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PROPERTIES OF VORTEX SYSTEM WITH RANDOM COLUMNAR DEFECTS AT ZERO TEMPERATURE*

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Abstract: The properties of a two-dimensional vortex system with random columnar defects were studied by cooling the vortex system to zero temperature. The vortex lattice became more and more disordered with increasing pinning strength f_p. At small f_p, a triangular vortex lattice away from the pins was observed. The peak of structure factor $S(G_0)$ of the vortex lattice decreased with pinning strength f_p, which accorded with the finding that the probability of vortex to be pinned increased with f_p. Some of our results agreed with experimental findings.

Key words: vortex lattice（VL），columnar defects，structure factor
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INTRODUCTION

Type II superconductors in a sufficiently strong magnetic field change to a mixed state and a vortex lattice（VL）forms. The influence of quenched random pinnings on the vortex lattice is a subject of longstanding interest. In application of high-Tc superconductors in strong magnetic fields, an effective VL pinning mechanism is essential in order to minimize the resistive losses through Lorentz-force induced vortex motion. Numerous experiments and simulations had been conducted to study vortex system in motion under an applied force produced by a current（Henderson et al., 1996; Higgins et al., 1996; Faleski et al., 1996）. Random pins lead to pinning of the VL and thereby produce a highly non-linear dynamic response of the system with current-voltage characteristic exhibiting a critical current density below which dissipation is strongly reduced. Specifically, columnar defects, i.e., linear damage tracks in the material usually produced by heavy-ion irradiation, had emerged as very effective pinning centers（Blatter et al., 1994）. For such systems, a continuous vortex localization transition at T_{BG} from an entangled flux liquid to a disorder-dominated Bose glass phase was predicted（Nelson et al., 1992）, and subsequently found in experiment（Blatter et al., 1994）. It is therefore important to describe the physical properties of a vortex system at low temperature, i.e., $T < T_{BG}$, especially the ground state at zero temperature（Wengel et al., 1997）. At low temperature, $T << T_{BG}$, thermal wandering of vortices can be neglected and the vortices will essentially be straight. Therefore, the problem can be mapped to a two-dimensional（2D）problem of interacting vortices and pinnings.

Many methods were developed to investigate the structure of VL in superconductors with columnar defects at low temperature. Behler et al.（1994）used scanning tunneling microscopy（STM）to image vortex pinning in an $NbSe_2$ crystal at 3 K. Dai et al.（1994）revealed pinning of vortices by columnar defects in $Bi_2Sr_2CaCu_2O_8$ at 4.2 K by using magnetic decoration method. It was found that the VL had remnant correlation despite the random distribution of pinning sites. At vortex density $B > B_\phi$, where B_ϕ was the field at which the densities of vortices and defects were equal, a triangular VL

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away from the defects was observed. While at $B = B_\phi$, only a small portion of vortices were pinned and the VL looked much more ordered than that of pins.

In this work, we numerically studied the ground state of a 2D vortex system interacting with random pins. The influence of pinning strength VL structure was investigated. Some of our results agreed well with the experimental findings of Behler et al. (1994) and Dai et al. (1994).

MODEL

Simulations were carried out for a 2D vortex system by numerically integrating the overdamped equation of vortex motion (Reichhardt et al., 1998),

$$\eta \frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} = \sum_{j \neq i}^{N_v} \boldsymbol{F}_{vv}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \sum_{k=1}^{N_p} \boldsymbol{F}_{pv}(\boldsymbol{r}_i - \boldsymbol{R}_k) + \boldsymbol{F}_i^{\mathrm{T}} \qquad (1)$$

Here η is the viscosity coefficient, $\{\boldsymbol{R}_k\}$ specifies the N_p pinning center positions, $\boldsymbol{r}_i$ denotes the location of the ith vortex. The parameter η is fixed to unity. The force between vortices is given by

$$\boldsymbol{F}_{vv}(\boldsymbol{r}_i - \boldsymbol{r}_j) = f_0 K_1(|\boldsymbol{r}_i - \boldsymbol{r}_j|/\lambda)\hat{\boldsymbol{r}}_{ij} \qquad (2)$$

where $K_1(r/\lambda)$ is a modified Bessel function, λ is the penetration depth, and $\hat{\boldsymbol{r}}_{ij} = (\boldsymbol{r}_i - \boldsymbol{r}_j)/|\boldsymbol{r}_i - \boldsymbol{r}_j|$. A cutoff is placed on $K_1(r/\lambda)$ after it reaches an extremely small value at $r = 6\lambda$. The pinning force is taken as

$$\boldsymbol{F}_{pv}(\boldsymbol{r}_i - \boldsymbol{R}_k) = (f_p/r_p)|\boldsymbol{r}_i - \boldsymbol{R}_k| \\ \Theta((r_p - |\boldsymbol{r}_i - \boldsymbol{R}_k|)/\lambda)\hat{\boldsymbol{r}}_{ij} \qquad (3)$$

Here, Θ is the step function, f_p is called pinning strength or the maximum pinning force, r_p is the pinning force range which is fixed as $r_p = 0.3\lambda$, and. $\hat{\boldsymbol{r}}_{ki} = (\boldsymbol{R}_k - \boldsymbol{r}_i)/|\boldsymbol{R}_k - \boldsymbol{r}_i|$. All lengths, fields, and forces are given in units of λ, Φ_0/λ^2, and $f_0 = \Phi_0^2/8\pi^2\lambda^3$, respectively. Here, Φ_0 is the flux quantum.

