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# Parametrically excited oscillation of stay cable and its control in cable-stayed bridges*

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**Abstract:** This paper presents a nonlinear dynamic model for simulation and analysis of a kind of parametrically excited vibration of stay cable caused by support motion in cable-stayed bridges. The sag, inclination angle of the stay cable are considered in the model, based on which, the oscillation mechanism and dynamic response characteristics of this kind of vibration are analyzed through numerical calculation. It is noted that parametrically excited oscillation of a stay cable with certain sag, inclination angle and initial static tension force may occur in cable-stayed bridges due to deck vibration under the condition that the natural frequency of a cable approaches to about half of the first model frequency of the bridge deck system. A new vibration control system installed on the cable anchorage is proposed as a possible damping system to suppress the cable parametric oscillation. The numerical calculation results showed that with the use of this damping system, the cable oscillation due to the vibration of the deck and/or towers will be considerably reduced.

**Key words:** Stay cable, Cable-stayed bridge, Parametric vibration, Vibration control, Support excitation
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## INTRODUCTION

Cable-stayed bridges are widely applied in modern bridge structural system. The largest span of cable-stayed bridge is close to 1000 m when many new materials and technologies are applied. The stay cables are important bridge structure components most prone to exhibit high amplitude oscillation, due to their great flexibility, small mass, and weak damping. Continued cable oscillations may be lead to fatigue and corrosion of strands in surprisingly short time and seriously decrease the safety and durability of the cable-stayed bridge. Therefore, reduction of the oscillation amplitude is considered by some researchers (Matsumeto et al., 1989a; 1989b) as a matter of vital importance for the acceptance of cable-stayed bridges in the future. The rain-wind induced cable vibrations have been observed in a number of cable-stayed bridges worldwide, and have been extensively studied in the past decades. In recent years, large amplitude cable oscillation caused by parametric excitation due to support (deck or tower) motions were observed in cable-stayed bridges (Pinto da Costa et al., 1996; Virlogeux, 1998; Yamaguchi et al., 1998). This kind of cable oscillation is produced by the deck and/or tower vibration induced either by traffic vehicles or by buffeting responses of gust wind and is referred to as parametric oscillation of the stay cables. The oscillation phenomena and control methods of bridge stay cable excited by the deck and/or tower motions are less studied than the rain-wind induced vibration. The parametric oscillation and dynamic stability of the stay cables subjected to support excitation were studied by Takahashi (1991) and Perkins (1992). Abdel-Ghaffar and Khalifa (1991) investigated in detail the interaction between the deck-dominated model and the cable local models. Fujino et al. (1993) and Warnitchai et al. (1995) studied theoretically and experimentally

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the internal resonance between the bridge global models and the cable local models involving linear and quadratic nonlinear couplings. Lilien and Pinto da Costa (1994), Pinto da Costa et al. (1996) and Takahashi et al. (1997) used a semi-discrete model for the studies of parametric oscillation of stay cables induced by periodic motions of the deck and/or towers. Kang and Zhong (1998) proposed a simple mathematical model MDOF for analyzing the parametric vibration of stay cable. Zhong and Peil (1999) used a spline finite element to study the parametric instability of stay cables under arbitrary dynamic excitation. The authors (2000) studied the parametric oscillation of the stay cables modeled by a simple vertical string. In the present work, the stay cables with inclination angle, initial sag and initial static tension force were studied deeply for parametric oscillation of the cable.

The research results revealed that when the vibration of the bridge deck or tower falls within certain frequency ranges, the stay cable oscillation caused by support motions can become unstable and exhibit large amplitude response. The most dangerous situations arise when the vibration frequency of the deck or tower is in the neighborhood of twice the first natural frequency of the stay cables. A new vibration controller installed on the cable anchorage is proposed as a possible control system to suppress the cable parametric oscillation. Numerical calculation results showed that with use of this vibration control system the cable oscillation due to the vibration of the deck/or towers of the cable-stayed bridge will be considerably reduced.

