ISSN 1009－3095 *Journal of Zhejiang University SCIENCE* V. 3，No. 5，P. 507－512，Nov. －Dec.，2002
http://www.periodicals.com.cn; http://www.zju.edu.cn/English
http://www.zjupress.com; http://lib.zju.edu.cn/eindex.htm; jzu_s@mail.hz.zj.cn

Consolidation behavior of cement- and lime/cement-mixed column foundations*

CAI Yuan-qiang(蔡袁强), XU Chang-jie(徐长节)
(*Department of Civil Engineering, Zhejiang University, Hangzhou 310027, China*)
E-mail: cyq@civil.zju.edu.cn; xucj@civil.zju.edu.cn

Received Dec. 21, 2001; revision accepted Feb. 26, 2002

Abstract: The consolidation behavior of mixed in place cement- and lime/cement-mixed column was studied. Consolidation of the composite foundation was modeled as a three-dimensional axi-symmetric problem. The authors used the finite difference method to obtain the pore pressure variation with time at any location below the surface. A computer program developed by the authors was used to draw some interesting conclusions about the consolidation behaviors of cement- and lime/cement-mixed pile foundation. Finally, a combined model including the permeability coefficients of cement-mixed piles and soil, was studied and its feasibility was evaluated.

Key words: Consolidation, Axi-symmetric, Cement-mixed column, Lime/cement mixed column, Composite foundations
Document code: A **CLC number:** TU472

INTRODUCTION

Lime and lime/cement columns have been very competitive in the Nordic countries, compared with other soil stabilization methods such as preloading, excavation and replacement as well as embankment piles. In many cases, it is not necessary for the columns to extend the bottom of the compressible soil deposit. A foundation consisting of an upper layer with reinforcing columns, and a lower, unstabilized soil layer, is called composite foundation. The deformation characteristics of such a foundation solution are complex, especially with respect to the dissipation of pore water pressure after loading.

Field tests indicated that lime columns function as vertical drains (Broms and Boman, 1979). The permeability of the stabilized soil often increases with time possibly due to long-term shrinkage. The permeability of lime and lime/cement columns, and its effect on the consolidation behavior of the reinforced foundation is an important design aspect.

COMPOSITE FOUNDATION

Composite foundation is a kind of artificial foundation consisting upper, strengthened layer, and unstablized lower stratum. The consolidation behavior of the columns and the adjacent soil is very complex. Gong (1992) did research on the deformation of cement-mixed pile foundations. Their work dealt with one-dimensional consolidation. However, the consolidation behavior of the columns and the surrounding soil is different. Therefore, it is necessary to treat this problem as a three-dimensional case. Yoshikuni and Nakanodo (1974) introduced an equation accounting for the consolidation process affected by the stress concentration in drain wells when compacted sand piles of large diameter are used. Scott (1963) used term radial consolidation for axi-symmetric problems.

Since the unique characteristics of the lime-mixed columns had not been studied in detail before, attention is given to the deformation and consolidation behavior of lime/cement mixed pile foundations.

* Project supported by National Natural Science Foundation of China (No. 59908012) and Education Ministry of China

CONSOLIDATION EQUATION AND ITS SOLUTION

The loading is regarded as instantaneous load and the pile and soil are treated as elastic models. Other assumptions are listed as follows:

1. Pile and soil are all homogeneous; soil is saturated.
2. The permeability of soil differs in horizontal and vertical directions. The permeability of pile is isotropic.
3. Soil particles and water can't be compressed.
4. Under the loading, there is not only vertical flow of seep and compression, but also horizontal seep, which also causes vertical compression.
5. Seepage in soil and pile follows Darcy's law.
6. The displacement is only caused by the seepage and depletion of the pore pressure.

This three-dimensional problem can be treated as an axi-symmetric problem. The model is shown in Fig. 1 and Fig. 2. The shaded area in Fig. 2 is a unit for analysis.

The consolidation equation can be expressed as:

$$C_v \frac{\partial^2 u}{\partial z^2} + C_{vr}\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) = \frac{\partial u}{\partial t} \quad (1)$$

where: r is the radial distance from the pile's axis;

C_v, C_{vr}, are vertical and horizontal consolidation coefficients;

z is the depth;

u is pore pressure.

