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The Q theory of investment, the capital asset pricing model, and asset valuation: a synthesis

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Abstract: The paper combines Tobin's Q theory of real investment with the capital asset pricing model to produce a new and relatively simple procedure for the valuation of real assets using the income approach. Applications of the new method are provided.

Key words: Investment theory, Asset pricing, Appraisal
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INTRODUCTION

This paper combines the economic theory of real investment and the standard financial model of asset pricing to produce a method for the valuation of real assets; and intentionally uses relatively simple versions of these two theories to link economics, finance, and appraisal. Numerical examples using data on real estate assets illustrate the valuation method.

The Q theory of investment, introduced by James Tobin (1969), is popularly accepted theory of real investment hypothesized to be a positive function of Q, defined as the ratio of the market value to the replacement cost of capital. Standard presentation of the theory, such as that of Romer (1996), shows that Q is the value to the firm of an additional unit of capital, which is the discounted value of its future marginal revenue products. Extensions of the Q theory of investment have added uncertainty about future returns. An important article by Craine (1989) examines the effect of total risk on the allocation of capital in a simple general equilibrium model; with the result that an exogenous increase in risk reallocates capital towards less risky investments. Tobin and William Brainard, in a series of papers in the late 1970s and early 1980s, developed what they called a "fundamentals" approach to asset valuation built on the capital asset pricing model. This approach links the standard financial theory of asset valuation and investment in the real economy, a theme in Tobin's (1952) work. The purpose of this work is to explore more fully the implications of the Tobin/Brainard approach to asset valuation, and to apply the model to real estate asset valuation and investment. The paper shows that the Q theory of investment implies a simple approach to the income method for the valuation of real estate.

The paper begins with an introduction to the Q theory of investment and a brief discussion of the capital asset pricing model (CAPM) followed by a presentation of the "fundamentals" approach to asset valuation that combines these two models and uses material in Tobin and Golub (1998), but goes beyond their results. The fundamental result is that the market value of an asset equals its expected earnings (net income plus capital appreciation) divided by the risk-adjusted discount rate. This section is followed by model extensions useful for understanding application of the method to real estate markets and investment. Property taxation is added to the model, and then the standard methods for real estate appraisal are examined in light of the model. The paper concludes with examples of real estate valuation using the new method.

The *Q* theory approach to the study of real estate investment would appear to be a good combination of theory and application for at least two reasons. There are real estate development firms that specialize in the task of planning, building, and selling real estate to permanent owners. Their basic calculation is whether *Q* is greater than 1.0. Also, the three standard methods for real estate appraisal all lead to a value of *Q* for an individual property. The sales comparison and income capitalization approaches to appraisal provide estimates of market value, while the cost approach provides an estimate of reproduction cost. In real estate appraisals the cost approach is usually implemented as reproduction cost for an existing property minus accrued depreciation. However, most appraisal reports combine these three approaches into one estimate of value. Among other things, it is suggested in this paper that the estimated cost should be reported separately along with an estimate of *Q*.

THE *Q* THEORY OF INVESTMENT

This section is a brief presentation of the *Q* theory of investment. The focus is on the returns to the marginal unit of real investment by a profit-maximizing firm. One of the standard assumptions in the theory is that there is a cost of installing additional capital. The marginal adjustment cost is assumed to increase with the size of the adjustment; i.e., with the amount of real investment. Following Romer (1996), a discrete-time version of the theory is presented so that the linkage can be made to the single-period capital asset pricing model.

The firm maximizes the present discounted value of profits subject to the constraint in each period that the capital stock in the next period equals the capital stock in the current period plus the amount of real investment undertaken. Depreciation is ignored in this presentation. The Lagrangian for the firm's maximization problem is

$$L=\sum_t[1/(1+r)^t][\pi(K_t)k_t(M,S,L)-I_t-C(I_t)] +\sum_t\lambda_t(k_t+I_t-k_{t+1}), \quad (1)$$

where $\pi(K_t)$ = the marginal revenue product of capital at time t; a function of the total stock of capital K in the industry; r = the discount rate for the firm; k_t = the firm's capital stock at time t, a function of machines M, structure capital S, and land L at time t; I_t = capital investment during time t; $C(I_t)$ = the firm's adjustment cost, a function of I_t; λ_t = Lagrange multiplier associated with the constraint relating k_t and k_{t+1}.

