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Sliding mode identifier for parameter uncertain nonlinear dynamic systems with nonlinear input*

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Abstract: This paper presents a sliding mode (SM) based identifier to deal with the parameter identification problem for a class of parameter uncertain nonlinear dynamic systems with input nonlinearity. A sliding mode controller (SMC) is used to ensure the global reaching condition of the sliding mode for the nonlinear system; an identifier is designed to identify the uncertain parameter of the nonlinear system. A numerical example is studied to show the feasibility of the SM controller and the asymptotical convergence of the identifier.

Key words: Nonlinear system, Sliding mode, Identifier, Input nonlinearity

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INTRODUCTION

Since publication of Utkin's paper (Utkin, 1977) on variable structure system (VSS) and sliding mode control, significant interest on them has been generated in the control research realm. One of the most intriguing aspects of sliding mode is the discontinuous nature of the control action, with primary function of each of the feedback channels being to switch between two different system structures in such a way that a new type of system motion (called sliding mode) exists in a manifold. In the ideal sliding mode, the stability of the closed system is dependent on a reduced order system that is determined by the nominal system and the choice of sliding surface. The ability to specify performance coupled with inherent robustness has attracted significant interest in sliding modes as offering a possible solution for dealing with the control problem of nonlinear systems.

It is well known that the sliding mode control system has two features: the high-gain feature near the switching surface, and the theoretically infinite switching feature that leads to the sliding mode. Based on the first feature, many kinds of SM control methods (Gao et al., 1993; Lu et al., 1999; Furuta et al., 2000; Hu et al., 2000) have been studied for the case of uncertain system with linear input which is indeed linearizable. However, in practice, due to physical limitations, there do exist nonlinearities in the control input and their effect cannot be ignored in analysis and synthesis. Therefore it is necessary to develop robust control methods to deal with the control problem of the nonlinear dynamic systems with nonlinear input. Hsu (1998) presented a variable structure control approach for uncertain linear system with nonlinear input, then he (1997) extended this method to uncertain linear large-scale systems. On the other hand, the chattering control property of variable structure control makes it possible to establish an SM identifier or SM state estimator (Misawa, 1988; Choi et al., 1999). Because the switching control input reflects the effect of system un-

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certainty, an SM identifier can be established. We know that while a system is in sliding mode, its behavior is completely detemined by the switching surface. Therefore, if there exist some operations such that the information from the switching control input is achievable, then the unknown parameters can be accurately identified. Xu (1993) developed an SM identification method for a class of uncertain affine nonlinear dynamic systems, and then use this method to develop a self-tuning SM controller (Xu, 1998).

In this paper, an SM identification method is considered for a class of parameter uncertain nonlinear system with nonlinear input. The rest of this paper is organized as follows: in Section 2, a general parameter uncertain dynamic nonlinear system with nonlinear input is described; then a SM controller is designed for this system in Section 3; based on this SMC, an identifier design algorithm is proposed in Section 4; finally, a numerical example is given to illustrate the results presented in this paper.

SYSTEM DESCRIPTION

Consider an SISO high-order nonlinear system with nonlinear input described by the following state equation:

$$\dot{x}_i = x_{i+1} \quad (i=1,2,\cdots,n-1) \tag{1}$$

$$\dot{x}_n = f(x,p,t) + b(x,t)\Phi(u) \tag{2}$$

where $x^{\mathrm{T}} = [x_1, x_2, \cdots, x_n] \in R^n$ is the measurable state vector, $u \in R$ is the control input, $b(x,t)$ is a known nonlinear function with $b(x,t) \neq 0$. $p^{\mathrm{T}} = [p_1, p_2, \cdots, p_{n_p}]$ is the unknown parameter vector in parameter space P $\{p_i \in [p_{i\min}, p_{i\max}],\ i=1,2,\cdots,n_p\}$. $\Phi(u)$ is a nonlinear continuous function. $f(x,p,t)$ is a smooth nonlinear function vector, and f, Φ satisfy the following assumptions.

Assumption 1:

$$\begin{aligned} f(x,p,t) &= \alpha^{\mathrm{T}}(p)\xi(x,t) \\ \alpha^{\mathrm{T}} &= [\alpha_1, \alpha_2, \cdots, \alpha_m] \\ \xi^{\mathrm{T}} &= [\xi_1, \xi_2, \cdots, \xi_m] \end{aligned} \tag{3}$$

where $\xi_i = \xi_i(x,t)$ are known nonlinear functions and linear indepemdent. $\alpha_i = \alpha_i(p)$ are the combinations of p; m denotes the dimension of α.

