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Intersections of two offset parametric surfaces based on topology analysis*

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Abstract: Conventional methods for solving intersections between two offset parametric surfaces often include iteratively using computationally expensive SSI (surface/surface intersections) algorithm. In addition, these methods ignore the relations between the intersection curves of parametric surfaces with different offset distances. The algorithm presented in this paper, makes full use of the topological relations between different intersection loops and calculates intersection loops with the help of previously calculated intersection loops. It first pre-processes two parametric surfaces to obtain the characteristic points, called topology transition points (TTPs), which can help in the subsequent finding of the topologies of the intersection curves. Then these points are categorized into several distinct groups, and we can determine the calculation strategy for searching initial points by analyzing the properties of these TTPs on the surfaces. Hence, all intersection curves can be marched from initial points by the tracing algorithm. The proposed algorithm could calculate intersection curves robustly and effectively, and has been tested to be capable of overcoming the degenerate conditions such as loop and singularities leaking that occur frequently in conventional algorithms.

Key words: Offset parametric surface, Topology transition point, Surface intersection
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INTRODUCTION

The OSSI (Offset Surface/Surface Intersection) problem is a sub-problem of general SSI. Although the OSSI problem can be resolved by applying general SSI algorithm, it is computationally expensive and exhaustive. In recent decades, efforts have been focused on computing SSI exactly, efficiently, and robustly in a general setting (Abdel-Malek and Yeh, 1997; Chang et al., 1994; Burke and Sabharwal, 1996; Mullenheim, 1991). These techniques can be classified mainly into five categories: algebraic, lattice evaluation, marching (tracing) method, implicit function method and recursive subdivision. Yu (1996) adopted a normal vector projection method, which could directly calculate intersection loops in the original surface parametric domain, without using any approximation method to solve the problem. Beginning with

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a series of initial intersection points, it used the tracing algorithm to calculate the intersect loops. However, his paper did not specify how to determine the initial intersection points and the topologic structure of the intersection curves. The loop detection method (Ma and Lee, 1998; Hohmeyer, 1992) received wide attention in the surface intersection domain as it can improve the efficiency and accuracy of tracing-based algorithm and ensure that all intersection curves and singularities can be found, and consequently has become a power tool for tracing-based SSI. Nevertheless, the algorithm cannot guarantee that the correct intersect curve topology is obtained, as the resulting topology is dependent on the tracing strategy employed. Hu et al.(2000) introduced a moving affine frame method for intersection calculation of parametric curves and surfaces. The numeric stability and efficiency of the method are better than traditional methods.

Jun et al.(2001) and Tait et al.(2002) discussed the relations between the TTPs and the topologies of intersection curves. Because the offset parametric surface is evolved from the original surface (Kulkarni and Dutta, 1995; Nackman, 1984), there should exist similarities between the intersection curves of surface with different offset distances. Our analysis of the characteristics of two offset parametric surfaces revealed the relations between the TTPs and the topologies of intersection curves of offset surfaces; and on the basis of the relations we proposed an optimized algorithm for OSSI. The algorithm is stated briefly as follows. To begin with, it pre-processes two initial surfaces to obtain the TTPs, which are then categorized into several groups by analyzing their properties. Secondly, according to the distribution of these TTPs, the initial point search strategy and intersection curve topology can be determined. Thus, the whole intersection loops can be marched by tracing algorithm.

Consider the case when two offset surfaces intersect. If the offset distance(s) of one(two) surfaces change, the shapes and topologies of the intersection curves will change. The topology changes include the emergence of new loops, the removal of existing loops, the merger of two open curves to form a closed loop, and so on. An example of the TTP is shown in Fig.1, in which $\boldsymbol{S}_1^0$ is the offset surface of $\boldsymbol{S}_1$, and the pair of points, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, is a TTP. When the distance between $\boldsymbol{S}_1$ and $\boldsymbol{S}_1^0$ increases, the intersection loops of $\boldsymbol{S}_1^0$ and $\boldsymbol{S}_2$ appear around $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$. According to the distance between $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$, we can quickly determine whether or not there exist intersection loops around $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$.

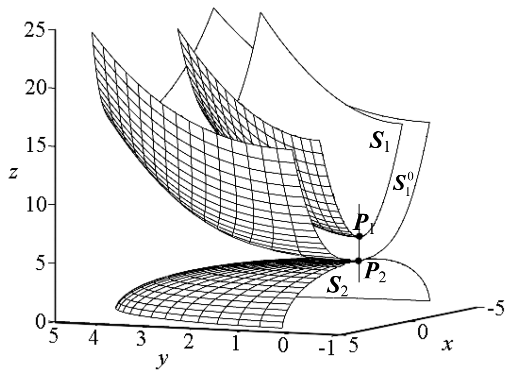

Fig.1 An example of TTP

The whole algorithm proposed consists of the following steps:

1. Calculate all the TTPs on two original surfaces and sort them in an ascending order with respect to the distances between them.

