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APPLICATION OF VIRTUAL LAMINATED ELEMENT IN THE TOPOLOGY OPTIMIZATION OF STRUCTURES

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Abstract: This paper presents the topology optimization design of structures composed of plane stress elements. The authors' proposed method of topology optimization by virtual laminated element is based on the Evolutionary Structural Optimization (ESO) method of linear elasticity, but dose not require formation of as many elements as the conventional ESO method. The presented method has the important feature of reforming the stiffness matrix in generating optimum topology. Calculation results showed that this algorithm is simple and effective and can be applied for topology optimization of structures.

Key words: virtual laminated element, evolutionary structural optimization, topology optimization, layer, segment, block
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INTRODUCTION

Bendsoe and Kikuchi initiated the homogenization method (Bendsoe et al., 1988) for continuous structures. A very important step in the topology optimization is the change from skeletal structure to continuous structure. Accordingly, topology design is changed from defining whether there is any node or pole to whether there is continuous sub area. The homogenization method is carried out by using microstructure cells, and through optimization, to calculate and define a materials density distribution to obtain the best topology structure.

Xie And Steven presented the evolutionary method (Xie et al., 1993) using life evolution philosophy for structural topology optimization, wherein material element with little contribution to the stiffness of the structure is gradually removed. This method is very simple and effective, and had been widely applied for static and dynamic problems (Xie et al., 1996).

All the above methods require division of the structure into a large number of elements. The virtual laminated element method applied in this paper will partially solve this problem. The following calculation cases give satisfactory results with much fewer elements compared with those applied in reference (Xie et al., 1993), so application of the virtual laminated element method for structural topology optimization is recommended.

The virtual laminated element is based on the principles of both single lamina theory and multiple lamina theory. In order to reduce the number of elements, the equivalent numerical integration method is applied to integrate multiply lamina different material into the different lamina of the single element. The reduction of the number of elements will effectively reduce the scale of the problem to be calculated. This improvement will significantly benefit large projects such as those in civil engineering, especially for the structural topology optimization required repeatedly for establishing the stiffness matrix.

VIRTUAL LAMINATED ELEMENT

The effectiveness of the virtual laminated element applied in the engineering field is discussed in references (Wu, 1997; Ling et al., 1998). The basic method is to apply the concept of virtual lamina and virtual segment. The virtual lamina and virtual segment have no physical material. In order to divide the element easily and increase the efficiency of computation, some of the material characteristics of the assumed lamina and segment, for example, elasticity tensor E, G, etc. are defined to zero, i.e. the lamina and segment are empty. Normal elements can then be obtained by placing the empty lamina or empty segment between the lamina. On one hand, one element can have different material in different lamina, and a lamina can be made of several materials or nothing. On the other hand, application of different lamina and different segment inside the element can be used to treat structures with complicated geometric figure. This method can go around problems created by local node connection of the element.

This article describes the application of the plane stress virtual laminated element, which is the same as the plane iso-parameter element in defining geometric and offset interpolation and in calculating strain. In calculating the element stiffness matrix, element mass matrix and stress, different lamina integration are required for different dimension and material characteristics on different lamina.

1. Description of the element geometric shape

Fig. 1 shows the 2D virtual laminated element. The boundary of the mother element is defined as $\xi, \eta = \pm 1$. The number of lamina n and number of segment m will be defined for $\eta = -1$ and $\eta = +1$, $\xi = -1$ and $\xi = +1$ according to the actual requirement.

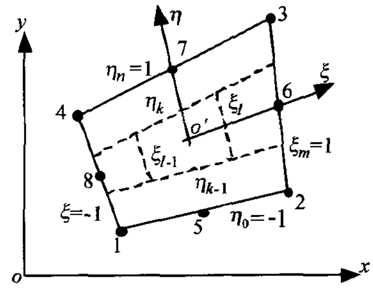

Fig. 1 2D virtual laminated element

The correlationship between the mother element and this virtual element is:

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \sum_{i=1}^{\infty} N_i(\xi, \eta) \begin{Bmatrix} x_i \\ y_i \end{Bmatrix} \tag{1}$$

2. Element stiffness matrix

Since the material modulus matrix $\boldsymbol{D}_{kl}$ in different lamina and segment is often different from each other, integration in different lamina and segment will be required in calculating the element matrix. such as the one below:

$$k_{ij} = \sum_{k=1}^{n} \int_{\eta_{k-1}}^{\eta_k} \left(\sum_{l=1}^{m} \int_{\xi_{l-1}}^{\xi_l} \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{D}_{kl} \boldsymbol{B}_j \mid \boldsymbol{J} \mid w_{kl}(\xi, \eta) \mathrm{d}\xi \right) \mathrm{d}\eta \tag{2}$$

In general, different lamina can have different number of segments. k and l in the formula represent the sequence number of lamina and segment, $w_{kl}(\xi, \eta)$ is the thickness of the element at the local coordinate (ξ, η).

