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Application of gap element to nonlinear mechanics analysis of drillstring*

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Abstract: This paper presents a nonlinear finite element method to resolve the problem of the nonlinear contact between the drillstring and hole wall by using a Multi-directional Contact Gap Element（MCGE）contacting at appropriate positions in each beam element. The method was successfully applied to the Daqing Oil Field GP1 well. It was shown that the drillstring's contact resistance at any well depth could be obtained by calculations and that as the error in the calculation of the hole top load is below 10%，the calculation result can provide theoretical basis for the design and operation of drillstrings.

Key words: Drillstring，Contact non-line，Finite element，Horizontal well
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INTRODUCTION

In the drilling of directional and horizontal well with large displacement，the drillstring's possible contact with the hole wall at many places in the inclined and horizontal sections increases the frictional resistance，force moment，hole top load，and drilling risk. When the frictional resistance is very high，the drilling weight and torque of the bit cannot be effective and the up and down drill pipe cannot be operated，so the drilling process stops. Since 1980，many scholars have researched the frictional resistance of drillstring using the method of drillstring mechanics，and among many models developed are three main types: flexible structure type（Brett et al.，1989），rigid structure type（Payne et al.，1997）and mixed structure type（Ma et al.，1996）. These models cannot consider both geometric and contact nonlinear effect，so their calculation of frictional resistance is imperfect. For this reason，the MCGE is constructed in this work and the contact and geometric nonlinear mechanics of drillstring is studied using the MCGE connection with beam element so that the frictional resistance can be properly taken into account.

THE NONLINEAR STATICS MODEL OF DRILLSTRING

The drillstring's force and deformation in the hole can be divided into two parts. The first part is the bending deformation of the straight drillstring caused by the hole trajectory，which is usually produced in the inclined section. The second part is the load induced drillstring assembly deformation in the hole during the drilling process. In the second part，the drillstring mechanics includes the geometric and nonlinear contact problem. The geometric non-line means the longitudinal bend of drillstring acted on by the more axial load，and the contact non-line means that the drillstring deformation is constrained by the hole wall and the contact is randomly distributed along the well depth and hole circumferential direction. Therefore，in the analysis of drillstring mechanics these force and deformation must be considered so that the contact frictional resistance can be properly taken into account.

According to the drillstring work state，the whole drillstring is selected as a research object and the nonlinear statics model is presented in this paper，as illustrated in Fig. 1. In the mod-

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el, according to the drillstring structure and hole trajectory, the drillstring can be discretized into several beam elements along the axis and then the MCGE is installed at every beam element. The beam element can describe not only the various drillstring construction and loads, hole top and toe boundary condition, but also consider the longitudinal bend and initial deformation of the drillstring. The MCGE can properly describe the random contact between the drillstring and hole wall. This shows that this model can correctly describe the real work state of the drillstring, which takes the drillstring mechanics as a geometric and nonlinear contact problem, not as the previous helical bend or the longitudinal and lateral bend. The model makes the drillstring mechanical analysis more rigorous, but the analysis method is more difficult and calculation time is increased. Because the drillstring work state is complicated and the statics analysis method is limited, the following hypotheses are adopted.

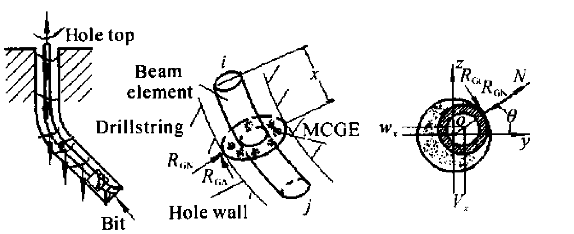

Fig.1 The nonlinear statics model of drillstring

1. The hole wall is a cylindrical shell. Measurement and the engineering experience may give the deformation of the hole wall and the thickness of the mud sheath.