2. By analyzing the offset distances, the original surfaces and the distance of TTPs, we get the distribution of intersection curves, the dynamic topology of intersect curves, and thus determine searching strategy for the initial intersection points.

3. Trace out all intersection curves from the initial points.

This paper mainly focuses on the first and second steps, and the third step adopts the algorithms given in Tang et al.(1999), Tang and Dong (2000), and Wu and Andrade (1999). We discuss the necessary and sufficient conditions of TTPs and analyze their effects on the topologies of the intersection curves in Section 2. Section 3 details how to find the initial intersection points by topology similarity. The last two sections, Section 4 and 5, discuss implementation of the algorithm with some examples, draw some conclusions, and touch on the direction of our future work.

PROPERTIES AND DETECTION OF TTPS

We assume all parametric surfaces discussed in this paper have tangent plane continuity (G^1).

Let $\boldsymbol{G}(u, v)$ and $\boldsymbol{F}(s, t)$, where $(u, v)\in[0, 1]\times[0, 1]$ and $(s, t)\in[0, 1]\times[0, 1]$, are the original surfaces of offset parametric surfaces $\boldsymbol{G}^0(u, v)$ and $\boldsymbol{F}^0(s, t)$, respectively, then

$$\boldsymbol{G}^0(u,v)=\boldsymbol{G}(u,v)+l*\boldsymbol{m}(u,v)$$
$$\boldsymbol{F}^0(s,t)=\boldsymbol{F}(s,t)+h*\boldsymbol{n}(s,t)$$

where l and h are the offset distances of $\boldsymbol{G}^0(u, v)$ and $\boldsymbol{F}^0(s, t)$ respectively.

Suppose $\boldsymbol{G}_u(u, v)\times\boldsymbol{G}_v(u, v)\neq 0$ and $\boldsymbol{F}_s(s, t)\times\boldsymbol{F}_t(s, t)\neq 0$ everywhere. The unit normal vectors of the original surfaces are

$$\boldsymbol{m}(u,v)=\frac{\boldsymbol{G}_u(u,v)\times\boldsymbol{G}_v(u,v)}{||\boldsymbol{G}_u(u,v)\times\boldsymbol{G}_v(u,v)||},$$
$$\boldsymbol{n}(s,t)=\frac{\boldsymbol{F}_s(s,t)\times\boldsymbol{F}_t(s,t)}{||\boldsymbol{F}_s(s,t)\times\boldsymbol{F}_t(s,t)||}.$$

Definition 1 If $\boldsymbol{P}(\boldsymbol{G}(u_0,v_0))$, $\boldsymbol{Q}(\boldsymbol{F}(s_0,t_0))$ are two points on surface $\boldsymbol{G}(u, v)$ and $\boldsymbol{F}(s, t)$, respectively, then define the directed distance between $\boldsymbol{P}$ and $\boldsymbol{Q}$ by

$$d_{\mathrm{qp}}(\boldsymbol{P},\boldsymbol{Q})=\mathrm{sign}(\boldsymbol{m}(u_0,v_0)\bullet\boldsymbol{QP})*||\boldsymbol{QP}||,$$

where $\bullet$ and $*$ are the cross products between two vectors and the multiplication between two real numbers, respectively.

Definition 2 If $\boldsymbol{G}(u_0,v_0)$, $\boldsymbol{F}(s_0,t_0)$ are two points on $\boldsymbol{G}(u, v)$ and $\boldsymbol{F}(s, t)$, respectively, such that $\boldsymbol{G}^0(u_0,v_0)$ and $\boldsymbol{F}^0(s_0,t_0)$ are tangent at (u_0, v_0) and (s_0, t_0), $\{\boldsymbol{G}(u_0,v_0), \boldsymbol{F}(s_0,t_0)\}$ is referred to as a TTP.

According to the TTP's location, it can be categorized into two groups: the Interior-Interior TTP, (I-I), that is inside both surface, and Interior-Boundary TTP, (I-B), that is at least on one surface boundary.

Interior-Interior TTP (I-I)

Theorem 1 $\{\boldsymbol{G}(u_0, v_0), \boldsymbol{F}(s_0, t_0)\}$ is an I-I TTP if and only if $\boldsymbol{m}(u_0, v_0)$ is bi-parallel to $\boldsymbol{n}(s_0, t_0)$, and $\boldsymbol{G}(u_0,v_0)\boldsymbol{F}(s_0,t_0)$ is parallel to $\boldsymbol{n}(s_0, t_0)$.

Proof This is immediately apparent from Definition 2.

