Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Stable response of axisymmetric two-phase water-saturated soil

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Received Jan. 16, 2003; revision accepted Jan. 1, 2004

Abstract: Biot's dynamic consolidation equations and Hankel transform were used to derive the integral solutions of stress and displacement for axisymmetric harmonic excitations in the two-phase saturated soil with subjacent rock-stratum. The influence of the coefficient of permeability and loading frequency on the soil displacement at the ground surface were studied. The results showed that higher loading frequency led to more dynamic characteristics; and that the effect of the soil permeability was more obvious at higher frequencies.

Key words: Biot's dynamic consolidation, Lamb's problem, Saturated subsoil
doi:10.1631/jzus.2004.1022 **Document code:** A **CLC number:** TU91

INTRODUCTION

In the analysis of the dynamic response of saturated subsoil, sometimes, especially under higher frequency dynamic force, it may be necessary to consider the inertia force of both the soil and the water. Moreover, instead of studying the subsoil as an elastic half-space model, study of subsoil with subjacent rock-stratum has more practical meaning. In this work, the vibration of saturated soil under axisymmetric cyclic loading was studied on the basis of the above background.

Biot (1956) proposed a three-dimensional consolidation theory of saturated soils and gave the equations of poroelasticity foundation under different boundary conditions with the use of Laplace Transform. Although inertia coupling of soil and water and other complex conditions were considered in the theory to make the equations more effective and practical, in most cases they were too difficult to solve. Only under few conditions could we get the precise solutions by means of integral transform, series, etc. Lamb (1904) studied the vibration of point sources and linear sources in single phased half-space, but for natural foundations it would be more appropriate to regard them as two-phased media. Halpern and Christiano (1986) and Jin (1999) studied the response of poroelastic half-space to steady-state harmonic surface tractions. Chen *et al*.(2002) also considered the vibration of point sources and linear sources in half-space.

However, all of the work mentioned above only considered the vibration of in half-space, which could not reflect the influence of the echoes generated by the bottom boundary. From her study of subsoil with subjacent rock-stratum, Zhang *et al*. (2002) got the integral solutions under harmonic force of low frequency, and Bougacha *et al*.(1986) got the solution of foundations with finite-element method. In this paper, Hankel transform was used to get the integral solution of the subsoil, and the

* Project (No. 81067) supported by Fok Ying Tung Education Foundation, Education Ministry of China

boundary conditions were combined to yield the analytic solutions of the soil with subjacent rock-stratum. The effects of the soil permeability and loading frequency on the soils were also studied in this work. Examples showed that the frequency of the force influences the results significantly and that the effect of the permeability of soil is influenced by the frequency.

EQUATIONS

When inertia coupling and the compressibility of soil and water are neglected, according to Biot's consolidation theory the axisymmetric balance equation of soil can be expressed as:

$$\frac{\lambda+G}{G}\frac{\partial e}{\partial r}+\nabla^2 u-\frac{u}{r^2}-\frac{1}{G}\frac{\partial\sigma}{\partial r}=\frac{\rho}{G}\ddot{u}+\frac{\rho_f}{G}\ddot{v}_r \tag{1a}$$

$$\frac{\lambda+G}{G}\frac{\partial e}{\partial z}+\nabla^2 w-\frac{1}{G}\frac{\partial\sigma}{\partial z}=\frac{\rho}{G}\ddot{w}+\frac{\rho_f}{G}\ddot{v}_z \tag{1b}$$

where λ and G are the Lame coefficients; u, w, v_r and v_z are the radial displacement of soil, the vertical displacement of soil, the radial displacement of water relative to soil, and the vertical displacement of water relative to soil respectively; σ, n, ρ_s and ρ_f are the porewater pressure, porosity, density of soil particles and density of water respectively;

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2},\ e=\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z},$$
$$\rho=(1-n)\rho_s+n\rho_f$$

The balance equation of water can be expressed as:

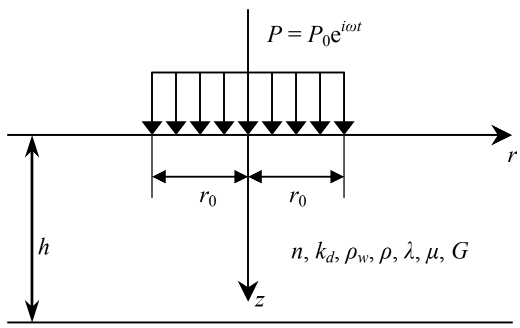

Fig.1 Description of the model and coordinate system

$$-\frac{\partial\sigma}{\partial r}=\frac{\rho_f g}{k_d}\dot{v}_r+\rho_f\ddot{u}+\frac{\rho_f}{n}\ddot{v}_r \tag{2a}$$