The summation is over time zero to infinity. The Lagrange multiplier gives the marginal value of relaxing the constraint; i.e., the marginal impact of an exogenous increase in k_{t+1} on the present discounted value of the firm's profits. Define

$$Q_t = (1+r)^t\lambda_t, \quad (2)$$

so the Lagrangian can be written as

$$L=\sum_t[1/(1+r)^t]\cdot [\pi(K_t)k_t-I_t-C(I_t)+Q_t(k_t+I_t-k_{t+1})]. \quad (3)$$

The first-order condition for investment by the firm at time t is

$$Q_t = 1+C'(I_t), \quad (4)$$

Which states that the marginal value of capital equals its purchase price (equal to 1) plus the marginal adjustment cost. It is assumed that $C'>0$; i.e., that there is a rising marginal adjustment cost. The first-order condition for capital stock at time t is

$$[1/(1+r)^t][\pi(K_t)+Q_t] - [1/(1+r)^{t-1}]Q_{t-1} = 0. \quad (5)$$

This can be rewritten as

$$\pi(K_t) = (1+r)Q_{t-1}-Q_t = rQ_{t-1}-\Delta Q. \quad (6)$$

Here ΔQ is the change in Q from time t–1 to time t. If we replace time t–1 with 0, then

$$\pi(K_1) = rQ_0-\Delta Q, \quad (7)$$

which states that the marginal revenue product of capital at time 1 equals the discount rate times the marginal value of capital at time 0 minus the change in the marginal value of capital. Alternatively, the discount rate for the firm is:

$$r = [\pi(K_1) + \Delta Q]/Q_0. \quad (8)$$

And the marginal value of capital at time 0 can be written as

$$Q_0 = [\pi(K_1)+\Delta Q]/r = 1 + C'(I_0). \quad (9)$$

The Q theory of investment states that investment is a positive function of Q, which summarizes all information needed to determine investment. In Eq.(9) a higher value for Q_0 means a larger value for C', and therefore a larger amount of investment I at time 0.

Eq.(9) provides a simple equation for Q_0 that involves only three variables; the marginal revenue product of capital, the change in Q, and the discount rate. This simple formula is implied by the standard discounted present value method of asset valuation. Consider a two-period example in which an asset earns π_1 in the first period and π_2 in the second period (and nothing thereafter). The discounted present value of the asset at the beginning of period one is

$$V_0 = \pi_1/(1+r)+\pi_2/(1+r)^2, \quad (10)$$

and the change in value from the beginning of period one to the beginning of period two is

$$\Delta V = \pi_2/(1+r)-V_0. \quad (11)$$

Substitution of Eq.(11) for V_0 and then insertion of this expression into the valuation formula

$$(\pi_1+\Delta V)/r \quad (12)$$

yields the discounted present value equation for V_0.

This result can easily be generalized to n periods. Write the discounted present value equation as

$$V_0 = \pi_1/(1+r) + \pi_2/(1+r)^2 + \ldots + \pi_n/(1+r)^n. \quad (13)$$

The change in value during period one can be written

$$\Delta V = \pi_2/(1+r) + \ldots + \pi_n/(1+r)^{n-1} - V_0 = -\pi_1/(1+r)+\pi_2[1/(1+r)-1/(1+r)^2]+ \ldots+\pi_n[1/(1+r)^{n-1} -1/(1+r)^n]. \quad (14)$$

The computation of V_0 via Eq.(9) is

$$v = \pi_1/r+\Delta V/r =(\pi_1/r)[1-1/(1+r)]+(\pi_2/r)\{[1/(1+r)] -[1/(1+r)^2]\} +\ldots+(\pi_n/r)\{[1/(1+r)^{n-1}]-[1/(1+r)^n]\}. \quad (15)$$

Each term in Eq.(15) simplifies to the corresponding term in Eq.(13).

THE CAPITAL ASSET PRICING MODEL

The presentation of the CAPM follows sources such as Hamada (1969) and Rubinstein (1973). Those who are completely familiar with the assumptions of this standard model can skip to Eq.(16) below. The assumptions used to develop the CAPM are:

There are perfect capital markets. Information is available to all at no cost, there are no transactions costs, and assets are infinitely divisible and fixed in supply. All investors can borrow and lend at the same rate of interest. The risk of default associated with borrowing is negligible.

Investors are risk-averse and maximize the expected utility of wealth at the end of the planning horizon, which is one period in length. Portfolios are assessed only by the expected rate of return and the standard deviation of return.