Given a pair of points, $\{\boldsymbol{G}(u_0, v_0), \boldsymbol{F}(s_0, t_0)\}$; suppose that $\boldsymbol{G}^0(u, v)$ and $\boldsymbol{F}^0(s, t)$ are tangent at (u_0, v_0) and (s_0, t_0), and the corresponding offset distances are l_0 and h_0, respectively. According to Theorem 1, the following equation

$$\boldsymbol{G}^0(u_0,v_0)=\boldsymbol{F}^0(s_0,t_0) \qquad (1)$$

holds and there should exist l_0 and h_0 satisfying the relation

$$d_{\mathrm{qp}}(\boldsymbol{G}(u_0,v_0),\boldsymbol{F}(s_0,t_0))\,\boldsymbol{m}(u_0,v_0)+l_0\boldsymbol{m}(u_0,v_0)=h_0\boldsymbol{n}(s_0,t_0). \qquad (2)$$

By modifying the equation above, we get

$$\begin{aligned}\delta&=d_{\mathrm{qp}}(\boldsymbol{G}(u_0,v_0),\boldsymbol{F}(s_0,t_0))\\&\quad+l_0-h_0\mathrm{sign}(\boldsymbol{m}(u_0,v_0)\bullet\boldsymbol{n}(s_0,t_0))=0\end{aligned} \qquad (3)$$

According to Eq.(3), a TTP can be classified into two main groups:

1. The unit normal vectors of a TTP are the same vectors.
2. The unit normal vectors of a TTP are reverse but parallel vectors.

The first group of TTP mentioned above is shown in Fig.2. In this case, Eq.(3) can be predigested to an abbreviated form as follows:

$$\delta=d_{\mathrm{qp}}(\boldsymbol{G}(u_0,v_0),\boldsymbol{F}(s_0,t_0))-h_0+l_0=0 \qquad (4)$$

The second group of TTP is shown in Fig.3, and Eq.(3) can be predigested to:

$$\delta = d_{qp}(\boldsymbol{G}(u_0,v_0),\boldsymbol{F}(s_0,t_0)) - h_0 - l_0 = 0 \qquad (5)$$

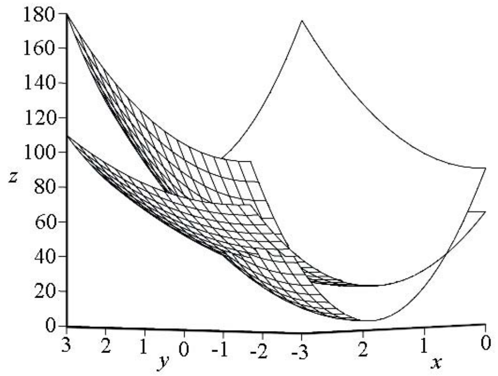

Fig.2 Unit normal vectors of a TTP are the same

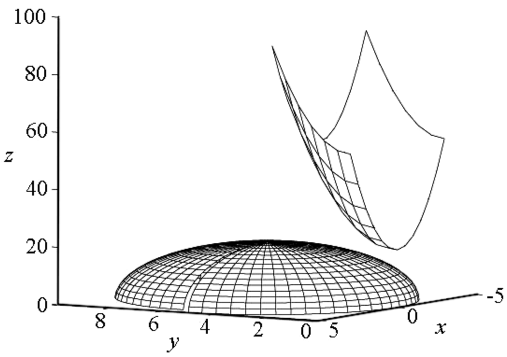

Fig.3 Unit normal vectors of TTP are reverse but parallel to each other

The relation between the TTP and the topology of the intersection curve is affected by the shape of the surface in the neighboring region of the TTP. Su et al.(1979) specified that the shape of the surface in the vicinity of the point $\boldsymbol{P}(\boldsymbol{G}(u_0, v_0))$ is characterized by its total curvature K and that the surface can be regarded as a quadric surface. Here, $K=Gk_u(u, v)*Gk_v(u, v)$, and $Gk_u(u, v)$, $Gk_v(u, v)$ are the principal curvatures, respectively. The total curvature K categorizes the points on surface in the vicinity of P into three groups: Elliptic Points: $K>0$; Hyperbolic Points: $K<0$; Parabolic Points: $K=0$.

Table 1 sums up the effect of the directed distance and the total curvatures on the topology of the intersection curve.

Interior-Boundary TTP (I-B)

As $\boldsymbol{G}(u, v)$ and $\boldsymbol{F}(s, t)$ is symmetrical, we only discuss the TTPs that locate on the boundary of $\boldsymbol{G}(u, v)$ and the interior of $\boldsymbol{F}(s, t)$. The conclusions obtained also hold for the TTPs that are on the boundary of $\boldsymbol{F}(s, t)$ and the interior of $\boldsymbol{G}(u, v)$.