$$-\frac{\partial\sigma}{\partial z}=\frac{\rho_f g}{k_d}\dot{v}_z+\rho_f\ddot{w}+\frac{\rho_f}{n}\ddot{v}_z \tag{2b}$$

k_d and g are the dynamic coefficients of permeability and acceleration of gravity, respectively. When the compressibility of soils and water is neglected, the compatible equations can be expressed as:

$$-\left(\frac{\mathrm{d}v_r}{\mathrm{d}r}+\frac{v_r}{r}+\frac{\mathrm{d}v_z}{\mathrm{d}z}\right)=\left(\frac{\mathrm{d}u}{\mathrm{d}r}+\frac{u}{r}+\frac{\mathrm{d}w}{\mathrm{d}z}\right) \tag{3}$$

SOLUTION

A flexible circular foundation rests on the layered soils. The z-axis of the cylindrical coordinate system coincides with the vertical axis of symmetry of the foundation. The vibration of the soils is induced by the loading $P_0e^{i\omega t}$, where ω is the angular velocity of the loading and $i=\sqrt{-1}$.

It is convenient to introduce the dimensionless constants and variables:

$$\tilde{r}=\frac{r}{r_0},\quad \tilde{z}=\frac{z}{r_0},\quad \tilde{u}=\frac{u}{r_0},\quad \tilde{w}=\frac{w}{r_0},\quad \tilde{w}_0=\frac{w_0}{r_0},\quad \tilde{v}_z=\frac{v_z}{r_0},$$

$$\tilde{v}_r=\frac{v_r}{r_0},\quad \tilde{\rho}_f=\frac{\rho_f}{\rho},\quad \tilde{\rho}_s=\frac{\rho_s}{\rho},\quad \tilde{\lambda}=\frac{\lambda}{G},\quad \tilde{M}=\frac{M}{G},$$

$$a_0=\sqrt{\frac{\rho}{G}}r_0\omega,\quad \tilde{b}=\frac{r_0}{\sqrt{\rho G}}b,\quad \tilde{\sigma}_z=\frac{\sigma_z}{G},\quad \tilde{\sigma}=\frac{\sigma}{G},$$

$$\tilde{\tau}_{rz}=\frac{\tau_{rz}}{G},\ \tilde{Q}=\frac{r_0 Q}{kG}$$

where r_0 is the radius of the foundation.

After introducing the dimensionless variables, Eqs.(1a)–(3) can be written as:

$$(\tilde{\lambda}+1)\frac{\partial e}{\partial\tilde{r}}+\nabla^2\tilde{u}-\frac{\tilde{u}}{\tilde{r}^2}-\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}=\tilde{\rho}a_0^2\tilde{u}+\tilde{\rho}_f a_0^2\tilde{v}_r \tag{4a}$$

$$(\tilde{\lambda}+1)\frac{\partial e}{\partial\tilde{z}}+\nabla^2\tilde{w}-\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}=\tilde{\rho}a_0^2\tilde{w}+\tilde{\rho}_f a_0^2\tilde{v}_z \tag{4b}$$

$$-\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}=\tilde{b}a_0\tilde{v}_r+\tilde{\rho}_f\tilde{u}a_0^2+\frac{\tilde{\rho}_f}{n}\tilde{v}_r a_0^2 \quad (5a)$$

$$-\frac{\partial\tilde{\sigma}}{\partial\tilde{z}}=\tilde{b}a_0\tilde{v}_z+\tilde{\rho}_f\tilde{w}a_0^2+\frac{\tilde{\rho}_f}{n}\tilde{v}_z a_0^2 \quad (5b)$$

$$-\left(\frac{\mathrm{d}\tilde{v}_r}{\mathrm{d}\tilde{r}}+\frac{\tilde{v}_r}{\tilde{r}}+\frac{\mathrm{d}\tilde{v}_z}{\mathrm{d}\tilde{z}}\right)=\left(\frac{\mathrm{d}\tilde{u}}{\mathrm{d}\tilde{r}}+\frac{\tilde{u}}{\tilde{r}}+\frac{\mathrm{d}\tilde{w}}{\mathrm{d}\tilde{z}}\right) \quad (6)$$

From Eqs.(5a) and (5b) we can get:

$$\tilde{v}_r=c(\tilde{u}-\frac{1}{\tilde{\rho}_f{a_0}^2}\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}),\ \overline{v}_z=c(\overline{w}-\frac{1}{\overline{\rho}_f{a_0}^2}\frac{\partial\overline{\sigma}}{\partial\overline{z}}),$$

$$c=\frac{na_0\tilde{\rho}_f}{\tilde{b}ni-\tilde{\rho}_f a_0}$$

Introduce the equations above into Eqs.(4a) and (4b):

$$(\tilde{\lambda}+1)\frac{\partial e}{\partial\tilde{r}}+\nabla^2\tilde{u}-\frac{\tilde{u}}{\tilde{r}^2}-(1+c)\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}=-a_0^2(\tilde{\rho}+c\tilde{\rho}_f)\tilde{u} \quad (7a)$$

$$(\tilde{\lambda}+1)\frac{\partial e}{\partial\tilde{z}}+\nabla^2\tilde{w}-(1+c)\frac{\partial\tilde{\sigma}}{\partial\tilde{z}}=-a_0^2(\tilde{\rho}+c\tilde{\rho}_f)\tilde{w} \quad (7b)$$