In order to apply Gauss integration to every lamina and segment, the Eq. (2) was linearly transformed to:

$$\xi = \frac{\xi_l - \xi_{l-1}}{2} \xi' + \frac{\xi_l + \xi_{l-1}}{2}$$

$$\eta = \frac{\eta_k - \eta_{k-1}}{2} \eta' + \frac{\eta_k + \eta_{k-1}}{2}$$

The element matrix (2) will then be:

$$k_{ij} = \sum_{k=1}^{n} \int_{-1}^{1} \left(\sum_{l=1}^{m} \int_{-1}^{l} \frac{\xi_l - \xi_{l-1}}{2} \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{D}_{kl} \boldsymbol{B}_j \mid \boldsymbol{J} \mid \cdot w_{kl}(\xi', \eta') \mathrm{d}\xi' \right) \frac{\eta_k - \eta_{k-1}}{2} \mathrm{d}\eta' \tag{3}$$

Every item in Eq. (3) can be obtained by Gauss integration. In the following calculation examples, the plane stress virtual laminated element has been divided into 2 laminas and 2 segments.

EVOLUTIONARY METHOD FOR THE TOPOLOGY OPTIMIZATION

The stress distribution of a structure can be obtained after applying finite element analyses, which, however, cannot be efficiently applied to some material. Based on full stress theory, rejection can be applied according to certain stress criterion. Material block with little stress will be gradually rejected, if the element satisfies the Eq. (4). The rejection of the block can be repeated using the same RR_i until there is no more rejectable block existing in the current step. Afterwards, evolution ratio ER is added to the rejection ratio RR_i. This starts again a new round of rejection until a proper topology structure is obtained. This is the main philosophy of the evolution method.

Since the process of optimization is dependent on the stress distribution characteristics for a specific case, the Eq. (4) does not use the maximum stress $\bar{\sigma}$ of all material blocks but uses the average stress of all material blocks. The formula is given below:

$$|\sigma^e| \leqslant RR_i \times |\bar{\sigma}| \tag{4}$$

$$RR_i = RR_{i-1} + ER \tag{5}$$

In the above formula, σ^e is the block stress, $\bar{\sigma}$ is the average stress of all material blocks, RR_i is the rejection ratio and ER is the evolution ratio.

$\bar{\sigma}$ shall apply different stress criterion with different characteristics of the material. In this article von Mises stress criterion is applied in case 1 and case 2; while the first main stress criterion is applied in case 3.

CASE STUDY OF OPTIMIZATION

1. Case 1

Fig. 2 shows a short cantilever. A vertical force P acts on the center of the free end. The design domain is 16 × 10 cm, the number of elements divided is 32 × 20 (Evolution method, number of elements 64 × 40, Xie et al., 1993). The Young's modulus E = 100 GPa, Poisson's ratio $v = 0.3$. Fig. 3 shows the results of optimization that is the same as those given in Suzuki et al. (1991) applying homogenization method.

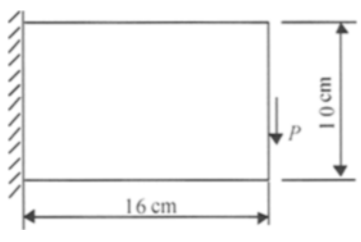

Fig. 2 Design domain for the short cantilever

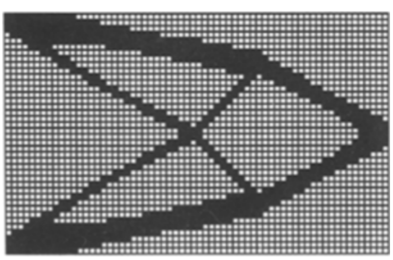

Fig. 3 Optimum configuration for the short cantilever

2. Case 2

Fig. 4 shows a design domain 30 × 15 cm. A vertical force P acts on the center of the bottom boundary. The two corners at the bottom are assumed fixed. The number of elements is 40 × 20, (50 × 25 for Evolution method, Xie et al., 1993). The characteristics of the material are the same as those in case 1. The results of the reference (Xie et al., 1993) have four bars under the arch. Our result shown in Fig. 5 is closer to the analytical solution of the Michell structure (Hemp, 1973) shown in Fig. 6.