Assumption 2:

$$u(\Phi)(u) \geqslant hu^2 \tag{4}$$

where h is a positive and non-zero constant. $\Phi(u)$ is measurable and $\Phi(0)=0$.

As the boundary of p are given, the boundary of $\alpha(p)$ can also be obtained and the following assumption can be introduced.

Assumption 3:

$$\alpha_i \in [\alpha_{i\min}, \alpha_{i\max}] \quad \forall p \in P$$

In the following, we will omit the elements of some functions for simplicity.

SLIDING MODE CONTROL

In order to force the states of the closed-loop system $x^{\mathrm{T}} = [x_1, x_2, \cdots, x_n]$ to track a desired trajectory $x_{\mathrm{d}}^{\mathrm{T}} = [x_{1\mathrm{d}}, x_{2\mathrm{d}}, \cdots, x_{n\mathrm{d}}]$, where $x_{(i+1)\mathrm{d}} = \dot{x}_{i\mathrm{d}}$ $i=1,\cdots,n-1$, we define the error vector as

$$E = x_{\mathrm{d}} - x = [e, \dot{e}, \cdots, e^{(n-1)}]^{\mathrm{T}}, \tag{5}$$

where $e = x_{\mathrm{d}1} - x_1$.

The switching surface S is selected as

$$S = C^{\mathrm{T}}E \tag{6}$$

where $C^{\mathrm{T}} = [c_1, c_2, \cdots, c_n]$ is the coefficient of the Hurwitz polynomial and $c_n = 1$.

Suppose $\hat{\alpha}$ is the adjustable parameter vector of the SM identifier and $\hat{\alpha}_0$ is a time-varying parameter vector which is calculated in terms of the adjustable parameter and its bounds:

$$\hat{\alpha}_{i0}(t) = \begin{cases} \alpha_{i\min} & \text{if } \hat{\alpha}_i(t) < \alpha_{i\min} \\ \hat{\alpha}_i(t) & \text{if } \alpha_{i\min} \leqslant \hat{\alpha}_i(t) \leqslant \alpha_{i\max} \\ \alpha_{i\max} & \text{if } \hat{\alpha}_i(t) > \alpha_{i\max} \end{cases} \tag{7}$$

Theorem 1: Consider the nonlinear system Eqs. (1) and (2) subjected to Assumptions 1 - 3. If the sliding surface is chosen as Eq. (6) and u is given by

$$u = \varphi(x,t)\mathrm{sgn}(Sb), \quad \varphi(x,t) > 0 \tag{8}$$

the reaching condition

$$S\dot{S} < 0 \tag{9}$$

is satisfied. Where $\varphi(x,t)$ is selected by

$$\begin{aligned} \varphi(x,t) = \frac{1}{h}(&|b^{-1}(v - \hat{\alpha}_0^{\mathrm{T}}\xi)| + \\ &|b^{-1}|d^{\mathrm{T}}|\xi|) + \varepsilon \end{aligned} \tag{10}$$

where ε is a small positive constant, $|\xi| = [|\xi_1| \cdots |\xi_m|]^{\mathrm{T}}$, and

$$v = \dot{x}_{nd} + \sum_{i=1}^{n-1} c_i (x_{(i+1)\mathrm{d}} - x_{i+1}) \quad (11)$$

$$d^{\mathrm{T}} = [d_1, d_2, \cdots, d_m], \ d_i = |\alpha_{i\max} - \alpha_{i\min}| \quad (12)$$

Proof From Eqs. (4) and (8), the following equation can be obtained

$$u\Phi(u) = \varphi(x,t)\mathrm{sgn}(Sb)\Phi(u) = \frac{Sb}{|Sb|}\varphi(x,t)\Phi(u) \geqslant hu^2 = h\varphi(x,t)^2$$

then

$$Sb\Phi(u) \geqslant h|Sb|\varphi(x,t) \quad (13)$$

From Eqs. (6), (12) and (13), we get

$$S\dot{S} = S(v - \alpha^{\mathrm{T}}\xi - b\Phi(u)) = S(v - \hat{\alpha}_0^{\mathrm{T}}\xi + (\hat{\alpha}_0 - \alpha)^{\mathrm{T}}\xi - b\Phi(u)) \leqslant |Sb||b^{-1}(v - \hat{\alpha}_0^{\mathrm{T}}\xi)| + |Sb||b^{-1}|d^{\mathrm{T}}|\xi| - h|Sb|\varphi(x,t) \quad (14)$$

Substituting Eq. (10) into Eq. (14), the reaching condition Eq. (9) is satisfied.