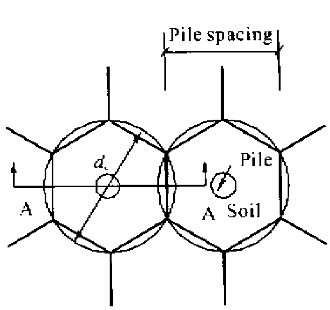

Fig. 1 Plan view of pile and soil influence area of A − A section line

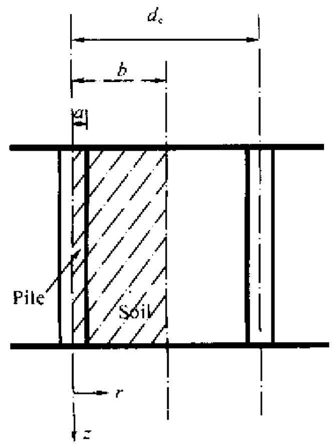

Fig. 2 A − A cross section of pile and soil influence area

The initial and boundary conditions of the unit can be expressed as follows, initial conditions:

$$u|_{t=0} = 0 \quad (2)$$

The boundary conditions are shown as follows:

$$r = 0 \quad \frac{\partial u}{\partial r} = 0 \quad (3)$$

$$r = b \quad \frac{\partial u}{\partial r} = 0 \quad (4)$$

$$r = a \quad \begin{cases} u_1 = u_2 \\ k_p \dfrac{\partial u_p}{\partial r} = k_{sr} \dfrac{\partial u_s}{\partial r} \end{cases} \quad (5)$$

$z = h$ for draining at one side (surface):

$$\frac{\partial u}{\partial r} = 0 \quad (6)$$

for draining at two sides (surface & bottom):

$$u = 0 \quad (7)$$

where: u_p, u_s are pore pressure in pile and soil respectively;

k_p, k_{sr} are the radial permeability coefficients of pile and soil.

The finite difference method is used to solve Eq. (1) (Li et al., 1995). By use of classic explicit difference formula, the following finite-difference solution of Eq. 1 can be obtained:

$$\frac{u_{j,m}^{n+1} - u_{j,m}^{n}}{\Delta t} = Cv \frac{u_{j+1,m}^{n} - 2u_{j,m}^{n} + u_{j-1,m}^{n}}{(\Delta h)^2} +$$

$$Cvr\left(\frac{u^n_{j,m+1} - u^n_{j,m}}{m(\Delta r)^2} + \frac{u^n_{j,m+1} - 2u^n_{j,m} + u^n_{j,m-1}}{(\Delta r)^2}\right) \tag{8}$$

$$u^{n+1}_{j,m} = \frac{Cv \times \Delta t}{(\Delta h)^2}(u^n_{j+1,m} - 2u^n_{j,m} + u^n_{j-1,m}) + Cvr \times \Delta t\left(\frac{u^n_{j,m+1} - u^n_{j,m}}{m(\Delta r)^2} + \frac{u^n_{j,m+1} - 2u^n_{j,m} + u^n_{j,m-1}}{(\Delta r)^2}\right) + u^n_{j,m} \tag{9}$$

Here, Δt is the time step length ($t_n = n \times \Delta t$, $n > 0$), Δh, Δr are the vertical and horizontal space step length of the grid which can be obtained by dividing the unit. $z_j = j \times \Delta h$; $r_m = m \times \Delta r$ ($j = 1, 2, \cdots, J$; $m = 1, 2, \cdots, M$; $J \times \Delta h =$ the length of pile; $M \times \Delta r =$ the radius of influence).

Therefore, from the initial and boundary conditions, the pore pressure of any locations at arbitrary time can be obtained step by step. The average degree of consolidation can be obtained by Carrillo (1942):

$$U = \frac{\int_0^a\int_0^h u_1 \mathrm{d}r\mathrm{d}z + \int_a^b\int_0^h u_2 \mathrm{d}r\mathrm{d}z}{\int_0^b\int_0^h u_0 \mathrm{d}r\mathrm{d}z} \tag{10}$$

Where, U is average percentage of consolidation;

u_p is pile's pore pressure;

u_s is soil's pore pressure;

u_0 is initial pore pressure.

It should be pointed out that in the analysis process, stability conditions of the finite difference method must be satisfied:

$$\begin{cases} \dfrac{Cv \times \Delta t}{(\Delta h)^2} \leqslant \dfrac{1}{2p} = 1/4 \\ \dfrac{Cv \times \Delta t}{(\Delta r)^2} \leqslant \dfrac{1}{2p} = 1/4 \end{cases} \tag{11}$$

Here, p is the number of dimensions.

BEHAVIOR OF CEMENT/LIME-MIXED PILE FOUNDATION FROM ANALYSIS

A computer program was developed to analyse the consolidation behaviors of the composite foundation. Using several numerical examples, some useful and reasonable results can be obtained, such as the distribution of pore pressure, the influence of parameters on the degree of consolidation and results of analysis of one widely-used model of combining soil's and cement-mixed pile's permeability coefficients, etc.