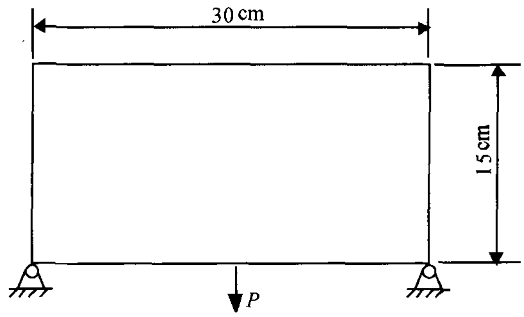

Fig. 4 Design domain for the Michell type structure

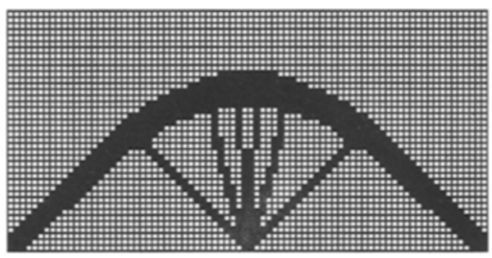

Fig. 5 Optimum configuration for the Michell type structure

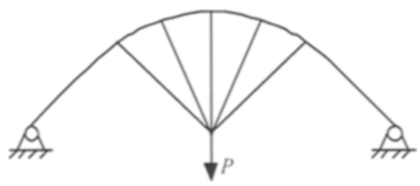

Fig. 6 A Michell type structure

3. Case 3

Fig. 7 shows a cable-stayed bridge model. The dark area is not design domain and the rest is the design domain. E = 210 GPa, $v = 0.3$, $\rho = 7.8 \times 10^{-3}$ kg/cm^3. Element dimension is 25

× 25 cm and has a total of 924 elements (3696 elements Guan et al., 1999). Two cases were studied for the topology optimization. Fig. 8 is the result of the structure under only its gravity load. Fig. 9 shows the result of applying a uniformly distributed load of 1.0 N/cm. Both cases give the same results as those in reference (Guan et al, 1999).

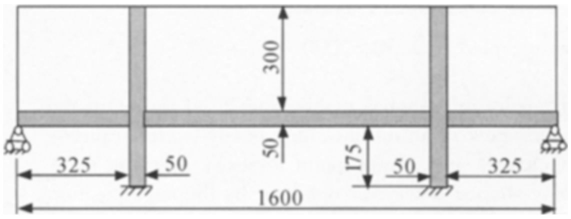

Fig. 7 Cable-stayed bridge model(m)

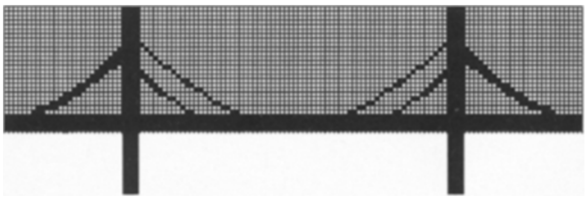

Fig. 8 Under gravity load

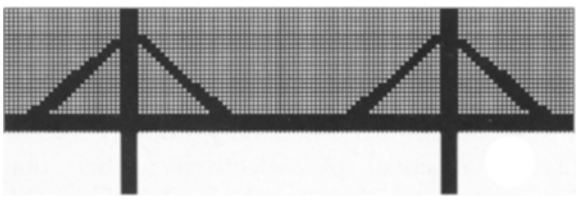

Fig. 9 Under uniformly distributed load

CONCLUSIONS

The above cases studied led to the following conclusions:

1. Plane stress was applied in all the above cases. Apparently it can be extended to general two-dimensional and three-dimensional problems.

2. Generally speaking, every rejection ratio can reach a steady state after 5 to 10 times iteration. At the beginning, the rejection ratio should not be too large so we selected 6%, the evolution ratio should also not be too large. A large rejection ratio and evolution ratio cannot produce a satisfactory topology structure. However, if we select too small a ratio, it will take too much computing time. According to our experience, in the beginning we can apply a larger evolution ratio and reduce the evolution ratio after certain iteration. In this article we used 2% to 6%.

3. Evolution method can provide a proper optimization scope and structure shape in the initial stage of engineering. It is limited to the establishment of topology structure. For the final engineering details, such as dealing with the accurate location and dimension of the bar in the topology structure, a strict shaping optimization method can be applied. However, the shaping optimization method cannot define the topology structure. Therefore, a complete structural optimization problem can be solved by combining both methods.

4. This is the first time that the virtual laminated element is applied in the structural topology optimization. This method is very effective for solving topology optimization design problems specially in areas such as civil engineering, where many elements are applied and needed for establishing the stiffness matrix repeatedly.

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