Let us investigate those points on the boundary, which meet the conditions of Theorem 1. Obviously, these points belong to the I-B TTP set, denoted as I-B TTP1. Table 2 shows the effect of the TTP on the topology of the intersection curve.

Let $\boldsymbol{b}_g(t)$ be the boundary curve of $\boldsymbol{G}(u, v)$, and $\boldsymbol{b}_g(t)$ the differential vector of $\boldsymbol{b}_g(t)$. According to Definition 1, the I-B TTPs can be identified by the following equation:

$$\boldsymbol{b}_{gt} \bullet \boldsymbol{n}(s,t) = 0 \qquad (6)$$

Here Eq.(6) is a sufficient condition for $\{\boldsymbol{G}(u_0, v_0), \boldsymbol{F}(s_0, t_0)\}$ to be a I-B TTP. Those points that satisfy the requirement of Eq.(6) can be categorized

Table 1 Relations between the I-I TTP and the topology of the intersection curve[a]

δ	Total curvature		Intersections curve of approximation shape of surface around the TTP	The variety trend of the intersection curves	Kind of TTP
$\delta>0$	$GK\geq0$	$FK\geq0$		Add an intersect loop	I-I_1
	$GK\geq0$	$FK<0$	Add an intersect loop	Add an intersect loop	I-I_2
	$GK<0$	$FK<0$	Two intersection regions turn into other two intersect regions	Two intersection regions turn into other two intersect regions[b]	I-I_3
$\delta<0$				No effect	

[a] GK, FK are the total curvatures of $\boldsymbol{G}(u, v)$, $\boldsymbol{F}(s, t)$ respectively;
[b] The variety trend of intersection curves summed in Table 1 occurred only in the TTP's neighboring regions

Table 2 Relations between the I-B TTP_1 and the topology of the intersection curve

δ	Total curvature		Intersection curves of approximation shape of surface around the TTP	Tangent line's continuity of approximation shape's intersection curves	The variety trend of intersection curves	Kind of TTP
$\delta>0$	$GK\geq 0$	$FK\geq 0$			Add an open loop	I-B_1
	$GK\geq 0$	$FK<0$	Add an intersect loop		Add an open loop	I-B_2
	$GK<0$	$FK<0$	Two intersection regions turn into other two intersect regions	Continuous	One intersect region turns into two intersect regions	I-B_3
				Discontinuous	Two intersect regions turns into one intersect region	I-B_4
$\delta<0$					No effect	

into different groups with respect to their normal vectors. According to the different relations of the normal vectors, we classify the boundary point as follow:

1. $\boldsymbol{m}(u_0,v_0)$ is parallel to $\boldsymbol{n}(s_0,t_0)$

In this case, if $\boldsymbol{G}(u_0, v_0)\boldsymbol{F}(s_0, t_0)$ is parallel to $\boldsymbol{n}(s_0, t_0)$, the pair of points $\{\boldsymbol{G}(u_0, v_0), \boldsymbol{F}(s_0, t_0)\}$ is a I-B TTP, and it belongs to I-B TTP_1 discussed above; otherwise, $\{\boldsymbol{G}(u_0, v_0), \boldsymbol{F}(s_0, t_0)\}$ is not a TTP, so there is no need to consider this condition during the search of the TTPs.

2. $\boldsymbol{m}(u_0, v_0)$ is not parallel to $\boldsymbol{n}(s_0, t_0)$

As $\boldsymbol{m}(u_0, v_0)$ is not parallel to $\boldsymbol{n}(s_0, t_0)$, there must exist a certain offset distance l_0, such that $\boldsymbol{G}^0(u_0, v_0)\boldsymbol{F}^0(s_0, t_0)$ is parallel to $\boldsymbol{n}(s_0, t_0)$, To make $\boldsymbol{G}^0(u,v)$ and $\boldsymbol{F}^0(s,t)$ tangent at (u_0, v_0), (s_0, t_0), we can only change the offset distance of $\boldsymbol{F}^0(s,t)$, and the following equation

$$\boldsymbol{G}(u_0,v_0)+l_0*\boldsymbol{m}(u_0,v_0)=\boldsymbol{F}(s_0,v_0)+h*\boldsymbol{n}(s_0,t_0) \quad (7)$$

holds. It can be predigested as follows:

$$\delta = d_{qp}(\boldsymbol{F}(s_0,t_0),\boldsymbol{G}(u_0,v_0))-h\,. \quad (8)$$

Those points that meet Eq.(6) and Eq.(8) also belong to I-B TTP set and we denote them as I-B TTP2. Table 3 sums up the effect of these TTPs on the topology of the intersection curves.