In order to analyse the variation of pore pressure in horizontal direction, the first lattice line below the surface ($z = 0$) is designated as the objective to study. And in order to analyse the variation of the pore pressure in vertical direction, the points along the pile's axis ($r = 0$) is designated. In this thesis only the cases of one-side drainage is calculated. But due to symmetry of double-side drainage, cases of one-side drainage can be converted into the cases of double-side drainage.

The materials used are listed below.

Table 1 Materials and their parameters

	Parameters	Values
	Loading p	100 kPa
Soil's	Vertical permeability coefficient K_s	1.0×10^{-9} m/s
	Horizontal permeability coefficient K_{sr}	1.0×10^{-8} m/s
	Modulus of compression E_s	4 MPa
Cement-mixed pile's	Permeability coefficient K_{cp}	1.0×10^{-11} m/s
	Modulus of compression E_{cp}	60 MPa
Lime-mixed pile's	Permeability coefficient K_{lp}	1.0×10^{-8} m/s
	Modulus of compression E_{lp}	60 MPa
	Radius of pile a	0.3 m
	Radius of influence b	1.0 m
	Length of pile H	8.0 m

1. Analysis of distribution of pore pressure and the variation of the pore pressure with time

Fig. 3 shows that in cement/lime-mixed pile foundations, the deeper the point, the larger the pore pressure, i.e., the pore pressure nearer the surface dissipates rapidly. Moreover, the ratio of pore pressure increment to depth increment ($\Delta u/\Delta h$) is smaller at the deeper points.

Also according to Fig. 3 and Fig. 4, the pore pressure dissipates with time. The degree of drainage is greater at shallow depth and larger at locations farther away from the axis of cement-mixed pile foundations. However, the drainage effect is smaller at locations farther away in lime-mixed pile foundations. After a certain time interval, the pore pressure values at the horizontal plane are almost equal. Then the degree of consolidation in pile and in soil is nearly homogenous and the axi-symmetric consolidation is almost equal to one-dimensional consolidation.

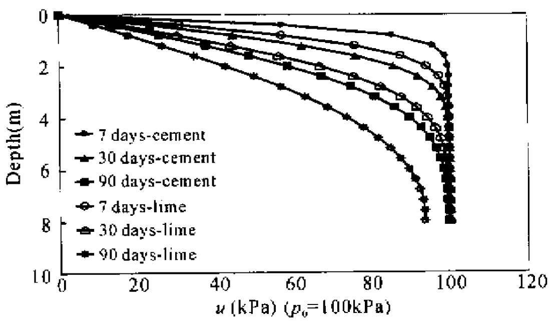

Fig. 3 Variation of the pore pressure with depth

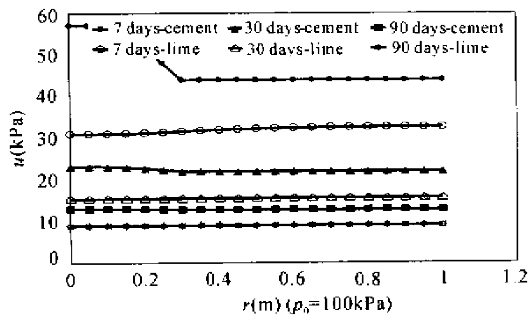

Fig. 4 The variation of the pore pressure with radial distance

Fig. 3 and Fig. 4 show that cement-mixed pile foundation differs from lime-mixed composite foundations with respect to consolidation behavior. It is apparent that the drainage of the cement pile is significantly slower than that of the lime/cement mixed pile. In fact, the lime-mixed columns' drainage process is faster than that of the soil, while that of the cement-mixed columns' drainage is much slower. This is due to the high permeability of the lime-mixed pile. Therefore lime-mixed columns can not only strengthen the foundations, but also quicken the consolidation of foundations. This is an important advantage of lime-mixed columns.

2. Influence of permeability coefficients of pile and soil

From Fig. 5, it can be concluded that the variation of the permeability of cement-mixed columns has little influence on the degree of consolidation. However, the permeability coefficient of the soil has great influence on the consolidation behavior of composite foundations. This aspect is reasonable, as the permeability of cement-mixed piles is significantly slower than that of the soil. In fact, the permeability of cement-mixed columns is so small that they can be treated as practically impermeable. Fig. 6 also supports this conclusion. Since the permeability of lime-mixed columns is higher than that of the soil, the effect of the change of the column parameters is greater than that of soils.