## NONLINEAR ANALYTICAL MODEL

Fig. 1 shows a bridge stay cable with its lower anchorage at the bridge deck. The stay cable is modeled as a nonlinear string with initial length $L$ and initial static tension force $T_0$. The cable also has initial sag and the initial static deflection is in parabolic shape with the curvature $\chi = mg\sin\gamma / T_0$; $m$ is mass per unit length of the cable, $g$ is gravity acceleration, $\gamma$ is the stayed cable's angle of inclination from the vertical axis. The cable has axial elastic stiffness $EA$, but the bending stiffness is omitted. The inclined stay cable is excited by a sinusoidal vertical motion with amplitude $X$ and the angular frequency $\Omega$ at the lower anchorage (point B of Fig 1.) on the bridge deck. Obviously, the nonlinear mechanical model represents the practical situation of the cable parametric oscillation excited by deck and/or tower of the cable-stayed bridge.

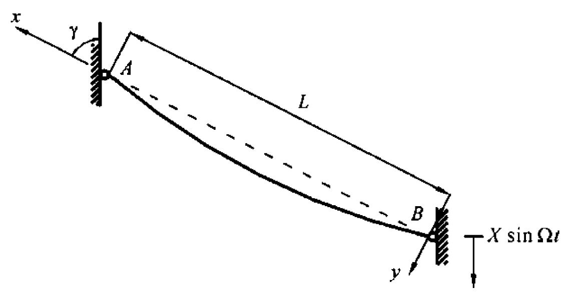


**Fig.1 An inclination stay cable excited by support motion**

The Hamilton principle was applied to establish the equation of the motion of the cable excited by support motion. If the axial inertia force is neglected, the kinetic energy of the stay cable can be expressed as

$$T(y) = \frac{1}{2}\int_0^L m\dot{y}^2 \mathrm{d}x \qquad (1)$$

and the potential energy of the cable (assuming that it is elastic and neglecting the effect of geometric non-linearity) can be written as

$$V(y) = \frac{1}{2}\left[T_0 + \frac{EA}{L}X\cos\gamma\sin\Omega t\right]\int_0^L (y')^2\mathrm{d}x + \frac{1}{2}\frac{EA}{2}\left[\int_0^L\left(\chi y - \frac{1}{2}(y')^2\right)\mathrm{d}\chi\right]^2 - \left[\chi\frac{EA}{L}X\cos\gamma\sin\Omega t\right]\int_0^L y\,\mathrm{d}\chi \qquad (2)$$

Where: $\dot{y}$ is partial derivative with respect to time $t$, i.e., $\frac{\partial y}{\partial t}$ and $y'$ is partial derivative with respect to $\chi$ axis, i.e. $\frac{\partial y}{\partial \chi}$. The Hamilton principle was used to derive the governing equation of the stay cable excited by support motion as

$$m\ddot{y} - \left[T_0 + \frac{EA}{L}X\cos\gamma\sin\Omega t\right]y'' + \frac{EA}{L}(y'' + \chi)\int_0^L\left(\chi y - \frac{1}{2}(y')^2\right)\mathrm{d}x = \chi\frac{EA}{L}X\cos\gamma\sin\Omega t \qquad (3)$$

The boundary conditions are

$$y(0,t) = X\sin\gamma\sin\Omega t, \quad y(L,t) = 0$$

If only the first vibration mode is considered, the solution of Eq. (3) can be expressed as

$$y(x,t) = (X\sin\gamma)\left(1 - \frac{x}{L}\right)\sin\Omega t + y_1(t)\sin\frac{\pi x}{L} \quad (4)$$

