$

After direct algebraic manipulations of (10), we have (7).

The equation (7) is called the robust modified return difference equality (RMRDE) for RLQRCPT. The significance of Theorem 1 is that it bridges a gap between the robust Riccati equation and frequency domain transfer function. Based upon RMRDE, we can derive the following results.

**Theorem 2** The robust stability margins GM of  $(1/2, \infty)$  and the PM of 60 degrees are

guaranteed for RLQRCPT, if there exists a positive matrix  $P > 0$  such that

$$(I) \quad Q - P\Delta A - \Delta A^T P \geq 0 \quad (11)$$

(II)  $L(s)$  as defined above is positive real, i. e.,  $E(s) = L(s) + L^T(-s) \geq 0$  for all  $s$  with  $R_e(s) \geq 0$ .

where  $Q \geq 0$  is the weighting matrix of cost function (4).

**Proof** If conditions (I) and (II) are satisfied, then it is clear that  $\sigma_i(I + G\Phi(s)B) \geq 1$  for all  $i$ . By the well-known frequency condition of optimum system, we complete the proof of the theorem.

**Remark 1** The conditions (I) and (II) are sufficient for RGS of RLQRCPT as well as of a standard robust LQ optimal regulator (when  $N = 0$ ). It can be used to see if an uncertain LQ optimal system guarantees the stability margins of the standard LQ optimal regulators, and also be used as guidelines to adjust matrices  $Q$  and  $P$  by solving inequality (11) including  $\Delta A$  and to select  $N$  for maintaining the robust guaranteed GM of  $(1/2, \infty)$  and PM of 60 degrees.

**Theorem 3** For the RLQRCPT problem, if there exists a positive matrix  $P > 0$  and some matrix  $F$  such that

$$Q - P\Delta A - \Delta A^T P = F^T F \quad (12)$$

and suppose  $\underline{\sigma}(F^T F) \geq f$ ,  $\underline{\sigma}(Q) = q > 0$ ,  $\bar{\sigma}(N^T N) = n$  satisfy  $\rho q - n \geq 0$ , then the RLQRCPT problem will have robust guaranteed stability with

$$GM = 1/(1 \pm \alpha_0), PM = \pm \cos^{-1}(1 - \alpha_0^2/2) \quad (13)$$

where  $\alpha_0 = \sqrt{1 - n/\rho f}$ ,  $Q \geq 0$  is the weighting matrix of cost function (4).

**Proof** The existence of the matrix  $F$  satisfying (12) is obvious. By the condition of (11) and the supposition, if we define  $T(s) = G\Phi(s)B$ , then it follows from (7) that

$$\begin{aligned} & (I + T(-s))^T (I + T(s)) \\ &= (1 + T(s))^H (I + T(s)) \\ &\geq I + 1/\rho B^T \Phi^T(-s) \underline{\sigma}(F^T F) \Phi(s) B + E(s) \end{aligned} \quad (14)$$

where  $(A)^H$  represents the complex conjugate transpose of matrix  $A$ . After simple algebraic

manipulations, we have

$$\begin{aligned} & (I + T(s))^H (I + T(s)) \geq I + f/\rho (\Phi(s)B \\ &+ N/f)^H (\Phi(s)B + N/f) - N^T N/\rho f \\ &\geq (1 - n/\rho f) I \end{aligned} \quad (15)$$

Therefore

$$\underline{\sigma}(I + T(s)) \geq \sqrt{1 - n/\rho f} = \alpha_0 \quad (16)$$

In view of the resulting Theorems (Lehtomaki et al., 1981), we complete the proof.

**Remark 2** From Theorem 3, we can see how the uncertainty in systems and the weighting matrices in the performance index degrade the robust guaranteed stability. Therefore, the significance of Theorem 3 is that it can be used as a guideline to choose proper parameters so that the designed system possesses the required robust guaranteed stability. Furthermore, when the bound of uncertainty and the weighting matrices are given, Theorem 3 provides us with the certain values of robust guaranteed stability margins.

## LQ REGULATOR DESIGN

We use a Riccati equation approach to construct a robust LQ regulator for parameter uncertain systems. The uncertainty under consideration is assumed to be norm-bound of the form

$$\Delta A = DF(t)E \quad (17)$$

where  $D \in R^{n \times r}$  and  $E \in R^{q \times n}$  are constant matrices of appropriate dimensions, which represent the structure of the uncertainty, and  $F(t) \in R^{r \times q}$  is an unknown matrix function satisfying

$$F^T(t)F(t) \leq I$$

By substituting the Riccati equation (6) for the nominal system into (11), we obtain

$$\begin{aligned} & P \left( A - \frac{1}{\rho} B N^T \right) + \left( A - \frac{1}{\rho} B N^T \right)^T P - \frac{1}{\rho} P B B^T P \\ & - \frac{1}{\rho} N N^T \leq 0 \end{aligned} \quad (18)$$