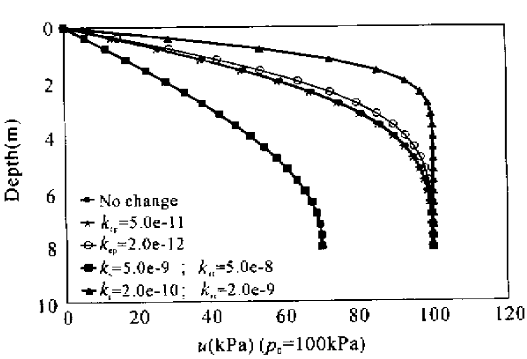

Fig. 5 The influence of the change of k_{cp} and k_s (after 90 days)

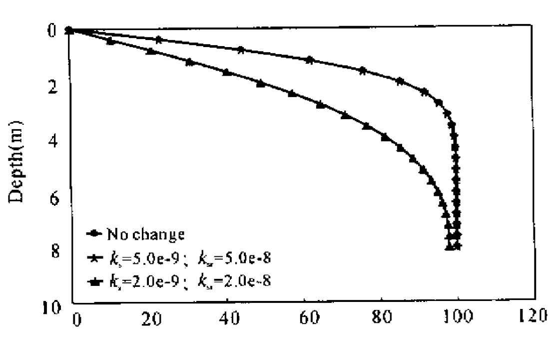

Fig. 6 The influence of the change of k_{cp} and k_s (after 30 days)

3. Influence of the change of radial and vertical parameters

Changes in the soil's radial or vertical permeability coefficients had obvious influence on the radial and vertical permeability. Fig.7 and Fig.8 show the results when our program was used to analyze these cases.

Fig.7 shows that the change of soil vertical permeability in the case of cement-mixed pile foundations has far more influence on foundation consolidation than the change of horizontal permeability. In fact, the effect of the change of horizontal permeability is so small that this influence can be neglected. Thus, the soil can be treated as isotropic material in the case of cement-mixed pile foundations.

However, from Fig. 8, it can be concluded that in the case of lime/cement mixed pile foundations, the change of horizontal permeability can't be neglected, although its influence is not as great as the influence of the change of vertical permeability. This effect is easy to understand since the radial drainage is more important when the column permeability increases. Therefore, the horizontal permeability of soil has relative greater influence on consolidation of foundations.

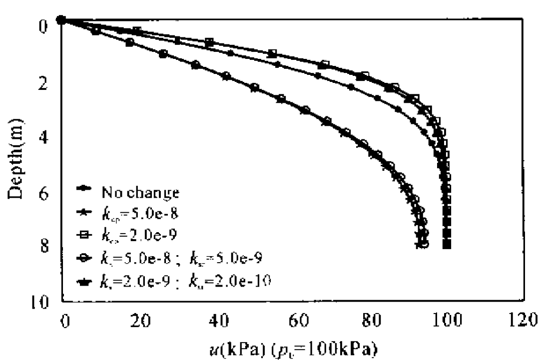

Fig.7 The influence of the change of k_s and k_{sr} (after 30 days)

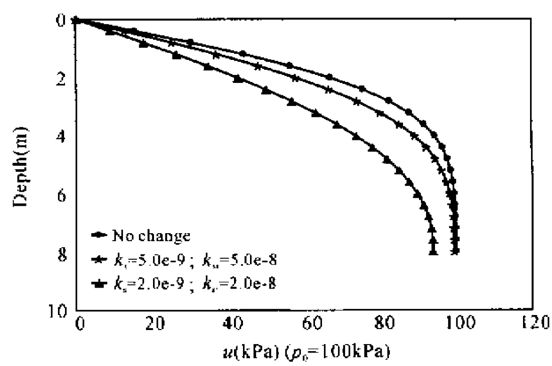

Fig.8 The influence of the change of k_s and k_{sr} (after 90 days)

4. Combination of soil permeability coefficient and cement-mixed pile permeability coefficient

The combined permeability coefficient can be estimated by

$$k_f = (k_p \times A_p + k_s \times A_s)/(A_p + A_s) \qquad (12)$$

where: k_f is the composite permeability coefficient;

k_p and k_s are permeability coefficient of cement-mixed pile and soil respectively; A_p and A_s are the pile's area and soil's area in the horizontal plane of model respectively. In this case, soil permeability is regarded as isotropic, which is a reasonable assumption considering 3.3.

Eq.12 is widely used in engineering practice to calculate the composite permeability coefficient of cement-mixed pile foundations in China. However there is no theoretical basis so far for Eq.12. In this paper, a computer program was used to study the precision of this composite model. The parameters are listed in Table 2.