Thermal noise is modeled as a stochastic term with properties

$$< \boldsymbol{F}_i^{\mathrm{T}}(t) > = 0 \qquad (4)$$

and

$$< \boldsymbol{F}_i^{\mathrm{T}}(t) \cdot \boldsymbol{F}_j^{\mathrm{T}}(t') > = 2\eta k_B T \delta_{ij} \delta(t - t') \qquad (5)$$

To find the vortex ground state, we gradually cooled a fixed number of randomly moving vortices from a high temperature $T_0 = 1$ to $T = 0$. Note that according to the two-fluid model, the penetration length diverges at the zero-field transition temperature Tc,

$$\lambda(T) = \lambda_0[1 - (T/T_c)^4]^{-1/2} \qquad (6)$$

But when $T << T_c$, the variation of λ is very small, e.g., for $T/T_c \approx 0.93$, one already has $\lambda(T)/\lambda_0 = 2$. Therefore, in this paper λ is fixed.

Simulations were performed on a system of size $L^2 = 20 \times 20$ with periodic boundary conditions. Each simulation was started with a different random initial configuration of the vortex positions and pinning center positions. We slowly decreased the temperature with constant ratio 0.9, and after it was less than 10^{-6} we then set $T = 0$. For each value of $T > 0$, 50 000 MD steps were used. At $T = 0$, 500 000 MD steps were used before data were obtained. Each MD step ensured every vortex attempted to occupy a new site on average. We typically took an average over 10 different configurations.

RESULTS AND SIMULATIONS

In Fig. 1 we show a series of vortex ground states after cooling from our simulation for system of $N_v = 100$ and $N_p = 50$, i.e., $B/B_\phi = 2$. In (a), $f_p = 0.1$, we find that the vortex arrangement is very ordered; and that an almost triangular VL is formed. That is consistent with the calculated vortex structure factor $S(k)$, which has a very sharp peak (see Fig. 2), and also in good agreement with experimental findings (Behler et al., 1994). Moreover, only a very small portion of the vortex is trapped by pins. With the increase of pinning strength f_p, the VL becomes more and more disordered. However, fluctuations in the spacing between pins are always much stronger than that for vortices. We find that if the spacing between two pins is much less than the average spacing between vortices, one of these two pins will not be occupied even for the case of a large f_p. This phenomenon was in fact found in experiments (Behler et al., 1994; Troyanovski et al., 1999).

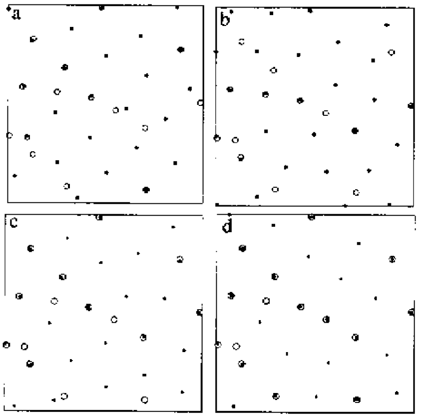

Fig. 1 Vortex ground states obtained from simulated cooling for different pinning strength: f_p = 0.1 in (a); 0.2 (b); 0.4 (c); 0.8 (d). Shown here is a 10 × 10 subset. Solid circles and open circles represent the vortices and pinning sites, respectively.

For the dynamical approach method, the ground state is that each vortex reaches its stable site where the total force, including vortex-vortex interactions and pinning force, is zero. The ground state of VL is governed by the competition between vortex-vortex interactions, which favors the formation of an ordered hexagonal lattice, and pinning by a random distribution of columnar defects, which favor a disordered VL. At small f_p, the vortex-vortex interaction is relatively strong, then the vortex system tends to be an ordered VL. At large f_p, the ground state of VL is governed mainly by pinnings, then a disordered VL forms.

Of course, to find the correct ground state requires infinitely long time. Therefore, the ground states found with dynamical method are "pseudo"-ground states. However, the above results showed that these "pseudo"-ground states accorded with experiment findings. In fact, the "true" ground state may be very difficult to reach for the corresponding real system as well. These "pseudo"-ground states may at least, at low temperature, satisfactorily describe the static properties of "real" ground states.