2. The hole wall may produce the key groove and the drillstring may produce the sticking at any well depth, which is not normal work state. In this case, the mechanical resistance acting on the drillstring can be obtained through simulation of the hole top load and turning moment.

3. There is no need to consider all dynamic loads caused by the motion of the drillstring. Thus it can seen that the nonlinear statics model presented in this paper should be solved by the MCGE and beam element.

MULTI-DIRECTIONAL CONTACT GAP ELEMENT (MCGE)

Description of the MCGE

The MCGE is a virtual element composed of drilling fluid, with outside linked to the hole wall, inside linked to the drillstring; and is in the form of a thick annulus. When the drillstring is not in contact with hole wall, the MCGE has compressive rigidity approximating to zero and has no effect on the free motion of the drillstring; once the drillstring contacts the hole wall, its compressive rigidity becomes so big as to prevent drillstring from getting into the hole wall, allow motion of the drillstring, produce the contact reaction force and the corresponding frictional resistance and force moment.

The local coordinate system oyz of MCGE is chosen so that it overlaps with that of the beam element. The MCGE may be installed at x distance to the left node of the beam element; its displacement should be the lateral displacement of the beam element and can be shown to be

$$\boldsymbol{f}_{\mathrm{G}} = [\nu_x,\ w_x]^{\mathrm{T}} = \boldsymbol{N}_{\mathrm{G}} \boldsymbol{d}_{\mathrm{e}} \qquad (1)$$

Where $\boldsymbol{N}_{\mathrm{G}}$ is the formal function of the beam element and also the transform matrix of 2×12, $\boldsymbol{d}_{\mathrm{e}}$ is the nodal displacement vector of the beam element. If the beam element is to make first contact with hole wall at maximum lateral displacement, the MCGE should be located at x_0 position which can be obtained as the extreme value of Eq. (1).

$$\boldsymbol{f}_{\mathrm{G}_0} = \max(\boldsymbol{f}_{\mathrm{G}}) = \boldsymbol{N}_G |_{x=x_0} \boldsymbol{d}_{\mathrm{e}} = \boldsymbol{N}_{\mathrm{G}_0} \boldsymbol{d}_e \qquad (2)$$

After the deformation of the drillstring, the MCGE may be compressed in any direction; and the relative compression is given by

$$\boldsymbol{\varepsilon}_{\mathrm{GN}} = \frac{2\boldsymbol{f}_{\mathrm{G}_0}}{\mathrm{D} - \mathrm{d}} \qquad (3)$$

Where d is the outside diameter of the drillstring, D is the inside diameter of the hole. When the MCGE produces the compressive deformation, the contact reaction force should be:

$$\boldsymbol{R}_{\mathrm{GN}} = G_k \boldsymbol{f}_{\mathrm{G}_0} \qquad (4)$$

Where G_k is the compressive rigidity of the

MCGE and its relation with relative compression is shown in Fig. 2. That proves the MCGE is a nonlinear element with varying rigidity. The motion of the drillstring produces frictional resistance and force moment as shown in Fig. 1.

$$\left.\begin{aligned} R_{\mathrm{Gt}} &= \mu_1 R_{\mathrm{GN}} \\ R_{\mathrm{GA}} &= \mu_2 R_{\mathrm{GN}} \\ M_{\mathrm{Gt}} &= \frac{d}{2} R_{\mathrm{Gt}} \\ M_{\mathrm{GA}} &= \frac{d}{2} R_{\mathrm{GA}} \end{aligned}\right\} \tag{5}$$