Table 2 Parameters and their values

Parameters	Values
Loading	100 kPa
Pile's k	1.0×10^{-11} m/s
The modulus of compression E_p	60 MPa
Soil's k	1.0×10^{-9} m/s
The modulus of compression E_s	4 MPa
Radius of pile	0.3 m
Radius of influence	1.0 m
Length of pile	8.0 m

Considering the results shown in Fig. 9, it can be seen that the above mentioned composite model is suitable when the variation of degree of consolidation with time is considered, although there is some difference between the two solutions as consolidation is developing.

The difference is so small that it can be neglected for practical purposes. However as far as the ratio of replacement is concerned, this model is not very accurate. The higher the ratio of replacement, the larger the discrepancy is between the two solutions. Therefore, one must be careful when using this composite model (Eq. 12). If the cement-mixed pile is large or the pile spacing is very small, i.e., the ratio of replacement is high, some errors emerge in this composite model.

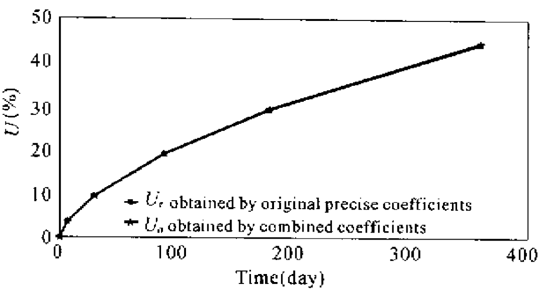

Fig. 9 Variation of average degree of consolidation with time (Comparing the results calculated from the composite model with the original method, $a = 0.05$)

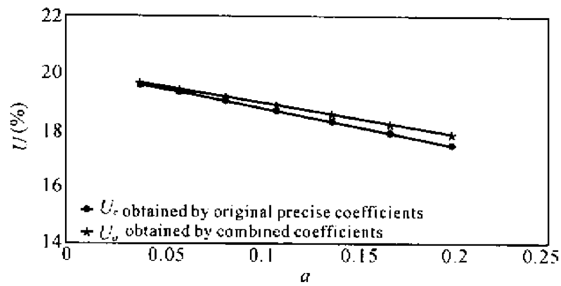

Fig. 10 Variation of average degree of consolidation with the ratio of replacement a (Comparing the results calculated from the composite model with the original method)

CONCLUSIONS

In this work to study axi-symmetric problems of three-dimensional consolidation of column-reinforced foundation, numerical methods were used to solve the equation of axi-symmetric consolidation, the consolidation behaviors of cement- and lime-mixed column (pile) foundations were also studied. The following conclusions can be drawn:

1. In the case of one-way drainage, the dissipation of pore pressure occurs faster at shallow depth. In the case of cement-mixed column foundations, the dissipation of pore pressure in the soil is faster than that in the column. However, after a certain period, the pore pressure will reach equilibrium. In the case of lime-mixed column foundations with high column permeability, the dissipation in the lime-mixed columns is faster than that in the soil.

2. The vertical permeability has great influence on the consolidation of foundations, especially in the case of cement-mixed column foundations. However, in the case of lime-mixed column foundations, when the column permeability increases, the radial drainage also increases. Thus, the soil can be regarded as isotropic material in cement-mixed case, but not in lime-mixed cases.

3. It is possible to use a composite model to calculate the permeability $k_f = (k_p \times A_p + k_s \times A_s)/(A_p + A_s)$, where the permeability of the cement-mixed columns and of the soil are combined. However, when the ratio of replacement is large, there are obvious errors when this composite model is applied.

References

Broms, B., Boman, P., 1979. Lime columns-a new foundation method. *Journal of the Geotechnical Engineering Division*, ASCE, **4**: 539-555.

Carrillo, N., 1942. Simple two- and three- dimensional cases in the theory of consolidation of soil. *J. of Math. and Phys.*, **21**(1): 6-9.

Gong, X. N., 1992. Composite Foundation. Zhejiang University Press, Hangzhou.

Li R. H., Fen G. Z., 1995, Numerical Methods for Deferential Equations. Publishing Press of Advanced Education, Beijing.

Scott, R. F., 1963. Principles of Soil Mechanics. Addison-Wesley Publishing Press., New York.

Yoshikuni, H., Nakanodo, H., 1974. Consolidation of soils by vertical drain wells with finite permeability. *Soils and foundation*, **14**(2): 36-43.

Zeng, G. X., Lu Z. J., 1988. Ground Improvement Methods. Chinese Publishing Press of Civil Engng, Beijing.