From the geometric viewpoint, only the isolated TTPs are discussed in this paper. However, the shape of a TTP can vary in several aspects. For example, it can be a line segment or a curve, a bounded region, a closed curve, a bounded region with a hole(s), or the combination of the previous cases. In general, a TTP can be in any shape that can emerge in general SSI problem. In this paper, we discuss the isolated TTPs only; and other type of TTPs will be discussed in other papers.

DETERMINING THE SEARCH STRATEGY OF INITIAL INTERSECTION POINTS

The TTP expresses the essential property of the relations between different intersection curves of offset surface. By analyzing the TTPs, we can find the similarities and the differences between the topologies of the intersection curves with different offset distances. With the topology similarities of intersection curves, the intersection problem can be simplified.

As well known, the geometrical and topological complexity of surface lead to the complexity of the geometrical and topological complexity of the intersection curves. Given the distribution of the intersection curves of the two original surfaces, we will try to solve the intersection problem of two offset surfaces when the offset distances are l_1 and h_1, respectively. One or more TTPs will emerge when the offset distances varies between the original

Table 3 Relations between the I-B TTP_2 and the topology intersection curve

δ	Total curvature		Intersection curves of approximation shape of surface around the TTP	Tangent line's continuity of approximation shape's intersection curves	Variety trend of intersection curves	Kind of TTP
$\delta>0$ and $l=l_0$	$K\geq0$	$K\geq0$			Add an open loop	I-B_1
	$K\geq0$	$K<0$	Add an intersect loop		Add an open loop	I-B_2
	$K<0$	$K<0$	Two intersection regions turn into other two intersect regions	Continuous	One intersect region turns into two intersect regions	I-B_3
				Discontinuous	Two intersect regions turn into one intersect region	I-B_4
$\delta>0$ and $l\neq l_0$					No effect	
$\delta<0$						

location and the location at the intersection curves that we want to calculate. The TTPs are used to find the intersection curve topology of the surfaces. In the following context, we classify the TTPs into several groups, and analyze the functionalities of different TTPs in ten cases. In the previous nine cases, one TTP at most is considered; the case of more than one TTP can be easily dealt with by considering each TTP separately. Fig.4 shows a possible distribution of the TTPs, and the following discussion will be based on it.

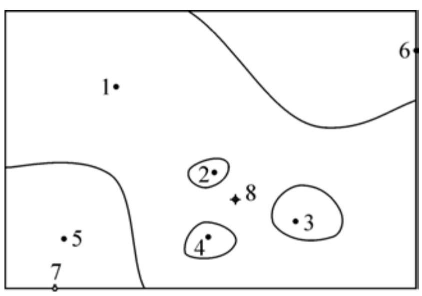

Fig.4 An example of TTP distribution

1. No TTP is disabled or enabled when the offset distances, l and h, vary from 0 to l_1 and h_1 respectively.

Because the number of the TTPs is not changed, the topologies of the intersection curves also remain unchanged. The intersection curves of the offset surfaces are obtained by simply expanding or shrinking the intersection curves of the old curves. If δ increases, the new intersection curves can be calculated by expanding the old intersection curves, and the initial points can be found in the outer normal direction of the curves at any point on old intersection curves. Otherwise, the initial points can be searched in the reverse direction.

2. One I-I_1 or I-I_2 TTP is enabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

The existing curves will not change in their topologies and are calculated as described in the case 1. A new loop is added to the structure of intersection curves, the initial points of the new loop is obtained in any direction from the TTP loop is obtained in any direction from the TTP enabled.

3. One I-I_1 or I-I_2 TTP is disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

The intersection curve that contains this TTP vanishes. There is no need to calculate the intersection curves around this TTP.

4. One I-I_3 TTP is enabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

According to the above analysis, the new TTP may turn two intersection regions into other two new intersection regions, but the new intersection regions may be enclosed out of the neighboring regions of the TTP. Even though the I-I_3 TTP merges regions, it is not certain, by looking at the TTPs only, which regions are merged. For instance, only by analyzing the 8th TTP, can we not know which pair of regions will be merged among the three intersection regions around the 2nd, 3rd and 4th TTPs. Thus, the new intersection curves around the old TTPs should be computed by using the first three methods mentioned above. Then inclusion test (O'Rourke, 1993) will be used for old regions to decide the intersection curve topology. For Fig.4, we can first calculate the intersection region around the 2nd TTP by method 1. Then inclusion test is done for the region obtained. If it does not contain the 3rd, 4th and 8th TTPs, the new region is only related to the 2nd TTP. Otherwise, if it contains the 3rd and 8th TTPs; the I-I_3 TTP merges the regions created by the 3rd and 2nd TTPs.

5. One I-I_3 TTP is disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

The intersection curve topology is obtained by reversing the process of the 4th case. The initial point of the intersection regions can be searched on the line of the disabled TTP and the TTPs that are associated with the merging process.