Because the bridge deck's vibration amplitude is relatively small, the first term on the right-hand side of the Eq. (4) is small also. Therefore, $y_1(t)$ dominates the amplitude of the first mode vibration of the stay cable. After some manipulations, a differential equation governing the motion of $y_1(t)$ with damping ratio $\xi$ can be obtained as

$$\ddot{y}_1 + 2\xi\dot{y}_1 + \omega_0^2\Big[1 + \frac{1}{2}\left(\frac{2}{\pi}\right)^4\frac{LEA\chi^2}{T_0} + \frac{XEA}{T_0}\left(\frac{1}{L}\cos\gamma - \frac{\chi}{2}\sin\gamma\right)\sin\Omega t + \frac{X^2EA}{2L^2T_0}\sin^2\gamma\sin^2\Omega t\Big]y_1 - \frac{3EA\chi\pi}{L^2m}y_1^2 + \frac{EA\pi^4}{4L^4m}y_1^3 = \frac{2X^2EA\chi}{L^2m\pi}\sin^2\gamma\sin^2\Omega t + \frac{4XEA\chi}{Lm\pi}\cos\gamma\sin\Omega t - \frac{2XEA\chi^2}{m\pi}\sin\gamma\sin\Omega t + \frac{2X\Omega^2}{\pi}\sin\gamma\sin\Omega t \quad (5)$$

Where $\omega_0$ is the natural frequency of the cable first vibration mode when not considering the effect of gravity on the cable (i.e. omitting the initial deflection), namely

$$\omega_0 = \frac{\pi}{L}\sqrt{\frac{T_0}{m}} \quad (6)$$

It is known that the term $\frac{XEA}{T_0}\left(\frac{1}{L}\cos\gamma - \frac{\chi}{2}\sin\gamma\right)\sin\Omega t$ in Eq. (5) represents the parametric vibration part of the dynamic response of the stay cable, while the term $\frac{X^2EA}{2L^2T_0}\sin^2\gamma\sin^2\Omega t$ represents the gravity function of the stay cable with initial deflection (parabolic shape). It is evident from the first two terms inside the brackets of Eq. (5) that the initial deflection of the stay cable affects the first mode natural frequency. The first mode natural frequency $\omega'_0$ of the stay cable with initial deflection is

$$\omega'_0 = \left[1 + \frac{1}{2}\left(\frac{2}{\pi}\right)^4\frac{LEA\chi^2}{T_0}\right]^{1/2}\omega_0 \quad (7)$$

According to Pinto da Casto et al. (1996), the initial deflection affects only the natural frequency of the first vibration mode, not the natural frequencies of second or higher vibration modes.

## PARAMETRIC AND FORCED VIBRATIONS

The parameter $\gamma$ (angle between the stay cable chord and vertical axis) significantly affects the vibration pattern of the stay cable. When $\gamma$ is equal to 90°, the stay cable becomes horizontal and its vibration excited by the deck vertical motion becomes a forced vibration without any parametric oscillation. When $\gamma$ equals to 0°, the initial deflection of the cable is zero and the excitation of the bridge deck acts along the stay cable axis. In this situation, the support-excited vibration of the cable becomes fully parametric oscillation without any forced vibration.

When only parametric oscillation of the stay cable occurs in the case of $\gamma = 0$ and $\chi = 0$, Eq. (5) reduces to

$$\ddot{u} + 2\mu\dot{u} + [1 + \varepsilon\sin\beta\tau + u^2]u = 0 \quad (8)$$

in which, $u = y_1/K$, $K = 2L^2/\pi^2 \cdot \sqrt{\frac{m}{EA}}$,

$$\dot{u} = \frac{\partial u}{\partial \tau}, \quad \tau = \omega_0 t,$$

$$\mu = \xi/\omega_0, \quad \varepsilon = XEA/T_0L, \quad \beta = \Omega/\omega_0 \quad (9)$$

By solving Eq. (8) in terms of the multiple scales method, the stable and unstable zones of the parametric oscillation can be obtained as shown in the Fig 2.