The planning horizon is the same for all investors, and all portfolio decisions are made at the same time.

All investors have identical estimates of expected rates of return and standard deviations of returns.

The rate of return of a portfolio or of a risky asset is denoted as random variable r.

From the second assumption above, the expected rate of return $E(r)$ and the standard deviation $\sigma(r)$ of portfolios are the objects of choice. This leads to the formation of an efficient set of risky portfolios. Introduction of a riskless asset with rate of return r_f leads to the conclusion that each investor will combine a single risky portfolio m (the market portfolio) with the risk-free asset. Risky portfolio m is combined by all investors in some proportion with the riskless asset. Market equilibrium requires that all risky assets be held in proportion to their market values; this condition determines the composition of the market portfolio m.

Given the above assumptions and in the absence of taxes, it is well known that the following equilibrium condition can be derived for any risky asset i in the market:

$$\begin{aligned} E(r_i) &= r_f + [\sigma(r_i, r_m)/\sigma^2(r_m)][E(r_m) - r_f] \\ &= r_f + \beta_i [E(r_m) - r_f], \end{aligned} \quad (16)$$

Where, $E(r_i)$ = the expected return for risky asset i; r_f = the risk-free borrowing and lending rate; $E(r_m)$ = the expected return on the market portfolio; $\sigma^2(r_m)$ = the variance of the returns of the market portfolio; $\sigma(r_i,r_m)$ = the covariance between the returns for asset i and the market portfolio, and $\beta_i = \sigma(r_i,r_m)/\sigma^2(r_m)$ = the "beta" of asset i.

This equation specifies a linear relationship between the required expected rate of return for an asset and its systematic risk, measured as β_i. The right-hand side of Eq.(16) is the conventional risk-adjusted discount rate.

Suppose the expected return for risky asset i can be written

$$E(r_i) = E(e_i)/V_i, \quad (17)$$

where $E(e_i)$ is expected earnings for the next period (including capital appreciation) and V_i is the current market value of the asset. Eq.(16) would then imply that value equals expected earnings divided by the risk-adjusted discount rate:

$$V_i = E(e_i)/\{r_f + \beta_i [E(r_m) - r_f]\}. \quad (18)$$

Does this result carry over to the Q theory of investment and asset valuation? It does, as we shall see in the next section.

Bossaerts (2002) used the method of dynamic stochastic programming to produce this central prediction of asset-pricing theory. Dynamic programming makes use of the Bellman principle that there exists a value function such that the sequence of optimal policies over time can be found by solving a particular sequence of one-period optimization problems. A non-stochastic version of this method was used above in the derivation of the Q theory of investment. Consequently, while the model in this paper is stated in terms of the conventional one-period CAPM, the model can be interpreted as the dynamic programming solution to a multi-period problem.

One difficulty with the application of CAPM to real estate is the evidence provided by Young and Graff (1995) that the returns to individual commercial real estate assets do not have finite variance. They used the Russell-NCREIF data on individual property returns for 1980−1992, and showed that a class of probability distribution functions called the stable Paretian distribution provides a better fit to the returns data than does the normal distribution. The stable Paretian distribution has the normal distribution as a special case, but the general case does not possess a finite variance. The stable Paretian distribution permits skewness and more weight in the tails of the distribution than does the normal distribution (kurtosis). Young and Graff (1995) stated that the total return on property of type m during year t is

$$R_t(m) = \mu_t[h(m)] + \varepsilon_t(m), \quad (19)$$

where $\mu_t[h(m)]$ is the expected return during year t for property type m, and $\varepsilon_t(m)$ is a stable random variable that possibly has infinite variance. The first term corresponds to systematic risk, while the second term is the non-systematic risk. Young and Graff (1995) were therefore concerned with the distribution of non-systematic risk, and specified a basic index model for systematic risk (based on time period and property type). They concluded that the well-diversified portfolio must contain more assets in the case of returns with infinite variance; as Fama (1965) demonstrated many years ago, the problem of portfolio analysis can still be solved. As we shall see below, the critical feature of the CAPM for real estate asset valuation is systematic risk. What matters for valuation is the coefficient that relates the returns for a particular asset to an index that represents the condition of the whole economy. In the CAPM this index is the return to the market portfolio. Researchers have debated the accuracy of the Russell-NCREIF returns data; especially that part of the returns based on annual appraisals of market value, but the basic message of this paper carries through even if non-systematic risk is of infinite variance. See Bond and Patel (2003) for more recent research on the skewness in the distribution of returns to real estate. As Campbell *et al.* (1997) indicated, the stable Paretian distribution has fallen out of favor with finance researchers in general in recent years. Instead researchers model returns as drawn from a distribution with finite higher moments that permit greater kurtosis than does the normal distribution. For example, returns might be normal with variance that varies randomly (or varies systematically with time such as in the autoregressive conditional heteroscedasticity (ARCH) model).