6. One I-B_1 or I-B_2 TTP is enabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

A new open intersection curve is created around the TTP, and the initial point on the intersection curve is found by searching along the boundary in any direction.

7. One I-B_1 or I-B_2 TTP is disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

The curve around the TTP is removed, and there is no need to calculate intersection curve around the TTP.

8. One I-B_3 is enabled or one I-B_4 is disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

A new curve is added to the structure of intersection curve. The initial point is found by searching along the boundary in different directions, and two open curves will be found.

9. One I-B_4 is enabled or one I-B_3 is disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

Two open curves join up at the TTP. A new curve is obtained by emerging two old open curves. Inclusion test is used to determine which TTP is included in the new curve, and the initial point can be found at certain direction of the TTP.

10. More than one TTP is enabled or disabled when the offset distances, l and h, vary from 0 to l_1 and h_1, respectively.

The analysis becomes complex, because this may combine all methods mentioned above. However, this case can be easily dealt with by considering each TTP separately. Firstly, all TTPs emerged are sorted according to their directed distance. Then we calculate the topologies of the intersection curves just when the TTP is enabled or disabled by employing the 1−9 methods above. Through this way, the topologies of the intersection curves are updated when all TTPs are processed.

In short, the calculation of initial intersection points can be simplified by taking into account the similarities of intersection curves with different offset distances and the TTPs.

EXAMPLES AND ANALYSIS

The proposed algorithm was implemented in Visual C++ on a Window2000 platform, and we applied the algorithm to some examples. Two B-spline surfaces in rectangle domain, denoted by $\boldsymbol{G}(u, v)$, $\boldsymbol{F}(s, t)$ are shown in Fig.5 and Fig.6, respectively. The distribution graph of TTPs is shown in Fig.7, in which there are 24 TTPs. Among them, we give an order number to each TTP that has some effect in our explanation. In Figs.8−18, we analyze the relations between the TTPs and the topologies of the intersection curves.

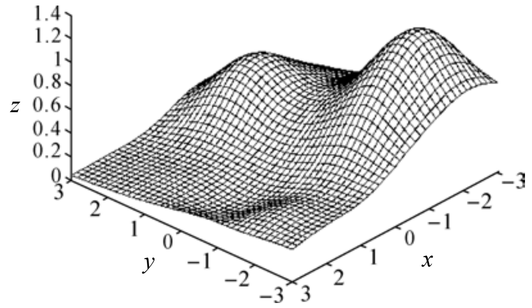

Fig.5 Surface *G*(*s*, *t*)

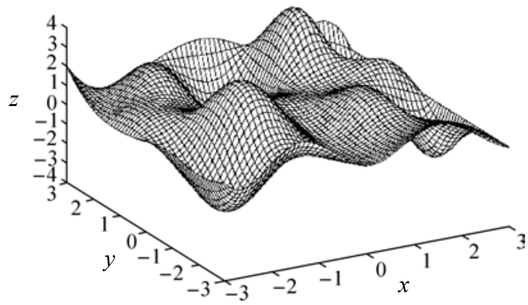

Fig.6 Surface *F*(*s*, *t*)

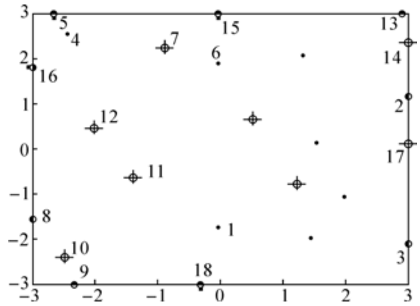

Fig.7 TTP's distribution graph

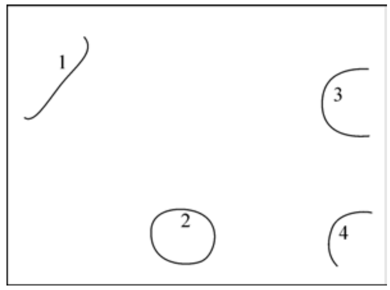

Fig.8 *h*=−1, *l*=−0.5. The 2nd and 4th together with 5th TTPs create the 1st intersection loop. The 1st TTP create a close intersection loop. The 2nd TTP and the 3rd TTP create the 3rd and 4th open curve, respectively

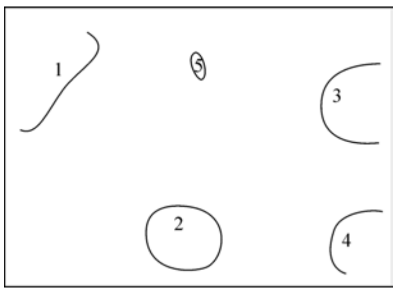

Fig.9 *h*=−0.6, *l*=−0.3. The 6th TTP create the 5th enclose intersect curve and original curve expand