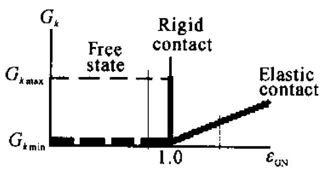

Fig. 2 The relation of compressive rigidity with relative compression of MCGE

Where R_{Gt} and R_{GA} are the frictional resistance along the hole circumference and axis direction respectively. μ_1 is the coefficient of sliding friction and $\mu_2=0.0$ during the rotary drilling; but $\mu_1=0.0$ and μ_2 is not zero, but some other value during the sliding drilling or the upward and downward drill pipe. M_{Gt} and M_{GA} are the torsional and bending moment caused by the frictional resistance. These forces and moments can be transformed into the equivalent nodal force of MCGE

$$\boldsymbol{R}_{\mathrm{G}}^{e} = \boldsymbol{N}^{\mathrm{T}} \begin{Bmatrix} R_{\mathrm{GA}} \\ R_{\mathrm{Gt}y} \\ R_{\mathrm{Gt}z} \\ M_{\mathrm{Gt}} \end{Bmatrix} + \frac{\mathrm{d}\boldsymbol{N}^{\mathrm{T}}}{\mathrm{d}x} \begin{Bmatrix} 0 \\ M_{\mathrm{GA}y} \\ M_{\mathrm{GA}z} \\ 0 \end{Bmatrix} \tag{6}$$

Where $R_{\mathrm{Gt}y}$ and $R_{\mathrm{Gt}z}$ are the components of the force $R_{\mathrm{G}t}$ in the y and z direction, $M_{\mathrm{GA}y}$ and $M_{\mathrm{GA}z}$ are the components of the bending moment M_{GA} in the y and z direction, N is the formal function of the beam element.

The contact state discriminant of MCGE

If the long drillstring is numerically analyzed using beam element, the global stiffness matrix may be a singular one because the constraint of the hole wall is not considered, which makes the equilibrium equation of drillstring unsolvable. In addition, the drillstring constrained by hole wall is also a random contact boundary that can not be correctly given. Therefore, the MCGE must be installed at every beam element and its compressive rigidity must be above zero during the iterative calculation. For the calculation efficiency and calculation error to satisfy the needs of drilling engineering, the relative compression and contact reaction force should be given a small value $\varepsilon_{\mathrm{N0}}$ and R_{N0} in the iterative calculation, and then the contact state discriminant of MCGE can be written as:

1. Free state

$$\begin{Bmatrix} \varepsilon_{\mathrm{GN}} - 1.0 < -\varepsilon_{\mathrm{N0}} \\ R_{\mathrm{GN}} - R_{\mathrm{N0}} \leqslant 0.0 \\ G_k = G_{k\min} \end{Bmatrix} \tag{7}$$

2. Elastic contact state

$$\left.\begin{aligned} -\varepsilon_{\mathrm{N0}} \leqslant \varepsilon_{\mathrm{GN}} - 1.0 \leqslant \varepsilon_{\mathrm{Nmax}} \\ R_{\mathrm{GN}} - R_{\mathrm{N0}} \geqslant 0.0 \\ G_k = G_{kc} \end{aligned}\right\} \tag{8}$$

Where $\varepsilon_{\mathrm{Nmax}}$ is the maximum relative compression of MCGE considering the deformation of rock and mud sheath; $G_{k\min}$ is the minimum compressive rigidity of MCGE and generally takes a small value; G_{kc} is the rock compressive rigidity used to simulate the rock deformation. When $\varepsilon_{\mathrm{Nmax}} = \varepsilon_{\mathrm{N0}}$ and $G_{kc} = G_{k\max}$ in Eq. (8), the elastic contact can be transformed into rigid contact, and $G_{k\max}$ is the maximum compressive rigidity used to simulate the hole wall rigidity.