Substitute into (18) the actual value of the  $A$  matrix  $A = A_0 + \Delta A$  to get

$$P \left( A_0 - \frac{1}{\rho} B N^T \right) + \left( A_0 - \frac{1}{\rho} B N^T \right)^T P$$

$$\begin{aligned}
 &+ PDF(t)E + (DF(t)E)^T P - \frac{1}{\rho} PBB^T P \\
 &- \frac{1}{\rho} NN^T \leq 0 \tag{19}
 \end{aligned}$$

Using the Petersen-Hollot bounds (Petersen et al., 1986), a sufficient condition for (19) is that there exists a constant  $\epsilon > 0$  and a positive matrix  $P > 0$ , such that

$$\begin{aligned}
 PA_0 + A_0^T P - \frac{1}{\rho} (B^T P + N^T)^T (B^T P + N^T) \\
 + \epsilon PDD^T P + \frac{1}{\epsilon} E^T E + Q = 0 \tag{20}
 \end{aligned}$$

Thus, to design a robust LQ regulator, we need to find the positive definite solution  $P$  by solving the modified robust Riccati equation (20) and apply the feedback

$$u = -Kx = -\frac{1}{\rho} (B^T P + N^T) x \tag{21}$$

If we find a solution  $P = P^T > 0$  in the robust Riccati equation (20), we can define a modified state weighting matrix  $\tilde{Q}$  by

$$\begin{aligned}
 \tilde{Q} = -PA_0 - A_0^T P + \frac{1}{\rho} (B^T P + N^T)^T (B^T P \\
 + N^T) = Q + \epsilon PDD^T P + \frac{1}{\epsilon} E^T E \tag{22}
 \end{aligned}$$

Then the RLQRCPT controller is the LQ optimal regulator when we are minimizing the cost functional

$$\begin{aligned}
 \tilde{J} = \int_0^\infty (x(t)^T \tilde{Q} x(t) + 2x(t)^T Nu(t) \\
 + \rho u(t)^T u(t)) dt \tag{23}
 \end{aligned}$$

Thus, the RLQRCPT can be interpreted as an LQR design for the nominal system of (1) with a suitably modified state weighting matrix  $\tilde{Q}$ . Therefore, the same robustness as in LQR designs is guaranteed because it is an LQR design itself.

### SYSTEM DESIGN AND SIMULATION

Suppose what is required is a refining process optimal regulator that efficiently regulates the uncertain system stable in the sense of robust guaranteed stability and, at the same time, minimizes  $P_h$  and adjusts the degree of stability of  $M$ . The cross-product term  $N$  in (4) can be se-

lected as follows

$$N = \begin{pmatrix} 0 & \beta_1 c_1 & 0 \\ 0 & \beta_2 c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1000c_1 & 0 \\ 0 & 159.07c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where  $c_1$  and  $c_2$  are two weighting coefficients.  $\rho$  of (4) is chosen to be  $10^5$ . In the next step, we choose the matrix  $F$  of (12) that serves our required control objectives: robust LQ optimum with robust guaranteed stability for the refining process with stable refining-pulp quality and least energy consumption.

In the practical refining process control, the stability margins GM of 15 (20 decibels) and PM of 50 degrees are minimum requirements. By (14),  $\alpha_0$  for Theorem 3 should be larger than or equal to 0.935. Since  $n = \bar{\sigma}(N^T N) = 1.0253 \times 10^6$ , the matrix  $F$  of Theorem 3 should be chosen to satisfy  $\underline{\sigma}(F^T F) \geq 81.52$ . Then solving the algebraic matrix equations of (6) and (12), we can obtain the expected results. These results, of course, are relevant to the uncertainties in systems, i. e., the resulting weighting matrix and positive solution  $P$  are functions of the uncertain parameters.

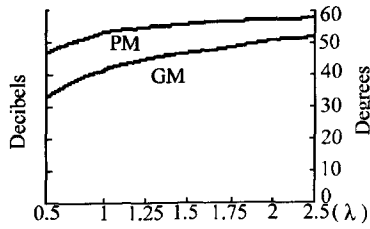
In order to guarantee that the feedback control system is robust optimal, the resulting  $Q$  must satisfy the condition  $Q - NN^T/P \geq 0$  (Anderson and Moore, 1989). Using this condition, we can obtain a robust bound of the weighting matrix  $Q$  for RLQRCPT. Let

$$Q - NN^T/\rho \geq \lambda I \tag{24}$$

for some constant  $\lambda > 0$ . Eq. (24) is satisfied if

$$Q \geq (\lambda + \bar{\sigma}(NN^T)/\rho)I = (\lambda + n/\rho)I \tag{25}$$

Thus Eq. (25) is the lower bound of  $Q$  for the uncertainties in the system. The condition of (25) can be used as guideline for designing RLQRCPT with robust guaranteed stability. Now the robust stability margins corresponding to uncertainty parameters  $k_{11}, k_{12}, k_{22}$  and  $k_{23}$  can be computed easily with the above mentioned approach and the condition of (25). For example, when  $k_{11} = 0, k_{12} = 0, k_{22} = -3.79, k_{23} = -0.43$ , the guaranteed stability margins which make the refining process control system robust optimal, corresponding to each  $\lambda$  are shown in Fig. 3.