When only forced vibration of the stay cable occurs in the case of $\gamma = 90°$, Eq. (5) becomes

$$\ddot{y}_1 + 2\xi\dot{y}_1 + \omega_0^2\Big[1 + \frac{1}{2}\left(\frac{2}{\pi}\right)^4\frac{LEA\chi^2}{T_0} + \frac{XEA}{T_0}\left(-\frac{\chi}{2}\right)\sin\Omega t + \frac{X^2EA}{2L^2T_0}\sin^2\Omega t\Big]y_1 - \frac{3EA\chi\pi}{L^2m}y_1^2 + \frac{EA\pi^4}{4L^4m}y_1^3 = \frac{2X^2EA\chi}{L^2m\pi}\sin^2\Omega t - \frac{2XEA\chi^2}{m\pi}\sin\Omega t + \frac{2X\Omega^2}{\pi}\sin\Omega t \quad (10)$$

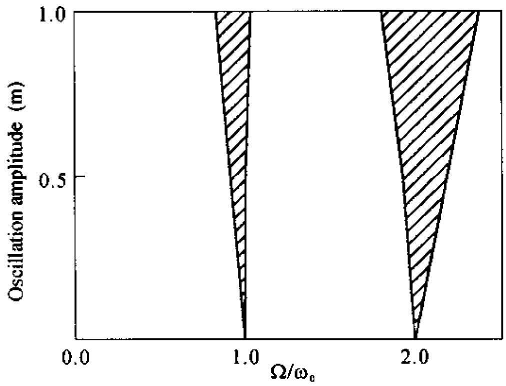


**Fig.2 Stable zone of cable parametric vibration without damping**

From the above analysis, an important conclusion can be obtained: when $0° < \gamma < 90°$, the parametric vibration of the first mode will exist in the first unstable region $\Omega/\omega_0 \approx 1$, and the parametric vibration will be coupled with the first mode forced vibration of the stay cable. Similarly, when parametric vibration of the first mode exists in the second unstable region $\Omega/\omega_0 \approx 2$, the parametric vibration will be coupled with the forced vibration of the second mode.

## NUMERICAL ANALYSIS OF CABLE PARAMETRIC OSCILATION

Numerical simulation studies were conducted to examine the influence of frequency matching between the deck motion and the cable first vibration mode, cable damping and cable static tension force on the amplitude of cable parametric oscillation. Previous studies revealed that when the vibration of the bridge deck falls in certain frequency ranges, the oscillation of stay cables caused by support (deck) motion can become unstable and exhibit large amplitude response. The most dangerous situations arise when the vibration frequency of the deck is in the neighborhood of twice the first natural frequency of the stay cables. This frequency-matching phenomenon is first checked here. As the stay cable's initial deflection (initial sag) was taken into account in this study, $\omega_0$ in Eq. (5) should be replaced by $\omega_0'$ for the cable response calculation.

In this example, the structural parameters are taken as: the mass per unit length of the cable $\rho A = 0.0113$ kg/m; the cable length $L = 12.0$ m; axial elastic stiffness of the cable $EA = 194000\ N$; static tension force $T_0 = 63\ N$; inclination angle $\gamma = 68.28°$; amplitude of the deck excitation motion $X = 0.5$ mm; and the cable damping is neglected. With these parameters, Eq.(5) is solved by numerical method for different exciting frequencies $\Omega$ of the deck motion. A set of cable oscillation amplitudes for different frequency ratios of the deck to cable are obtained as shown in Fig.3, in which the upper abscissa is the ratio of exciting frequency $\Omega$ to the first natural frequency $\omega_0$ of the cable without initial deflection, and the lower abscissa is the ratio of the exciting frequency $\Omega$ to the first natural frequency $\omega_0'$ of the cable with initial deflection.