For the control law Eq. (8), chattering phenomenon may occur in the closed-loop system due to the function sgn(Sb). To reduce the effect of chattering, the discontinuous function sgn (Sb) can be replaced by a proper continuous function (Chern et al., 1991).

$$\mathrm{sgn}_\delta(Sb) = \frac{Sb}{|Sb| + \delta} \quad (15)$$

where δ is selected as a function of the error e, i. e.

$$\delta = \delta_0 + \delta_1|e| \quad (16)$$

where δ_0, δ_1 are positive constants.

DESIGN OF SM-BASED IDENTIFIER

Based on the above SMC, an identification algorithm could be given by the the following theorem.

Theorem 2 Suppose the closed-loop nonlinear system Eq. (1) and Eq. (2) is in sliding surface, then the adjustable parameters can be made to converge to their real value asymptotically by constructing the identifier as follows

$$\dot{\hat{\alpha}}(t) = \Gamma\xi\omega \quad (17)$$

$$\hat{\alpha}(0) = \frac{1}{2}(\alpha_{i\min} + \alpha_{i\max}) \quad i = 1, 2, \cdots, n \quad (18)$$

$$\omega = v - \hat{\alpha}^{\mathrm{T}}\xi - b\Phi(u_{\mathrm{eq}}) \quad (19)$$

where u_{eq} is equivalent control input, $\Gamma = \Gamma^{\mathrm{T}} > 0$.

Proof When the closed-loop system (1) and (2) is in the sliding surface, we have

$$\dot{S} = 0, \quad u = u_{eq} \quad (20)$$

From Eq. (6) and Eq. (20), we obtain

$$\dot{S}(u = u_{eq}) = v - \alpha^{\mathrm{T}}\xi - b\Phi(u_{\mathrm{eq}}) = 0 \quad (21)$$

therefore, the following relationship is obtained

$$\omega = v - \hat{\alpha}^{\mathrm{T}}\xi - b\Phi(u_{eq}) = (\alpha - \hat{\alpha})^{\mathrm{T}}\xi \quad (22)$$

In order to prove the convergence of the identification algorithm, the following Lyapunov function is introduced

$$V = \frac{1}{2}(\alpha - \hat{\alpha})^{\mathrm{T}}\Gamma^{-1}(\alpha - \hat{\alpha}) \geqslant 0 \quad (23)$$

Substituting Eq. (17) and Eq. (22) into the time derivative of V, we get

$$\dot{V} = -(\alpha - \hat{\alpha})^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\alpha}} = -(\alpha - \hat{\alpha})^{\mathrm{T}}\xi\omega = -\omega^2 \leqslant 0 \quad (24)$$

since ξ is linearly independent, it is obvious that $\omega = 0$ if and only if $\hat{\alpha}(t) = \alpha$.

Usually there does not exist a general method to check the linear independence of nonlinear function vector ξ. What we can say is that, if the system can be linearized about an operating point (x_{e}, α_{e}), then based on the linearization, the local identifiability can be checked around the neighbourhood of (x_{e}, α_{e}) (Srewal et al., 1976).

NUMERICAL EXAMPLE

In this section, the SMC and SM-based identifier are applied to a parameter uncertain nonlinear dynamic system with nonlinear input to illustrate the obtained results above. Consider the following second-order nonlinear system with $\Phi(u)$ of the form Eq. (2) and α_1, α_2 being unknown parameters.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \alpha_1 \frac{x_2^2}{1+|x_2|} + \alpha_2 x_1(1-x_2) + \Phi(u)$$

$$\Phi(u) = (h + |sin(5u)|)u \quad h = 0.6$$

where the real value of unknown parameters are

$$\alpha_1 = 0.5 \quad \alpha_2 = 1$$

and their boundaries are

$$\alpha_{1max} = 4.5 \qquad \alpha_{2max} = 7.5$$
$$\alpha_{1min} = -1.5 \qquad \alpha_{2min} = -1.5$$

Let the desired eigenvalue be selected as $\lambda = 2$, the sliding surface is selected as

$$S = \dot{e} + 2e$$

The control objective is to make x tracking x_d, here a step function with amplitude equal to 1 adopted as the desired trajectory x_{1d}. Then $e = x_{1d} - x_1 = 1 - x_1 (\forall t \geqslant 0)$ and the control law of the closed-loop system can be calculated according to the method presented

$$u = \varphi(x,t)\frac{Sb}{|Sb| + \delta_0 + \delta_1|e|}$$

$$\varphi(x,t) = \frac{1}{0.6}\left\{\left| -2x_2 - \hat{\alpha}_{10}\frac{x_2^2}{1+|x_2|} - \hat{\alpha}_{20}x_1(1-x_2)\right| + 6\left|\frac{x_2^2}{1+|x_2|}\right| + 9|x_1(1-x_2)|\right\} + \varepsilon$$

where ε is a small positive constant.

