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Dynamic balancing of dual-rotor system with very little rotating speed difference*

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Abstract: Unbalanced vibration in dual-rotor rotating machinery was studied with numerical simulations and experiments. A new method is proposed to separate vibration signals of inner and outer rotors for a system with very little difference in rotating speeds. Magnitudes and phase values of unbalance defects can be obtained directly by sampling the vibration signal synchronized with reference signal. The balancing process is completed by the reciprocity influence coefficients of inner and outer rotors method. Results showed the advantage of such method for a dual-rotor system as compared with conventional balancing.

Key words: Field dynamic balancing, Dual-rotor system, Speed difference
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INTRODUCTION

In typical rotating machines, such as jet engines, gas turbines, lathes, centrifuges and other high speed rotating machines, one way to decrease vibration is to balance the unbalance defects of the rotors. Unbalance that causes synchronous vibrations is one of the most important defects in a dual-rotor system. The dynamic balancing technique can prevent machine noise, wear, and fatigue caused by the unbalanced shaking force and enable rotating machines to work safely and reliably. The trend of rotating machines toward increasingly high power requires powerful balancing techniques (Alauze et al., 2001; Lakatos et al., 1998; Xi, 1999).

The theory of field balancing of single-rotor rotating system is widely known. Conventional balancing methods are divided into modal methods and influence coefficient methods. According to measured method, balancing method can also be divided into single-plane, dual-plane and multi-plane dynamic balancing method (Mitty, 2000). The dynamic balancing method for dual-rotor system and the reciprocal influence of the rotors had been researched (Arakelian et al., 2000; Chakraborty, 2000), especially for a system with little rotating-speed difference, such as that of the dual-rotor screw type centrifuge (Fig.1), where the speed between its inner rotor and its outer rotor is only 7 rpm difference at 1500 rpm rotating speed. Dynamic balancing for such a system is very difficult. This paper presents the results of our recent research.

The composite signal of the two vibration signals forms a beat (Fig. 2). To obtain information on the magnitude and phase values of the unbalanced weight, we must separate the beat. The conventional method to separate the beat is based on cross-correlation theory wherein the beat signal is separable by a cross-correlation wave filter (He et al., 2002). If the frequency difference between two signals is large, the method will be effective. However, the less the difference is, the less effective is the method. Moreover, the amount of calculation would increase significantly for field dynamic balancing with microprocessor by using this method. Therefore, a new simple vibration signals separating method is proposed for dual-rotor system

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with very little difference in rotating speed.

BASIC PRINCIPLES

The modal balancing method based on a rotating structure model, determines experimentally the disturbing unbalance associated with a specific mode. The influence coefficient method uses an experimental model that represents the machine's sensitivity to unbalances. Here, the dual-plane influence coefficient method is adopted, and it is based on the assumption of system linearity between unbalances $\{M\}$ and the vibrations vector $\{X\}$ expressed as complex quantities

$$\{X\} = [R]\{M\} \qquad (1)$$

where $\{M\}$ is the unknown unbalance weight on each rectifying plane, $[R]$ is the sensitivity matrix of unbalance, $\{X\}$ is the vibration vector of the measured points at the bearing. The vibration of such system in two rectifying planes is measured as shown in Fig.1.

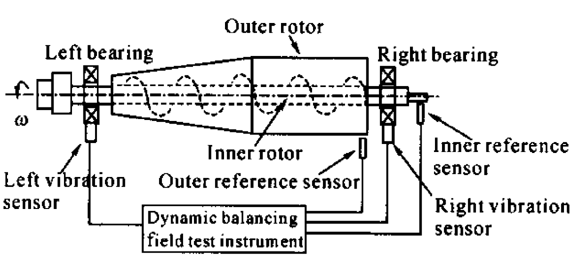

Fig.1 The measured system of dual-rotor in two rectifying planes

The coefficients are calculated after the addition of a test weight in the rectifying plane. For such two rotors and two rectifying planes, the matrix $[R]$ is square, the unbalance correction vector $\{M\}$ is determined from Eq.(1) by inverting the matrix

$$\{M\} = -[R]^{-1}\{X\} \qquad (2)$$

Obviously the key of such a balancing problem is how to obtain information on the magnitude and phase values of the unbalanced weight by the composite signals of measured vibration. According to the vibration theory of rotating machine, let the composite signals be expressed as

$$\boldsymbol{X} = \boldsymbol{X}_1 + \boldsymbol{X}_2 = A_1 e^{(\omega_1 t + \phi_1)i} + A_2 e^{(\omega_2 t + \phi_2)i} \qquad (3)$$

where $\boldsymbol{X}$ is the vector of the composite signal, $\boldsymbol{X}_1$ is the signal of the outer rotor, $\boldsymbol{X}_2$ is the signal of the inner rotor, A_1 and A_2 are the magnitudes of the rotor vibration, which can be the values of displacements, velocity or acceleration, respectively. ω_1 and ω_2 are the angular frequencies of the rotors, ϕ_1 and ϕ_2 are the phase of the rotors, respectively. The composite signal of the left reference pulse and the right reference pulse of such a measured system are shown in Fig.2.