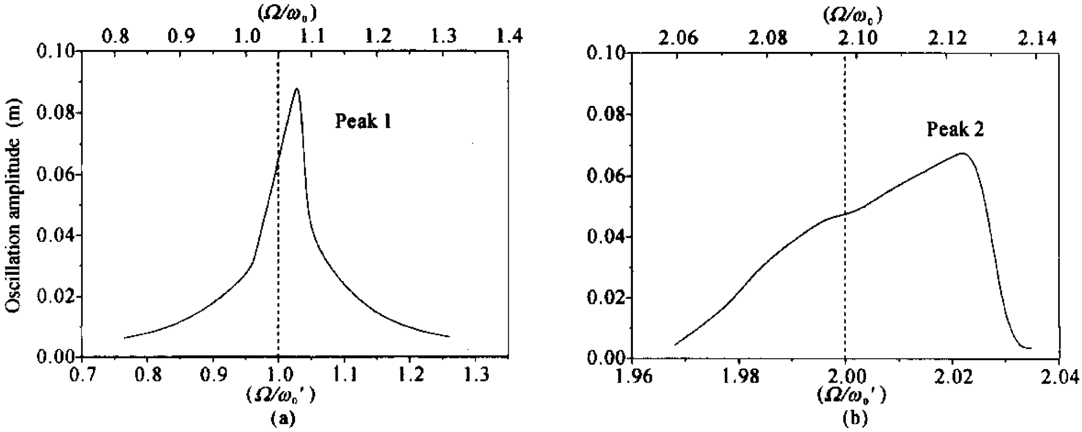


**Fig.3 Cable oscillation amplitude under different frequency ratios**

(a) for $\Omega/\omega_0' \approx 1$; (b) for $\Omega/\omega_0' \approx 2$

Fig.3 shows that the dynamic responses of the cable have two peaks, one at $\Omega/\omega_0' \approx 1$ (Fig.3a) and one at $\Omega/\omega_0' \approx 2$ (Fig.3b). However, the ratio is not exactly equal to 1 or 2 and becomes slightly higher than the corresponding $\Omega/\omega_0$ without considering initial deflection (Sun, 2000). This phenomenon is due to the increasingly non-linear property of the system governed by Eq. (5). Fig 4 shows the time history of the cable dynamic response when $\Omega/\omega_0 \approx 1.08$ within the first 40 seconds. It reveals "beating" characteristic for the cable parametric oscillation.

Fig. 3 shows that the frequency ratio range covering the peak 1 is larger than that covering the peak 2. Actually, the first mode forced vibration dominates the resonant response of the stay cable with $\gamma = 68.28°$ as in Fig.3. Considerably wide frequency ratio range for the peak 1 is caused by the frequency lock-in phenomenon of the first mode forced vibration. But the first mode forced vibration has only small effect on the second parametric resonance peak. Fig. 5 shows the vibration response amplitude of the cable under the same condition as above but with $\gamma = 90°$. In this case, the cable becomes a horizontal one. The excitation force alone causes the stay cable forced vibration. The results reveal that, in this case, the forced vibration has an important effect on the cable oscillation primary resonance, but no effect on the second resonance.

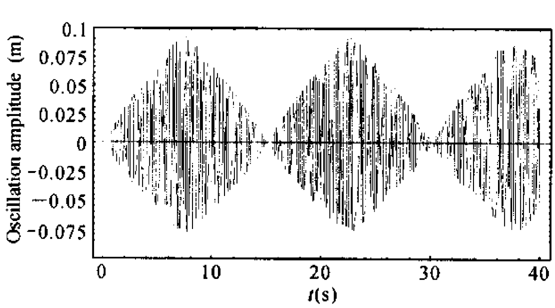


**Fig.4 Time history of cable dynamic response when $\Omega/\omega_0 = 1.08$**

Fig. 6 shows the vibration response amplitude of the cable with inclination angle $\gamma = 0°$. In this case, the cable becomes a vertical one. The jumping phenomenon of the cable oscillation is observed in Fig. 6. It is caused by the increasing non-linearity of Eq.(5). The numerical results also showed that in the vicinity of $\Omega/\omega_0' = 1$ the parametric vibration of the cable only occurs in a very narrow frequency ratio zone from $\Omega/\omega'_0 = 0.995$ to 1.0. The cable maximum response within this zone is about 0.02 m at $\Omega/\omega_0' \approx 1$. It should be noted that in this case ($\gamma = 0°$) $\Omega/\omega_0' = \Omega/\omega_0$. These numerical results accord well with the predicted stable zone depicted in Fig.2.