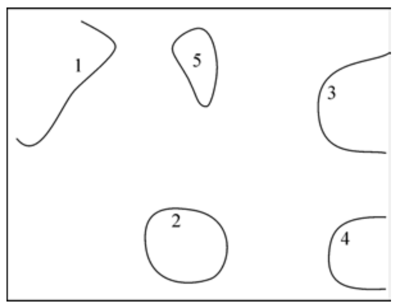

Fig.10 *h*=−0.3, *l*=−0.15. No new TTP emerge, and the new intersect curves are searched by expanding the old curves

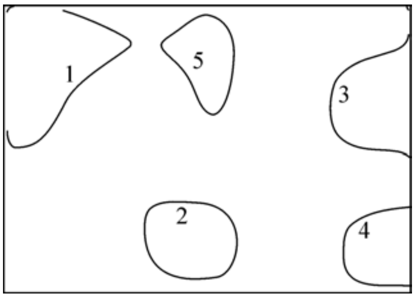

Fig.11 *h*=−0.2, *l*=−0.1. The 13th TTP creates the 6th new open curve. The 1st and 5th curves trend to merge with each other

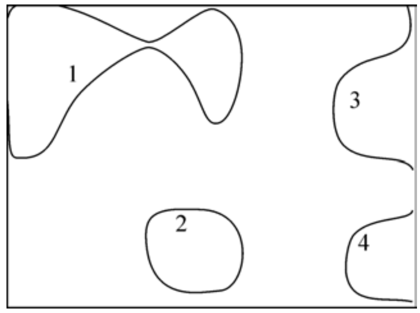

Fig.12 *h*=0, *l*=0. The 7th TTP merge the 1st and the 5th curves into a curve

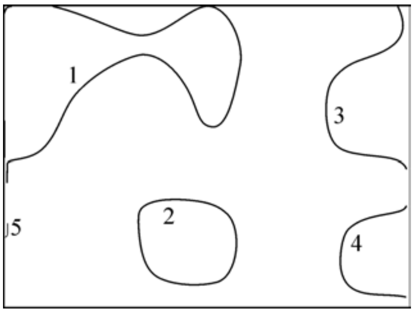

Fig.13 ***h*=0.1, *l*=0.05. The 8th TTP creates the 5th open curve**

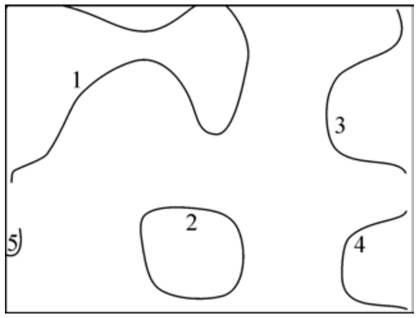

Fig.14 ***h*=0.2, *l*=0.1. The 15th TTP makes the 1st curve open around the 15th TTP**

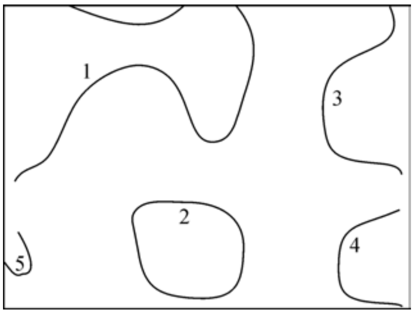

Fig.15 ***h*=0.4, *l*=0.2. No new TTP take effect, and the 3rd and 4th curves will be merged**

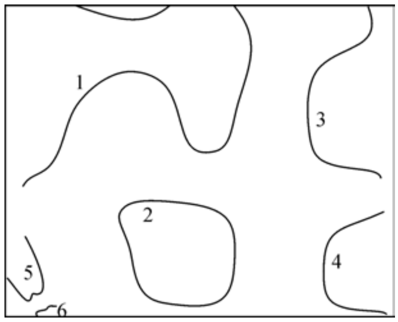

Fig.16 ***h*=0.6, *l*=0.3. The 9th TTP create the 6th new open curve**

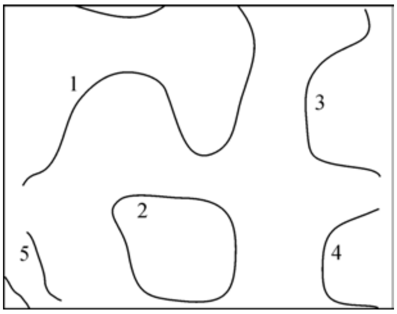

Fig.17 ***h*=0.7, *l*=0.35. The 10th TTP merges the 5th and 6th intersection curves into the 5th intersection curve**