To learn more about the structure of the vortex ground state, we have calculated the structure factor of the vortex system,

$$S(\boldsymbol{k}) = \frac{1}{N_v}\left|\sum_{i=1}^{N_v}\exp(i\boldsymbol{k}\cdot\boldsymbol{r}_i)\right|^2 \qquad (7)$$

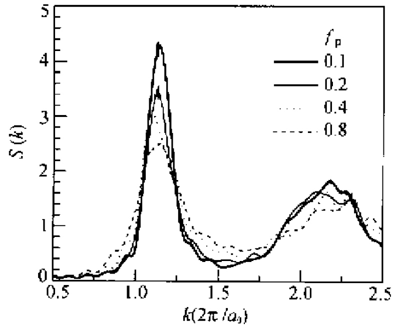

Fig. 2 Plots of the structure factor $S(k)$ calculated for different pinning strengths versus k (in unit $2\pi/a_0$) for system with $N_v = 100$ and $N_p = 50$, where a_0 is the mean vortex spacing.

The structure factor $S(\boldsymbol{k})$ averaged over angles and ten different random pinning arrays are shown in Fig. 2 for some systems. Parameter a_0 is the average vortex spacing, if the VL is assumed to be triangular. The first evident peak of $S(\boldsymbol{k})$ is near the reciprocal principal vector length $G_0 = 4\pi/\sqrt{3}a_0$ of a triangular lattice of lattice spacing a_0.

With the increase of f_p, the value $S(G_0)$ decreases. Fig. 3 shows the dependence of the value $S(G_0)$ on the pinning strength f_p. $S(G_0)$ can be expressed approximately as

$$S(G_0) = \alpha \cdot f_p^{-\beta} \qquad (8)$$

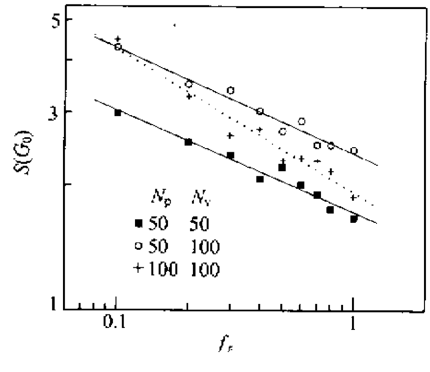

Fig. 3 Log-log plots of the peak of structure factor $S(G_0)$ versus the pinning strength f_p

G_0 is the reciprocal principal vector length, $G_0 = 4\pi/\sqrt{3}a_0$. The solid lines are the linear fit for each system.

for f_p in a range of 0.1 – 1.0. The peak of structure factor $S(G_0)$ is usually greater than 1.0, indicating that the vortex must preferentially occupy only certain pinning sites and that the correlation among vortices remains despite the random pinnings. This was what we found in our simulation as shown in Fig. 1a. We must point out that the power law relation between $S(G_0)$ and f_p will not be valid for large f_p range because the minimum of $S(G_0)$ is 1.0.

Fig. 1 shows that the number of pinned vortices goes up with f_p. In this paper, the probability of a vortex to be pinned, P_{pin}, is also calculated. It is clear that P_{pin} increases with f_p (see Fig. 4). Due to the random distribution of pinnings, the VL becomes more and more disordered with increasing f_p, i. e., its peak of structure factor $S(G_0)$ decreases. Especially, for the case $B = B_\phi$, we found P_{pin} was not equal to 1; which means repulsive vortex interactions destroy the Mott insulator phase predicted to occur at $B = B_\phi$ (Nelson et al., 1992). Wengel et al. (1997) found this phenomena using static method. They simulated the ground state of 2D vortex system by minimizing the system energy.

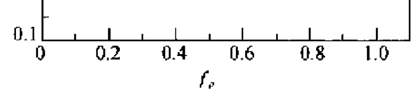

Fig. 4 The probability of vortex to be pinned P_{pin} vs. pinning strength f_p for $N_p = 50$ and $N_p = 100$, when the vortex number is $N_v = 100$.

In conclusion, we performed dynamical simulations on the properties of the ground state of a 2D vortex system with random pinnings by cooling the vortex system to zero temperature. A peak of structure factor $S(k)$ of vortices was found near the reciprocal principal vector length G_0 of a triangular lattice. $S(G_0)$ decreased with increasing pinning strength f_p, indicating that the VL became more and more disordered with increasing f_p. At small f_p, a triangular VL away from the pins was observed. At $B = B_\phi$, only some of the vortices were pinned, indicating repulsive vortex interactions destroyed the Mott insulator phase predicted to occur at $B = B_\phi$. Our numerical study was consistent with some experimental findings. The simulations showed the great influence of pinning strength on the VL at zero temperature. Therefore, we expect it will also affect VL's thermal property, such as heat capacity C, melting temperature T_m, etc.. Further study will be carried out on these problems.

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