By selecting Γ as a 2×2 identity matrix, then the identifier can be constructed as

$$\begin{cases} \dot{\hat{\alpha}}_1 = \dfrac{x_2^2}{1+|x_2|}\omega & \hat{\alpha}_1(0) = 1.5 \\ \dot{\hat{\alpha}}_2 = x_1(1-x_2)\omega & \hat{\alpha}_2(0) = 3.0 \end{cases}$$

$$\omega = -2x_2 - \hat{\alpha}_1\frac{x_2^2}{1+|x_2|} - \hat{\alpha}_2 x_1(1-x_2) - \Phi(u)$$

When the closed system is not in the sliding surface, $u \neq u_{eq}$. However, the controller can bring the system to the sliding surface, and make $u = u_{eq}$. Therefore, u can be used to replace u_{eq} in the above equation.

The simulation results are given in Figs. 1-5. Fig. 1 shows the tracking performance and Fig. 2 shows the phase plane between e and $\dot{e}$ under the

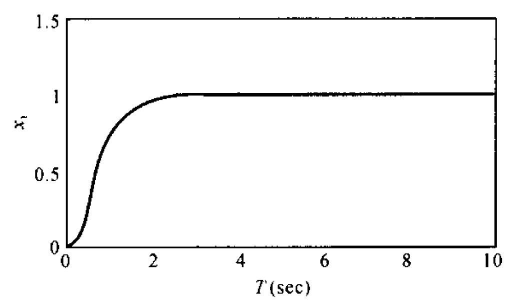

Fig. 1 Tracking performance of state x_1

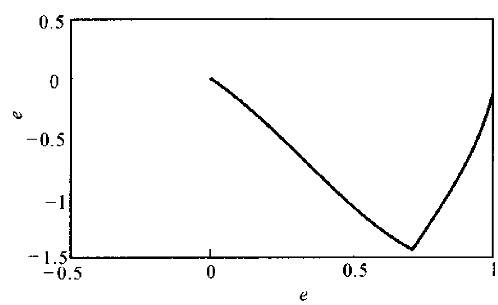

Fig. 2 Phase plane between e and $\dot{e}$

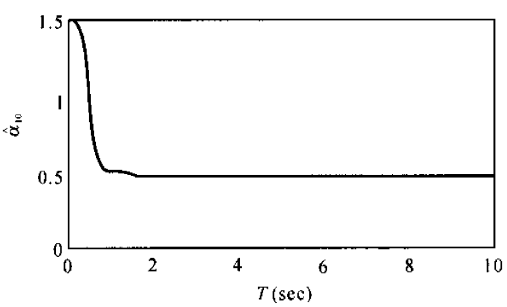

Fig. 3 Identification result of α_1

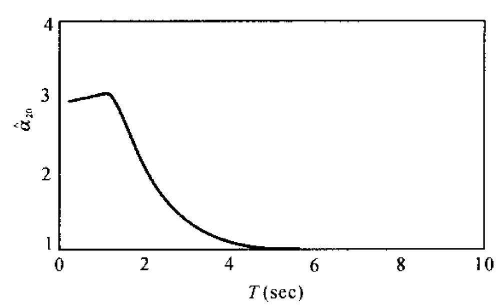

Fig. 4 Identification result of α_2

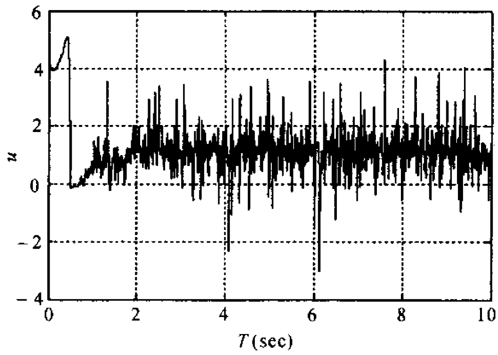

Fig. 5 The control input of the closed system

proposed SMC. Fig. 3 and 4 show the identification results of the unknown parameters α_1 and α_2 respectively. The control input of the closed system is shown in Fig. 5. The simulation results showed that the proposed SMC and SM-based identifier work well for the parameter uncertain nonlinear dynamic system with nonlinear input.

CONCLUSIONS

A novel SM-based identification method has been developed to deal with the parameter identification problem of parameter uncertain nonlinear dynamic system with nonlinear input. With the proposed control method, the global stability of SMC for the closed-loop system is guaranteed. The convergence of the parameters identifier is attained by Lyapunov theory. The validity of the proposed algorithm is confirmed through a numerical simulation example.

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