THE Q THEORY OF INVESTMENT AND THE CAPITAL ASSET PRICING MODEL

The Q theory of investment and the CAPM can be combined into one model of asset pricing and investment. The development of this combined model follows and extends the presentation of the "fundamentals" method of asset valuation in Tobin and Golub (1998). This method is identical to the Q theory presented above where the value of an investment is based on the stream of earnings, rather than on speculative movements in asset prices. Ownership of a unit of real investment is title to one unit of capital at replacement cost, and that unit of real capital produces a stream of real earnings, the stream of marginal revenue products in Eq.(1) above.

The following definitions are needed: $E(e_i)$=$E[\pi_i(K_1)+\Delta Q_i]$ from Eq.(8), so $E(r_i) = E(e_i)/Q_i$. Note: If $Q_i = 1$, $E(e_i) = E(r_i)$. $V_i^2 = Var(e_i)$, so $\sigma_i^2 = V_i^2/Q_i^2$, and $V_{im} = E(e_i,e_m)$, so $\sigma_{im}= V_{im}/Q_iQ_m$. $b_i = V_{im}/V_m^2 = \beta_iQ_i/Q_m$. Note the obvious equalities if $Q_i = Q_m = 1$.

Substitution of these terms into the CAPM from Eq.(16) produces an equation for the expected return to investment i, written as

$$[E(e_i)/Q_i]-r_f =[(V_{im}/Q_iQ_m)/(V_m^2/Q_m^2)] \{[E(e_m)/Q_m]-r_f\}. \quad (20)$$

This equation can be solved for Q_i to produce

$$Q_i=\{[E(e_i)]-b_i[E(r_m)-r_f]Q_m\}/r_f. \quad (21)$$

Substituting for b_i from above produces the even simpler result that

$$Q_i=E(e_i)/(r_f+\beta_i\theta), \quad (22)$$

where $\theta=E(r_m)-r_f$, the market risk premium. Eq.(22) states that the marginal value of capital equals e_i (the expected marginal revenue product plus the expected "fundamental" capital gain), divided by the risk-adjusted discount rate. This result is symmetric with the usual result in the CAPM that the expected rate of return for asset i is positively associated with systematic risk. Note that Q_m drops out of the equation for Q_i, a fact not stated by Tobin and Golub (1998). The Q theory of investment implies that knowledge of the "fundamental" capital gain is central to the simplicity of the valuation formula in Eq.(22).

The comparative-statics of the model are quite straightforward. The marginal value of capital varies as follows:

$$\partial Q_i/\partial E(e_i)=1/(r_f+\beta_i\theta)>0; \quad (23a)$$

$$\partial Q_i/\partial\beta_i=-\theta E(e_i)/(r_f+\beta_i\theta)^2 = -\theta Q_i/(r_f+\beta_i\theta)<0. \quad (23b)$$

$$\partial Q_i/\partial r_f=-E(e_i)/(r_f+\beta_i\theta)^2 = -Q_i/(r_f+\beta_i\theta)<0. \quad (23c)$$

$$\partial Q_i/\partial\theta=-\beta_i E(e_i)/(r_f+\beta_i\theta)^2 = -\beta_i Q_i/(r_f+\beta_i\theta). \quad (23d)$$

The first result simply states that the marginal value of capital increases if the expected payoff to capital increases, thereby increasing investment. The second result is that the marginal value of capital decreases if the covariance of its expected return with the expected market return increases. In other words, the result is that an increase in systematic risk reduces Q_i and therefore reduces this particular investment. The third result shows that an increase in the risk-free rate of return will reduce the marginal value of capital and reduce investment in the particular risky investment. The fourth result shows that a reduction in market risk θ will increase Q_i, provided that β_i is greater than zero – i.e., returns to the asset in question are positively correlated with the market. This last result is potentially very important in that policies that reduce market risk will stimulate investment in particular sectors such as real estate to the extent that their returns are positively correlated. Here is a simple demonstration of the more fundamental point made by Rajan and Zingales (2003) that the development of financial markets and financial infrastructure (proxied here by a reduction in θ) has a strongly positive impact on the growth of an economy.