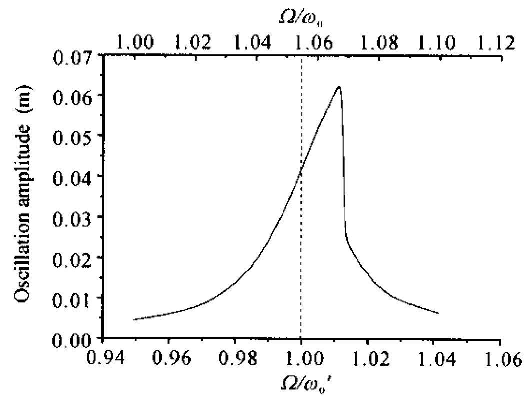


**Fig.5 Cable oscillation amplitude in the case of $\gamma = 90°$**

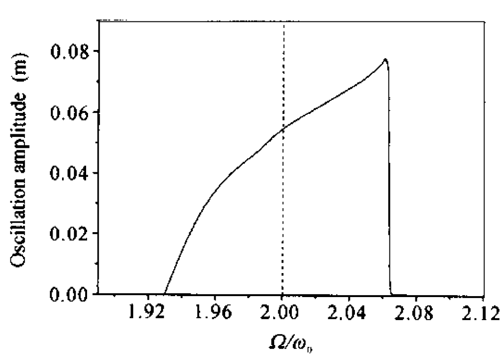


**Fig.6 Cable oscillation amplitude in the case of $\gamma = 0°$**

Fig.7 and Fig.8 show the dynamic response amplitude of the stay cable near the frequency ratio $\Omega/\omega' = 2$ when the static tension force $T_0$ is 50 N and 100 N respectively. Comparison of these figures with Fig.3b shows that with the increase of the cable static force, the unstable zone and maximum dynamic response of cable parametric oscillation reduce. Similar conclusions can be obtained based on the analysis of the definition of parameter $\varepsilon$ in Eq.(9). As a result, increasing the cable static tension force will help to suppress the dynamic response of the stay cable.

Fig.9 and Fig.10 illustrate the dynamic response time histories of the stay cable with inclination angle $\gamma = 68.28°$ when the damping ratio $\mu$ defined in Eq.(8) is 0.1% and 0.05% respectively. As shown in the figures, certain structural damping is conducive to reducing the cable dynamic response amplitudes. When the damping ratio reduces to a certain value, however, the cable will break through the damping effect and will exhibit large-amplitude oscillation, i.e. there is a critical damping ratio for effective suppression of the cable oscillation. Numerical simulations showed that a larger static tension force can lower this critical damping ratio, thereby suppressing the cable dynamic response more efficiently.