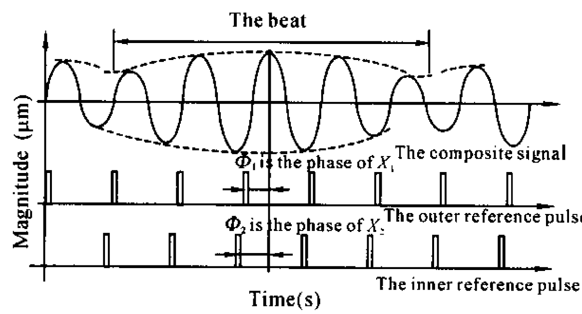

Fig.2 The composite signal and the reference pulse of the dual-rotor

As shown in Fig. 2, the composite signal of the two vibration signals forms a "beat". If such two signals have a large difference in frequency, the "beat" signal can be separated by a cross-correlation wave filter, but the method cannot separate two signals with very little frequency difference. For example, for the vibration of the dual-rotor screw type centrifuge in Fig. 1, its speed is 1500 rpm, the speed difference between its inner rotor and its outer rotor is about 7 rpm. For Eq.(3), it means that the frequency of the rotors are, f_1 = 25Hz and f_2 = 24.883Hz, respectively. Obviously, there is little difference between such two signals in the cross-correlation functions. So the values of the magnitude and phase obtained by cross-correlation wave filter method are not accurate enough to be used in dynamic balancing.

THE NEW SEPARATING METHOD

Here, the new separating method is based on the idea of relative coordinate. Just like a man who stands on the earth and watches the rotating track of the moon, we can also use the synchronal sample with one of the two rotors and find the vibration track of another rotor. The results will include the magnitude and phase of each rotor's track.

For example, in one rotating period, we sample the composite signal once it is synchronized with the outer rotor reference pulse. Eq. (3) can then be expressed as

$$\begin{aligned} \boldsymbol{X}^*(k) &= \boldsymbol{X}(t = 2k\pi/\omega_1) \\ &= A_1 e^{(2k\pi + \phi_1)i} + A_2 e^{(2k\pi\omega_2/\omega_1 + \phi_2)i} \\ &= A_1 e^{\phi_1 i} + A_2 e^{(2k\pi\omega_2/\omega_1 + \phi_2)i} \end{aligned} \quad (4)$$

where $k = 0, 1, 2, \cdots, n$, and n is the total rotating times.

The result of the Eq. (4) is shown in Fig. 3. Note that $A_1 e^{\phi_1 i}$ is a constant, then the wave of the composite signal is only related to the inner rotor, i.e. $A_2 e^{(2k\pi\omega_2/\omega_1 + \phi_2)i}$, especially the phase of the composite signal is ϕ_2, same as that of the inner rotor.

Furthermore, the max. module of $|\boldsymbol{X}^*(k)|$ is

$$|\boldsymbol{X}^*|_{\max} = A_1' |e^{\phi_1 i}| + A_2 \quad (5)$$

where $2k\pi\omega_2/\omega_1 + \phi_2 = 2m\pi$, $m = 0, 1, 2, \cdots, n$. Note that A_1' is the constant of $A_1 e^{\phi_1 i}$ at the sample. Similarly, the min. module of $|\boldsymbol{X}^*(k)|$ is

$$|\boldsymbol{X}^*|_{\min} = A_1' |e^{\phi_1 i}| - A_2 \quad (6)$$

where $2k\pi\omega_2/\omega_1 + \phi_2 = (2m+1)\pi$, $m = 0, 1, 2, \cdots, n$.

By solving the equation group of Eq. (5) and Eq. (6), we can obtain

$$A_2 = (|\boldsymbol{X}^*|_{\max} - |\boldsymbol{X}^*|_{\min})/2 \quad (7)$$

Notice that we can also use the same sampling method for the inner reference signals. Obviously, as shown in Fig. 3a, the difference between the max. position of the "beat" and the inner reference signal is the unbalanced weight phase of the inner rotor, i.e. ϕ_2.

Next, we sample the composite signal synchronized with the inner rotor reference. We can obtain

$$A_1 = (|\boldsymbol{X}'^*|_{\max} - |\boldsymbol{X}'^*|_{\min})/2 \quad (8)$$

Similarly, the difference between the max. position of the "beat" and the outer reference signal is the unbalanced weight phase of the outer rotor, i.e. ϕ_1, shown in Fig. 3b.