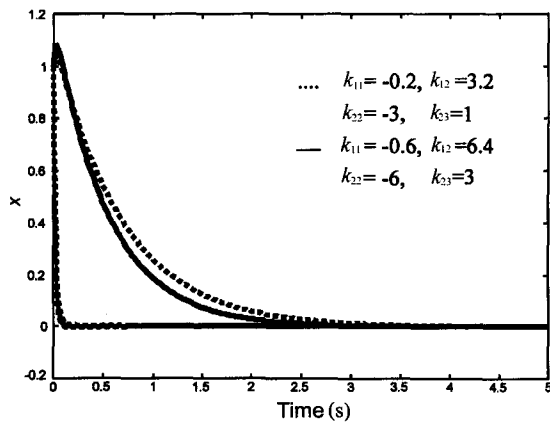


**Fig. 3 GM and PM of the robust optimal system**  
when  $k_{11} = 0, k_{12} = 0, k_{22} = -3.79, k_{23} = -0.43$

To design the PLQRCPT regulator, we consider a weighting matrix  $Q = \text{diag}(10^3, 10^3, 10^3)$  satisfying (25). The resulting regulator gains are:

$$G = \begin{pmatrix} 0.3495 & 0.0152 & 0.0007 \\ -0.1081 & 24.7686 & 0.6194 \\ -0.0304 & -1.2243 & -13.9809 \end{pmatrix}$$

As basis for comparison based on the same



**Fig. 4 Responses of robust LQ optimal system**

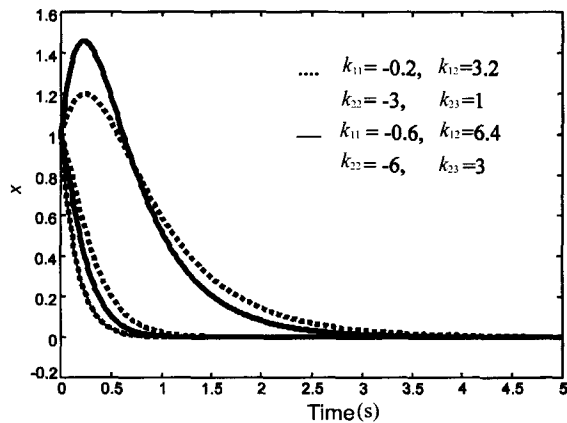
widely depending on the actual value of the uncertain parameters. It is shown that the obtained system is by no means guaranteed optimal although the system still remains stable. This indicates the unrobust performance of the system. However, the robust LQ optimal regulator yields similar transient responses for all values of the permissible uncertainties in the refining process. It is apparent from Fig. 4 that we achieved a level of performance robustness with the robust LQ optimal system. Additionally, the errors of the robust LQ optimal control system are less than those of the standard LQ optimal control system but the norm of the gains of the robust LQ optimal control system is higher.

conditions as above, we design a standard LQ optimal regulator for the nominal system characterized by the midpoint values of the parameters  $k_{11} = 1.75, k_{12} = 1.25, k_{22} = -3.12, k_{23} = 4.42$  and apply the control to the system with different values of the uncertainties. The resulting LQ optimal regulator gains are

$$G_0 = \begin{pmatrix} -0.0025 & -0.0001 & -0.0000 \\ 0.0007 & -0.0493 & -0.0139 \\ 0.0017 & 0.0274 & 0.0122 \end{pmatrix}$$

When the initial value of the inlet pulp consistency is about 3.8% and the temperature differential is 9.3(°C), the state transient responses of the robust LQ optimal system and the standard LQ optimal system of refining process are shown in Fig. 4 and Fig. 5 respectively.

Note from Fig. 5 that the transient responses of the mismatched LQ optimal regulator can vary



**Fig. 5 Responses of standard LQ optimal system**

**CONCLUSIONS**

Experiments proved that the given multivariable model of refining process matches the real plants accurately. The robust optimal control of refining process has shown its ability to ensure the system with both robustness and guaranteed stability margins, to guarantee stable refined-pulp, and to decrease energy consumption greatly. For such a refining process with large dynamic uncertainty and high performance requirement, the combination of robust control and optimal control seems to be a new available scheme,

which has great potential value and is worthy to be considered.

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