APPLICATION OF THE ASSET VALUATION MODEL TO REAL ESTATE

This section adds property taxation to the fundamental valuation model, and then conventional real estate appraisal methods are examined in light of the model. Property taxation is not normally included in the CAPM, but virtually all real estate is subject to this tax. Consider the marginal unit of real estate investment, measured in physical terms. For example, the marginal unit of real estate could be a housing unit with standard features or a standardized amount of commercial real estate. Investment was purchased up to the point at which Q is equal to one plus the marginal adjustment cost, as in Eq.(4), so Q is the price of the marginal investment.

Consider first the effect of the property tax. In a world of certainty the market value of an asset of infinite life subject to the property tax is

$$V=(R-tV)/r, \quad (24)$$

where R is annual earnings, t is the property tax rate, and r is the discount rate. The solution for V is

$$V=R/(r+t) \quad (25)$$

An increase in the property tax rate reduces V according to

$$\partial V/\partial t=-R/(r+t)^2=-V/(r+t). \quad (26)$$

If no debt is used to purchase the marginal unit, then the expected after-tax earnings in Eq.(22) are $E[(e_i-tQ_i)]$. If the amount of the tax bill tQ_i is known, then valuation can proceed as in Eq.(22). However, if only the tax rate t is known, then an adjustment in the valuation formula is needed. Insertion of this expression for earnings into Eq.(22) produces:

$$Q_i=E(e_i)/(r_f+\beta_i\theta+t), \quad (27)$$

where θ is the market risk premium $E(r_m)-r_f$. The property tax rate is now included in the risk-adjusted discount rate. Eq.(27) is the fundamental equation for the valuation of a real estate asset, and will be used in the numerical examples below (unless the actual tax bill is known). A change in the property tax rate has the following effect on Q:

$$\partial Q_i/\partial t = -E(e_i)/(r_f+\beta_i\theta+t)^2 = -Q_i/(r_f+\beta_i\theta+t). \quad (28)$$

The effect of an increase in the property tax rate on Q is negative, and analogous to the case of certainty. In both cases the percentage change in value equals –1 divided by the discount rate.

Now suppose the marginal investment is made by borrowing amount $B=mQ_i$ at rate r_f. The total value of the property is $Q=S+B$, where S is the value of the equity share. As is well known, without corporate income taxation, the amount borrowed has no effect on the value of the property. The value of equity is reduced dollar-for-dollar with an increase in the amount borrowed. The Modigiliani Miller theorem holds if there is property taxation. Furthermore, as McDonald (2002) showed, a necessary condition for borrowing ($m>0$) in the absence of corporate income taxation is that the borrowing rate must be less than the risk-free rate – an impossible condition. Borrowing in the presence of the corporate income tax is not included in the model presented here. An extension along these lines would appear to be useful.

Lusht (1997) advocated a definition of market value for real estate that is the expected selling price of the real property rights in an arm's length transaction, assuming reasonable exposure to the market. He provided a clear intermediate level presentation of the methods used by appraisers to estimate market value. The first of these methods is the sales comparison approach, and little need be said about it if data are available on the sales of a sufficient number of comparable properties. The basic valuation model of Eq.(27) above indicates points of comparability beyond location, size, condition, and so on. Valuation varies across properties at the same point in time because of differences in the property tax rate and non-diversifiable risk. The property tax is easily measured, but direct measures of differences in non-diversifiable risk across properties are not available. The usual approach is to focus on type of property and location in assembling data on sales of comparable properties.

The second method used by real estate appraisers is the cost approach, where market value is estimated as:

Reproduction (or replacement) cost of the subject property's improvements as if they were new, "minus" accrued depreciation on the improvements, "plus" land (site) value.

The methods for estimating reproduction (or replacement) cost and accrued depreciation for the improvements are highly developed and very detailed. Appraisal texts such as in Ring and Boykin (1986) and Lusht (1997) instruct the appraiser to value the land as if it is vacant and then put to its best use or most probable use. They advocated using the market comparison method for land valuation whenever feasible. In essence, the idea is to estimate the most that someone else would pay for the site – its opportunity cost.