THE EQUILIBRIUM EQUATION OF NONLINEAR FINITE ELEMENT IN THE DRILLSTRING MECHANICAL ANALYSIS

If the nodes of beam element produced any virtual displacement $\delta\boldsymbol{d}_{\mathrm{e}}$, the beam element would produce virtual displacement $\delta\boldsymbol{f}$ and virtual strain $\delta\varepsilon$, the MCGE would produce virtual displacement $\delta\boldsymbol{f}_{\mathrm{G}}$, and then the virtual work equation can be derived according to the virtual work principle (Liu et al., 1996).

$$\int_{V_e} \delta\varepsilon^{\mathrm{T}} \sigma \mathrm{d}V + \delta\boldsymbol{f}_{\mathrm{G}}^{\mathrm{T}} \boldsymbol{R}_{\mathrm{GN}} = \int_{V_e} \delta\boldsymbol{f}^{\mathrm{T}} \boldsymbol{P}_V \mathrm{d}V + \int_{A_e} \delta\boldsymbol{f}^{\mathrm{T}} \boldsymbol{P}_A \mathrm{d}A + \delta\boldsymbol{d}_{\mathrm{e}}^{\mathrm{T}} \boldsymbol{P}_{\mathrm{e}} + \delta\boldsymbol{d}_{\mathrm{e}}^{\mathrm{T}} \boldsymbol{R}_{\mathrm{G}}^{e} \tag{9}$$

Where σ, $\boldsymbol{P}_V$, $\boldsymbol{P}_A$ and $\boldsymbol{P}_e$ are the stress, distribution of body force, distribution of surface force and nodal force for the beam element, respectively. The second term on the left hand side of Eq. (9) stands for the virtual work produced by the displacement of MCGE, and the last term on the right stands for the virtual work by the equivalent nodal force of MCGE. The finite element method was used to derive the equilibrium equation of element as:

$$(\boldsymbol{K}_0^e + \boldsymbol{K}_N^e + \boldsymbol{K}_\sigma^e + \boldsymbol{K}_G^e)\boldsymbol{d}_e = \boldsymbol{F}_e + \boldsymbol{R}_G^e \quad (10)$$

Where $\boldsymbol{K}_0^e$ is the linear stiffness matrix that is not related with nodal displacement, $\boldsymbol{K}_N^e$ is the large displacement stiffness matrix that is related with nodal displacement, $\boldsymbol{K}_\sigma^e$ is the geometric stiffness matrix that is related with the stress of element, $\boldsymbol{F}_e$ is the equivalent nodal force of element. $\boldsymbol{K}_G^e$ is the MCGE stiffness matrix given by

$$\boldsymbol{K}_G^e = N_G^T G_k N_G \quad (11)$$

Through the transformation of coordinate system and assembly of every element stiffness matrix on overall MCGEs and beam elements, the global equilibrium equation of the drillstring can be obtained by nonlinear statics analysis as:

$$(\boldsymbol{K}_0 + \boldsymbol{K}_{N(d)} + \boldsymbol{K}_{\sigma(d)} + \boldsymbol{K}_{G(d)})d = \boldsymbol{F} + \boldsymbol{R}_{G(d)} \quad (12)$$

Where $\boldsymbol{K}_0$, $\boldsymbol{K}_N$, $\boldsymbol{K}_\sigma$ and $\boldsymbol{K}_G$ are the global linear stiffness matrix, large displacement stiffness matrix, geometric stiffness matrix and the stiffness matrix of MCGE, respectively; $\boldsymbol{F}$ and $\boldsymbol{R}_G$ are the equivalent nodal force of beam element and MCGE, respectively. After the nonlinear Eq. (12) is solved, the contact state discriminant of MCGE must be distinguished and adjusted, and the geometric stiffness matrix must be also modified according to the beam element stress during the same iterative calculation. This process must be repeated until all conditions are met, and the accurate solution of generalized displacement and internal force of drillstring can be obtained.