Finally, we can see that the new method is much simpler and requires less sampling (i.e. faster). As shown in Fig. 3, only 213 times of sampling are needed for a repetition. Especially with enough sampling, the less the difference between two speeds, the higher the precision will be.

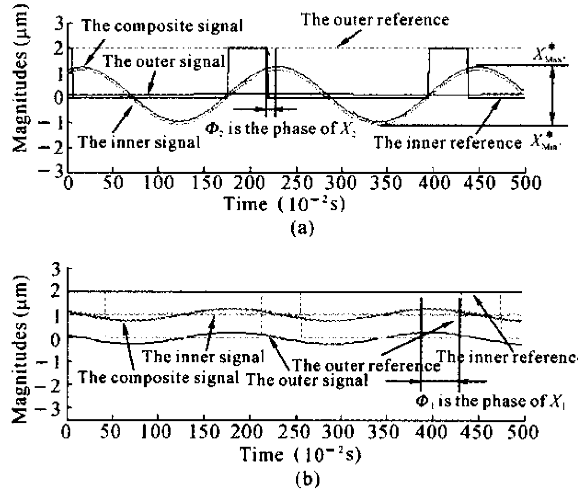

Fig.3 The separated signals by synchronal sampling of the inner and outer rotor, respectively.
(a) the times of the outer synchronal sampling; (b) the times of the inner synchronal sampling

DYNAMIC BALANCING

After information on the magnitudes and phase values of the unbalanced weight are obtained from the composite signals of the measured vibration, the dual-rotor system can be dynamically balanced. The sensitivity matrix can be constructed a column at a time by measuring the influence of a trial weight applied to one balancing plane on the vibration measured at each of the points of measurement. (Li et al., 2000; Wang and Zheng, 1996) The two rectifying planes method is presented below. Let M_R and M_L be the unbalancing values of the right and left plane respectively, ϕ_R and ϕ_L be their phase angles.

For simplification, the original vibrations of two planes are expressed as $\boldsymbol{X}_1 = A_1 \angle \phi_1$ and $\boldsymbol{X}_2 = A_2 \angle \phi_2$, respectively.

After addition of the trial weight $\boldsymbol{m}_L = m_L \angle \theta_L$ to the left plane, the measured vibrations of the two planes become $\boldsymbol{X}_{1L} = A_{1L} \angle \phi_{1L}$ and $\boldsymbol{X}_{2L} = A_{2L} \angle \phi_{2L}$, respectively.

Removing the left trial weight $\boldsymbol{m}_L = m_L \angle \theta_L$, and after addition of the trial weight $\boldsymbol{m}_R = m_R \angle \theta_R$ to the right plane, the measured vibrations of the two planes become $\boldsymbol{X}_{1R} = A_{1R} \angle \phi_{1R}$ and $\boldsymbol{X}_{2R} = A_{2R} \angle \phi_{2R}$, respectively.

Then in Eq. (1) and Eq. (2), matrix R is

$$[R] = \begin{bmatrix} R_{1L} & R_{1R} \\ R_{2L} & R_{2R} \end{bmatrix} \tag{9}$$

Thus, the Eq. (1) in each step above can also be expressed

$$\begin{bmatrix} R_{1L} & R_{1R} \\ R_{2L} & R_{2R} \end{bmatrix} \begin{Bmatrix} M_L \\ M_R \end{Bmatrix} = \begin{Bmatrix} X_L \\ X_R \end{Bmatrix} \tag{10a}$$

$$\begin{bmatrix} R_{1L} & R_{1R} \\ R_{2L} & R_{2R} \end{bmatrix} \begin{Bmatrix} M_L + m_L \\ M_R \end{Bmatrix} = \begin{Bmatrix} X_{1L} \\ X_{2L} \end{Bmatrix} \tag{10b}$$

$$\begin{bmatrix} R_{1L} & R_{1R} \\ R_{2L} & R_{2R} \end{bmatrix} \begin{Bmatrix} M_L \\ M_R + m_R \end{Bmatrix} = \begin{Bmatrix} X_{1R} \\ X_{2R} \end{Bmatrix} \tag{10c}$$

respectively. Where, M_L and M_R are the unbalancing weights.

Solution of Eqs. (10a, b, c) yields R_{1L} of $[R]$ as

$$R_{1L} = \frac{X_{1L} - X_1}{m_L} \tag{11}$$

Note that R_{1R}, R_{2L}, R_{2R} are calculated in the same way as R_{1L} above, and that all of them are vectors with magnitude and direction, but are represented by complex numbers.