Appraisers recognize that the cost approach produces an accurate estimate of market value only for cases in which the market is in long-run equilibrium, which corresponds to a value for Q of 1.0. Ring and Boykin (1986) stated that, "There is absolutely no reason for the estimated depreciated cost of a property to set its upper limit on value." Furthermore, appraisal methodology does not appear to recognize the notion of adjustment cost, denoted $C(I)$ above. Reproduction (or replacement) cost consists of direct costs (materials, labor, and so on), indirect costs (financing costs, fees, and other "soft" costs), and developer's profit, normally determined by rule of thumb for a "normal" profit. It seems fair to say that the purpose of the cost approach is to estimate the purchase price of real estate capital excluding the adjustment cost. Appraisal texts (Lusht, 1997; Ring and Boykin, 1986) do not entertain the idea that cost may be a function of the quantity of investment undertaken in a particular time period. They recognized that the cost approach has disadvantages; it is not well suited for older buildings, can be overly complicated; and fails to provide a completely independent estimate of value because the value of land must be estimated from market data. Nevertheless, because the cost approach is a well-developed method designed to provide an estimate of the purchase price of real estate capital, conventional cost estimates can be used as the denominator in the computation of Q. However, Q theory also makes it clear that it is difficult to justify the use of the cost estimate as another estimate of market value to be reconciled with estimates from the market comparison and income methods.

The income approach to valuation is often used for commercial real estate. In this method the income concept is net operating income (NOI), which is defined as effective gross income minus operating expenses, maintenance and repair costs, and reserves for replacement. Operating expenses include fixed and variable expenses (those that vary with the occupancy level in the building). Fixed expenses normally include insurance premiums and property taxes. This treatment of property taxes is potentially problematic because the property tax bill depends upon the value of the property, which depends upon the property tax rate. As Eq.(27) shows, unless the actual property tax bill is known, the more correct procedure is to include the property tax rate as part of the discount rate used to discount expected earnings adjusted for risk.

The basic valuation equation for a unit of real estate in the income approach is

$$V=\sum_{\tau} NOI_{\tau}/(1+\rho)^{\tau}+NSP/(1+\rho)^{n}, \tag{29}$$

where τ runs from 1 to n, the holding period, NSP is the net selling price of the real estate at time n, and ρ is the risk-adjusted discount rate. If the holding period n is very long, then Eq.(29) is analogous to the firm's objective function in Eq.(1) above. This valuation method has high information requirements; a forecast of net operating income for n periods, a forecast of the net selling price at the end of the holding period, and the risk-adjusted discount rate. In contrast, the Q theory method

$$E(e_i)=E[\pi_i(K)+\Delta Q], \tag{30}$$

requires only data on the expected current income plus the expected change in Q. Both methods require the risk-adjusted discount rate, so the information requirements for the implementation of the Q theory are smaller.

NUMERICAL EXAMPLES

A simple example can illustrate the basic method implied by Eq.(27). Consider the marginal investment with a purchase price (i.e., replacement or reproduction cost) of \$1.00, and assume the following values: $E(e_i)$=\$.10, β_i=0.5, θ=0.05, r_f= 0.03, and t=0.02. These values imply that the current market value of the asset is

$$Q_i=0.10/(0.03+0.025+0.02)=1.33.$$

The market value of the asset is 1.33 times the purchase price. In this example if β_i=1.0, Q_i=1; the market value of the asset is 1.0 times the purchase price if the returns to the asset are as volatile as the market.

A good realistic numerical example is the suburban office building case that appears throughout in Corgel *et al*.(2001), whose basic data for this case were as follows:

Appraisal via cost approach: \$1 050 000; Reproduction cost: \$1 205 000; Accrued depreciation: −504 600; Site value (opportunity cost of site): 349 600; Net operating income: 86 600; Property tax: 15 900; Selling price after five years (est.): \$974 700.

The annual income (plus the amount paid as property taxes) was 0.0976 of the cost of \$1 050 000, so π_i=0.0976. The market value of the property can be written as 1 050 000(Q_i), so the property tax rate is 15 900/1 050 000(Q_i)=0.015/Q_i. The annual change in Q can be written (assuming a constant dollar change per year) as:

$$\Delta Q=0.2[(974\,700-1\,050\,000Q_i)/1\,050\,000Q_i]=0.1857/Q_i-0.20.$$

The adjustment for risk for this example is based on data reported by Corgel *et al*.(2001). The proxy for the market as a whole is the U. S. S&P

500 stock index. The proxy for the market price of risk over the decade of the 1990s is the difference between the return to the S&P 500 and the 10-year U. S. Treasury bond, which is 9.5%. The value of "beta" for the National Association of Real Estate Investment Trusts (NAREIT) index of real estate investment trusts with respect to the S&P 500 was 0.56 for the decade of the 1990s, and "beta" for the NCREIF index of returns to individual real estate properties was 0.13 for the same time period[a]. The risk-free rate is the 3.0%, which is the return to the Treasury bond minus the rate of inflation (computed for the years 1999 and 2000).