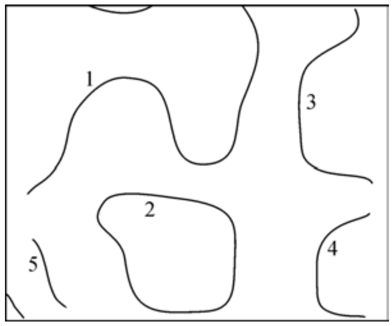

Fig.18 ***h*=0.9, *l*=0.45. The 2nd intersection curve will become a open curve near the 18th TTP**

CONCLUSION AND FUTURE WORKS

Traditionally, the SSI algorithms are often used to solve OSSI problems. In those algorithms, the characteristics of offset parametric surface were not used to simplify the OSSI problem. The algorithm proposed in this paper is an optimized algorithm based on the similarities of the topologies of the intersection curves. When we intersect one offset surface with another offset surface, the changes in topologies of the intersection curves are caused by the TTPs of surfaces. In this paper, the observed properties of TTPs were applied to devise a method for detecting TTPs, and the strategy to locate the initial intersection points is provided. The presented algorithm makes full use of the relations between TTPs and the topologies of intersection curves to simplify the calculation of the intersection problem. Firstly, the algorithm preprocesses the surfaces to get the TTPs. Then through analyzing the distribution graph of the TTPs, the topologies of the intersection curves and the searching strategy of initial points can be determined to reduce the scope of searching intersection curves.

The properties of the surfaces and the feasibility of the algorithm are analyzed and clarified in a number of illustrative samples. The presented algorithm has been tested and proved capable of overcoming the degenerate conditions such as loop and singularity leaking. It can calculate the intersection curves robustly and effectively in that it calculates the initial points with the information of TTPs and the topologies of the curves, decreasing the search ranges of the initial points and speeding up the calculation. Feature work should be directed at the efficiency for computing of all kinds of degenerate TTPs.

References

Abdel-Malek, K., Yeh, H.J., 1997. On the determination of starting points for parametric surface intersections. *Computer Aided Design*, **29**(1):21-35.

Burke, T.M., Sabharwal, C.L., 1996. Data Parallel Implementation of Surface-to-Surface Intersection between Parametric Surfaces. *In*: Symposium of Applied Computing. Philadelphia, Pennsylvania, p.353-357.

Chang, L.C., Bein, W.W., Angel, E., 1994. Surface intersection using parallelism. *Computer Aided Geometric Design*, **11**(1):39-69.

Hohmeyer, M.E., 1992. Robust and Efficient Intersection for Solid Modeling: [dissertation]. *In*: Computer Science Division, Department of Electrical Engineering and Computer Science, University of California, Berkeley.

Hu, S.M., Sun, J.G., Jin, T.G., Wang, G.Z., 2000. Computing the parameters of points on nurbs curves and surface via moving affine frame method. *Journal of Software*, **11**(1):49-53(in Chinese).

Jun, C.S., Kim, D.S., Lee, H.C., Hwang, J., 2001. Surface slicing algorithm based on topology transition. *Computer Aided Design*, **33**:825-838.

Kulkarni, P., Dutta, D., 1995. Adaptive Slicing of Parameterizable Algebraic Surfaces for Layered Manufacturing. *In*: Proceedings of the 1995 ASME Design Technical Conference. Boston MA.

Ma, Y., Lee, Y.S., 1998. Detection of loops and singularities of surface intersections. *Computer Aided Design*, **30**(14):1059-1067.

Mullenheim, G., 1991. On determining start points for a surface/surface intersection algorithm. *Computer Aided Design*, **8**(5):401-408.

Nackman, L.R., 1984. Two-dimensional critical point configuration graphs. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, **8**(5):401-408

O'Rourke, J., 1993. Computational Geometry in C. Cambridge. Cambridge University Press.

Su, B.Q., Hu, H.S., Shen, C.L., Pan, Y.L., Zhang, G.L., 1979. Differential Geometry. Higher Education Press.

Tait, S.S., Rada, T.F., Mohammad, A.K., Helmut, P., 2002. Optimal slicing of free-form surfaces. *Computer Aided Geometric Design*, **19**:43-64.

Tang, M., Dong, J.X., Li, H.L., He, Z.J., 1999. Boolean operation of non-regular precise geometric models. *Journal of Software*, **10**(12):1291-1297 (in Chinese).

Tang, M., Dong, J.X., 2000. An accurate intersection method for sculptured solids. *The Chinese Journal of Computer*, **23**(4):434-439(in Chinese).

Wu, S.T., Andrade, L.N., 1999. Marching along a regular surface/surface intersection with circular steps. *Computer Aided Geometric Design*, **16**:249-268.

Yu, W., 1996. Intersection of offsets parametric surfaces. *Computer Aided Geometric Design*, **13**:453-465.