Finally, the unbalancing weights of the left and the right plane are

$$\begin{cases} M_L = \dfrac{X_1 R_{2R} - X_2 R_{1R}}{R_{1L}R_{2R} - R_{2L}R_{1R}} \\ M_R = \dfrac{X_2 R_{1L} - X_1 R_{2L}}{R_{1L}R_{2R} - R_{2L}R_{1R}} \end{cases} \tag{12}$$

Furthermore, with the mutual influence of the inner and outer rotors ignored, we should calculate the unbalance of outer rotor and that of inner rotor respectively, and the dimension of each matrix $[R]$ is 2×2. Here, with the mutual influence of inner and outer rotors considered, we can get the unbalance of the outer rotor and that of the inner rotor at one time, but the dimension of the matrix $[R]$ is 4×4.

NUMERICAL EXAMPLE AND EXPERIMENTS

1. Using the new proposed method, the determined values from the example shown in Fig.1 are $A_1 = 0.2500\ \mu m$, $A_2 = 1.100\ \mu m$, $\phi_1 = 55.00°$ and $\phi_2 = 18.00°$. The composite signal obtained by sampling with 1kHz is

$$\boldsymbol{X} = \boldsymbol{X}_1 + \boldsymbol{X}_2 = 0.25e^{\left(\frac{1500}{60 \times 1000} \times 2\pi \times t + 55 \times \pi/180\right) i} + 1.1e^{\left(\frac{1493}{60 \times 1000} \times 2\pi \times t + 18 \times \pi/180\right) i} \tag{13}$$

The numerical simulation results are: $A_1 = 0.2460\ \mu m$, $A_2 = 1.1057\ \mu m$, $\phi_1 = 54.92°$. and $\phi_2 = 18.03°$, etc.

Fig.4 shows the results of numerical simulation of the magnitude values versus speed difference for fixed number of samples.

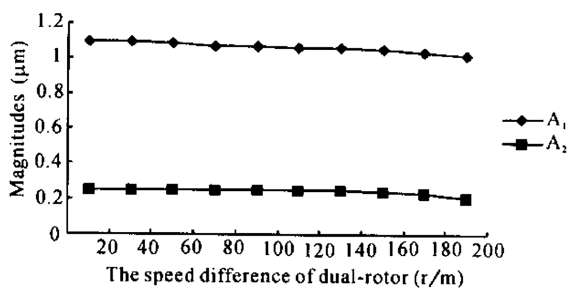

Fig.4 The determined precision with the different speed difference at 250 samples.

These numerical results indicate that the new method is effective and highly precise for double compounded vibration signals with little frequency difference. Obviously the number of samples needed is less than that of all conventional methods. This advantage enables it to be employed in instruments with microprocessor. Fig. 4 shows that the method is especially suitable for very little speed difference. If enough samples are used, the less the difference is, the higher the precision will be.

2. Table 1 and Table 2 give the dynamic balancing experiment results obtained for the dual-rotor screw type centrifuge by the dynamic balancing method. The vibration sensor was a CD-1 velocity sensor.

Comparison of the results listed in above two tables shows that for the outer rotor, the original measured vibration of 23.52 μm at the left plane reduced to 1.83 μm after balancing. The original measured vibration of 55.12 μm at the right plane reduced to 4.46 μm after balancing. The ratio of the reduction was 92.2% and 91.9%, respectively. On the other hand, for the inner rotor, the original measured vibration of 31.15 μm at the left plane reduced to 4.84 μm after balancing. The original measured vibration of 16.22 μm at the right plane reduced to 2.13 μm after balancing. The ratio of the reduction was 84.5% and 86.9%, respectively. We can see that the dynamic balancing method is effective.

Table 1 The balancing result of outer rotor

Balancing process	Left plane		Right plane	
	Magnitude(μm)	Phase(°)	Magnitude(μm)	Phase(°)
Original vibration	23.52	204	55.12	18
Left trial vibration	29.02	231	83.24	48
Right trial vibration	38.30	124	57.32	27
Unbalancing weight	65.5(g)	112	19.9(g)	66
Vibration after balancing	1.83	304	4.46	354

Table 2 The balancing result of inner rotor

Balancing process	Left plane		Right plane	
	Magnitude(μm)	Phase(°)	Magnitude(μm)	Phase(°)
Original vibration	31.15	177	16.22	154
Left trial vibration	29.02	178	15.13	149
Right trial vibration	32.30	182	17.42	160
Unbalancing weight	43.3(g)	72	22.2(g)	91
Vibration after balancing	4.84	176	2.13	152

CONCLUSIONS

A new vibration signals separating method is proposed for dynamic balancing of dual-rotor system with very little difference in rotating speed. Sampling the vibration signals synchronized with the reference, can yield information on the magnitudes and phase values of unbalanced weights. The method does not need to separate by wave filter the composite "beat" signals with very little rotating speed difference. It avoids the difficulty of separating the "beat". Results showed, the new proposed method is easier and faster than other balancing methods, especially for field dynamic balancing of the overall machine system.

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