These data mean that Q for suburban office building is:

$$Q_i = (\pi_i + \Delta Q)/(r_f + \beta_i \theta + t)$$
$$= [0.0976 + (0.1857/Q_i) - 0.20] / [0.03 + 0.095 beta + (0.015/Q_i)]$$

=0.947 using beta of 0.56 (risk-adjusted discount rate of 0.0832 for REITs)

=1.125 using beta of 0.13 (risk-adjusted discount rate of 0.0424 for NCREIF individual properties),

where the equation to be solved is quadratic in Q_i. The market value of the building is estimated to be \$994 000 or \$1 181 000 depending upon whether the earnings of the property in question behave as a real estate investment trust or as an individual property. These estimates are comparable to the standard appraisal estimates reported by Corgel *et al*.(2001), which varied from \$946 000 to \$1 069 000. The higher estimate of \$1 181 000 arises because of the very low value of beta of 0.13, which virtually turns real estate into what is known as the "zero beta" investment.

A third example is an actual shopping center development in suburban Chicago, data for which were made available to the author. The shopping center has 76 000 gross square feet and 75 000 net leasable square feet of space on a site of 3 550 000 square feet. The shopping center is used for general retailing and for a restaurant, and had the following development costs:

Hard costs: \$17 948 000; Soft costs: 3 190 000; Total cost: 21 138 000.

Net operating income in the first year of full operation was estimated at \$1 837 000, and the property tax rate in the jurisdiction is 2.1% of market value, so NOI plus property tax payments (in thousands) equals \$1837+0.021(\$21 138Q). NOI plus property tax as a fraction of cost is 0.0869 +0.021Q. No increase in market value is expected, so ΔQ=0. The equation for Q can therefore be written as:

$$Q = [0.0869 + 0.021Q] / [0.03 + 0.095 beta + 0.021].$$

If beta is set at 0.56, then Q=1.044 and estimated value is \$22 068 000. Estimated value is greater if beta is smaller. The property has an estimated market value that exceeds its cost, and the greater the estimated value the lower is the systematic risk.

CONCLUSION

This paper combined the popularly accepted theory of investment (Tobin's Q theory) with the capital asset pricing model to obtain a reasonably simple equation for Q, the market value of an asset divided by its reproduction cost, in a world in which returns are uncertain. As applied to real estate, the basic valuation formula is

Market value=Q times reproduction cost
=[expected earnings] divided by [risk-adjusted discount rate plus property tax rate].

The property tax is expressed as a percentage of market value or as the amount of the property tax bill. Numerical examples show that the method can be applied, and that the data requirements are less than those of the traditional discounted cash flow

[a] Gyrouko and Nelling (1996) estimated beta for equity REITs over the period 1988−1992 using the S&P 500 as the market index. Mean value of asset beta for 56 REITs was 0.33, and mean values by type of REIT are: Health care, 0.39; industrial and warehouse, 0.27; office, 0.22; residential, 0.37; and retail, 0.42

approach to real estate appraisal. In my opinion, the *Q* theory approach to appraisal can be added to the list of real estate appraisal methods.

Further development of the model presented in this paper is warranted. Debt financing of real estate is not included, and corporate taxes (with deduction of interest payments permitted) are ignored. The introduction of debt financing also would introduce the possibility that the borrower might default on the loan. Lusht (1997) noted that many appraisers use finance-explicit models in which the debt and equity portions of the property are valued separately (before taxes) and added together.

More importantly, perhaps, empirical testing of the model is needed. The valuation formula in Eq.(27) is based on carefully articulated theories of real investment and asset pricing. How well does the model explain prices of commercial real estate assets ex ante and ex post? Does the *Q* theory of investment, where *Q* is measured as estimated market value divided by reproduction cost as estimated by an appraiser, explain construction activity? Does the model explain the value of REIT stocks? These are topics for further research.

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