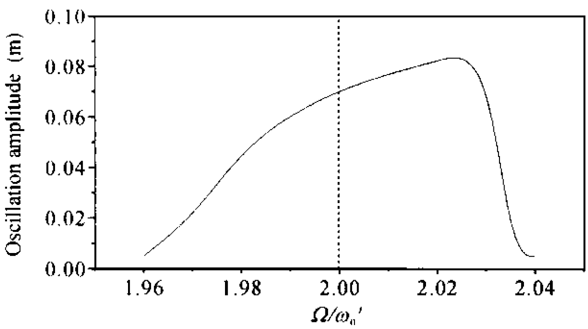


**Fig.7 Maximum dynamic response of cable when static tension force is 50 N**

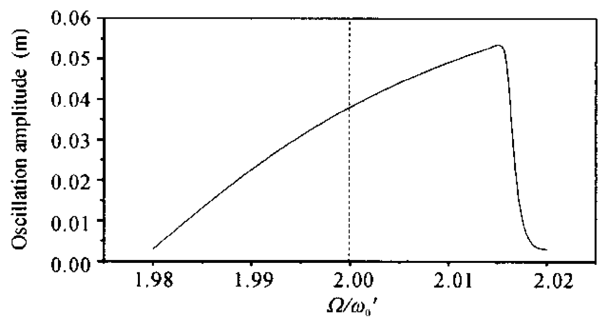


**Fig.8 Maximum dynamic response of cable when static tension force is 100 N**

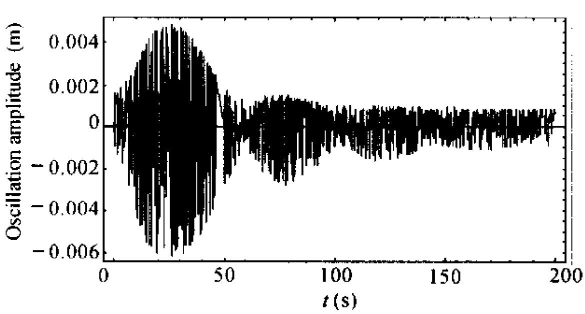


**Fig.9 Time history of cable dynamic response when $\mu = 0.1\%$ and $\Omega/\omega' = 2.025$**

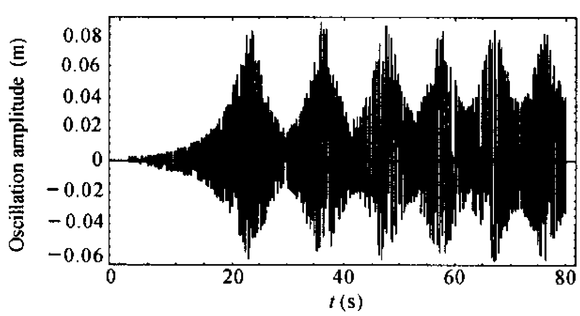


**Fig.10 Time history of cable dynamic response when $\mu = 0.05\%$ and $\Omega/\omega_0' = 2.025$**

## CONTROL OF CABLE PARAMETRIC OSCILLATION

The cable parametric oscillation can be controlled by a damping system installed on the cable or bridge deck. In recognizing that the cable parametric oscillation is induced by the anchorage (bridge deck) motion, we propose a cable oscillation suppression measure by installing a controller between the cable and bridge deck as shown in Fig 11. The controller is composed of springs and a mass with parameters $k_c$ and $m_c$. If the parameters are selected appropriately, the motion of controller will always keep in opposite direction with the vibration of the deck, and the parametric oscillation of the cable can be effectively reduced. This is easy to achieve by selecting

$$\Omega^2/\omega_c^2 = A, \quad A > 1 \quad \text{(it is determined by controller mass)} \tag{11}$$

in which $\Omega$ is the exciting angular frequency of the support (deck) motion and $\omega_c^2 = k_c/m_c$; the value of $A$ governs the vibration amplitude of the controller mass.

The proposed controller is obviously a passive mass-spring control system. Because the controller is installed on the cable anchorage, the controller motion depends on the vibration of bridge deck and does not directly affect the cable oscillation. The controller exerts an excitation force in opposition to the deck motion, and thereby suppresses the dynamic response amplitude of the cable. The positive effect of the controller can be easily understood by substituting $T_0$ in Eq.(5) with $(T_0 - T_c \sin\Omega t)$ (here $T_c$ is the amplitude of the force exerted by the controller). Fig. 12 shows the dynamic response time history of the cable when the inclination angle is $\gamma = 68.28°$ ($\Omega/\omega'_0 = 2.05$) and the control force amplitude is 5% of the static tension force. Since the control force is small, its influence on the cable natural frequency can be neglected. Comparison of Fig.12 with Fig.3b (the peak value equals to 2), shows that the dynamic response amplitude reduces from 0.07 m into 0.0025 m. It indicates that the proposed controller can efficiently suppress the cable parametric oscillation.