APPLICATION IN ENGINEERING

The Daqing Oil Field GP1 horizontal well with hole trajectory as shown in Fig. 3 was selected as an example. The predicted capability of the bit acted on by the drilling weight, and the operation risk of upward and downward drill pipe were analyzed in order to

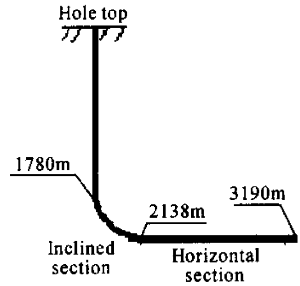

Fig. 3 The hole trajectory of GPI well

provide theoretical basis for the drillstring design and operation in GP1 well. The frictional resistance must be calculated for various load cases. According to the above drillstring mechanics, the frictional resistance was calculated at any inclined and horizontal sections under four load cases: up, down, rotary and sliding drill. Part of the calculated results are listed in Table 1 showing that the frictional resistance in the upward drill pipe was greater than that in the downward drill pipe; frictional resistance in the horizontal section was greater than that in the inclined section, that in the sliding drill pipe was greater than that in the downward drill pipe. And the longer the horizontal section was or the greater the well depth was, the greater was the frictional resistance. These increasing trends of frictional resistance were observed because the contact reaction force was increased by the drilling weight acting on the bit, the inclination of hole trajectory, contact section, etc.. In the sliding drilling at the longest horizontal section, the resistance was the maximum value and the hole top load of 432.5 kN was the minimum value that could meet the needs of the 100.0 kN drilling weight acting on the bit. In addition, the maximum calculated value of the hole top load was 768.4 kN in all the load cases, which was lower than the allowable load of the drilling machine. This showed that the design of the hole trajectory and drillstring construction were reasonable; that the drilling machine could safely work under the various load cases; that the weight of the drillstring could overcome the various frictional resistances and ensure the drilling weight acting effectively on the bit; and that the whole drilling process was safety and reliable.

In order to test and verify the calculation precision of the drillstring mechanics, actual measurements of the hole top load are also listed in Table 1 showing that the calculated hole top loads basically conform to the actually measured values, with relative error below 10%. The results showed that drillstring mechanical analysis could be used in drilling engineering. This conclusion was confirmed by drilling practice.

Table 1 The calculated and measurement results of frictional resistance in the GP1 well

Section and well depth	Load case	Drilling weight (kN)	Calculated hole top load (kN)	Measured hole top load (kN)	Relative error (%)	Frictional resistance (kN)	Force moment (kN·m)
Inclined section (well depth of 2075 m and inclination of 65°)	up	0.0	733.4	745.0	1.56	−17.29	−
	down	0.0	699.6	707.0	1.05	16.37	−
	rotary drilling	150.0	568.2	564.0	0.75	−	13.69
	sliding drilling	170.0	524.3	511.0	2.60	29.84	−
Horizontal section (well depth of 2770 m and inclination of 85°)	up	0.0	755.3	800.0	5.59	−63.30	−
	down	0.0	631.3	680.0	7.16	61.70	−
	rotary drilling	77.0	616.7	646.0	4.54	−	25.89
	sliding drilling	90.0	536.4	590.0	9.08	73.50	−
Horizontal section (well depth of 3190 m and inclination of 86°)	up	0.0	768.4	828.0	7.20	−131.50	−
	down	0.0	494.1	504.0	1.96	129.20	−
	rotary drilling	40.0	582.8	630.0	7.49	−	65.50
	sliding drilling	60.0	432.5	444.0	2.60	137.50	−

CONCLUSIONS

1. The nonlinear statics model of drillstring presented in this paper considers the random contact and the geometric nonlinear problem of the drillstring, and adequately describes the deformation and force of the drillstring.

2. The MCGE may be placed at any beam element position, which maximizes the lateral displacement of the beam element previously in contact with hole wall. A new element is provided so as to solve the contact friction between the drillstring and hole wall along the well depth and hole circumferential direction.

3. Engineering tests showed that this finite element composed of MCGE and beam element is effective for nonlinear statics analysis of drillstring. This model can be used not only in frictional resistance analysis of drillstring but also in mechanics analysis of tubing string, rod string, etc. in production practice; and has high potential for application in petroleum engineering.

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