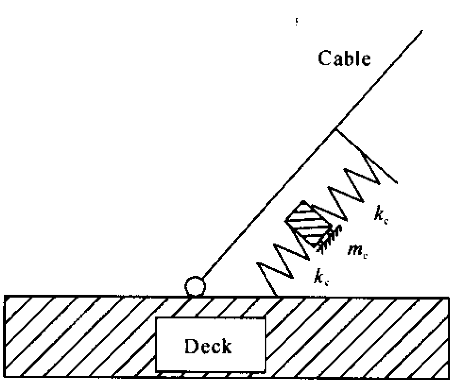


**Fig.11 Controller of the cable parametric oscillation**

Since the many stay cables in an actual stayed-bridge have different lengths, cross-sections, inclination angles and static tension forces, the natural frequencies of some cables might be in the frequency range of the lower modes of the bridge deck. In addition, a few cables may have frequency near half of the first mode frequency of the deck, and induce large amplitude parametric oscillation. Because many factors in fluence the parametric oscillation in a cable-stayed-bridge, selection of controller parameters

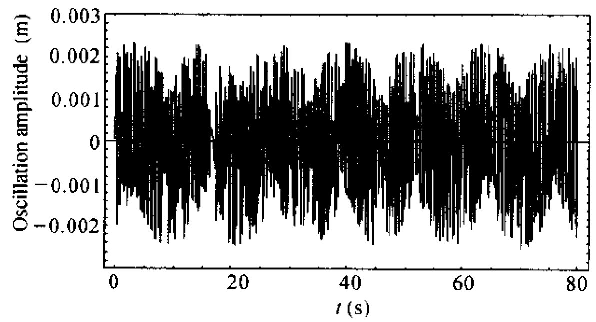


**Fig.12 Time history of dynamic response for the cable with controller**

and positions for suppressing the parametric oscillation is a challenging task and deserve further study.

## CONCLUSIONS

The following conclusions were drawn based on an analytical model proposed in this paper and the corresponding numerical conclusions are summarized as follows:

1. In cable-stayed bridge, if the ratio of natural frequencies of stay cable to bridge deck falls into certain range, serious large amplitude parametric oscillation of the cable may occur and have significantly adverse effect on the bridge safety and durability.

2. The parametric vibration and forced vibration of the stay cables are generally coupled. When the first mode parametric vibration occurs in the first unstable region with $\Omega/\omega_0 \approx 1$, first mode forced vibration will exist simultaneously. Similarly, when the first mode parametric vibration occurs in the second unstable region with $\Omega/\omega_0 \approx 2$, second mode forced vibration will occur simultaneously.

3. The cable parametric oscillation can exhibit obvious "beating" characteristic. The "beat" frequencies relate to the initial static tension force of the stay cable. The larger the static tension force is, the lower the "beat" frequency and the higher the cable vibration resistance. The matching frequency, which causes large amplitude cable parametric oscillation, also relates to the vibration amplitude of the bridge deck.

4. The increasing non-linearity of the cable dynamic motion equation accounting for the ini-

tial deflection (sag), initial static tension force and inclined angle will cause "beat" frequency ratios $\Omega/\omega_0'$ to shift to higher values in comparison with the straight string case. The first mode forced vibration dominates the primary parametric resonance of an inclined cable and has only little effect on the second parametric resonance. The cable static tension force will enlarge the unstable zone and the dynamic response in a certain range of the damping ratio. For a critical damping ratio, the cable will have large amplitude oscillation.

5. Installation of a controller beneath the cable anchorage can effectively suppress the deck-motion-induced cable parametric oscillation when appropriate controller parameters are selected. Even without damping in the control system, the controller can considerably reduce the cable response amplitude. When the control system contains damping as well, the dynamic responses of both the cable and the bridge deck can be significantly reduced and the "beat" phenomenon is weakened.

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