$\frac{\partial}{\partial r}$ (7a)+ $\frac{1}{r}$ (7a)+ $\frac{\partial}{\partial z}$ (7b), then we can get：

$$\alpha\nabla^2 e-\nabla^2\tilde{\sigma}+\beta e=0$$

$$\alpha=\frac{\tilde{\lambda}+2}{1+c},\quad \beta=\frac{\omega^2(\rho+c\rho_f)}{(1+c)G} \quad (8a)$$

Introduce Eqs.(7a) and (7b) into Eq.(6):

$$\nabla^2\tilde{\sigma}=Ee,\quad E=\frac{(1+c)\rho_f\omega^2}{cG} \quad (8b)$$

Then from Eqs.(8b) and (8a) we can get:

$$\nabla^2 e=De,\quad D=\frac{\rho_f+2c\rho_f-c\rho}{(\lambda+2G)c}\omega^2 \quad (8c)$$

We denote the Hankel transform of the first order and the zero order of a function $\tilde{f}(r,z)$ by $\overline{f}^1(p,z)$ and $\overline{f}(p,z)$ respectively.

$$\overline{f}(p,z)=\int_0^\infty r\tilde{f}(r,z)J_0(pr)\mathrm{d}r\,;$$

$$\overline{f}^1(p,z)=\int_0^\infty r\tilde{f}(r,z)J_1(pr)\mathrm{d}r\,.$$

And sometimes we translate $\overline{f}^1(p,z)$ into $\tilde{f}(r,z)$ to reduce the number of the unknown variables.

By means of zero order Hankel transform, Eqs. (7b), (8b) and (8c) yield

$$\frac{\mathrm{d}^2\overline{e}}{\mathrm{d}\tilde{z}^2}-q^2\overline{e}=0,\quad q^2=\xi^2+D \quad (9a)$$

$$\frac{\mathrm{d}^2\overline{\sigma}}{\mathrm{d}\tilde{z}^2}-\frac{1}{G}\xi^2=E\overline{e} \quad (9b)$$

$$\frac{\mathrm{d}^2\overline{w}}{\mathrm{d}\tilde{z}^2}-s^2\overline{w}=(1+c)\frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\tilde{z}}-(\tilde{\lambda}+1)\frac{\mathrm{d}\overline{e}}{\mathrm{d}\tilde{z}} \quad (9c)$$

$$s^2=\xi^2-\omega^2(\rho+c\rho_f)/G$$

And by means of first order Hankel transforms, Eq. (7a) and $e=\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}$ yield

$$\frac{\mathrm{d}^2\overline{u}^1}{\mathrm{d}\tilde{z}^2}-s^2\overline{u}^1=(\tilde{\lambda}+1)\xi\overline{e}-(1+c)\xi\overline{\sigma} \quad (9d)$$

$$\overline{e}=\xi\overline{u}^1+\frac{\mathrm{d}\overline{w}}{\mathrm{d}\tilde{z}} \quad (9e)$$

Then from Eqs.(9a)–(9e), we can get:

$$\overline{e}=A_1\mathrm{e}^{q\tilde{z}}+B_1\mathrm{e}^{-q\tilde{z}}, \quad (10a)$$

$$\overline{\sigma}=C_{21}A_1\mathrm{e}^{q\tilde{z}}+C_{21}B_1\mathrm{e}^{-q\tilde{z}}+A_2\mathrm{e}^{\xi\tilde{z}}+B_2\mathrm{e}^{-\xi\tilde{z}} \quad (10b)$$

$$C_{21}=\frac{E}{D}$$

$$\begin{aligned}\overline{u}^1=&C_{31}A_1\mathrm{e}^{q\tilde{z}}+C_{31}B_1\mathrm{e}^{-q\tilde{z}}+C_{32}A_2\mathrm{e}^{\xi\tilde{z}}\\&+C_{32}B_2\mathrm{e}^{-\xi\tilde{z}}+A_3\mathrm{e}^{s\tilde{z}}+B_3\mathrm{e}^{-s\tilde{z}}\end{aligned} \quad (10c)$$

$$C_{31}=\frac{\xi}{(q^2-s^2)G}[(\lambda+G)-(1+c)C_{21}]$$

$$C_{32}=-\frac{(1+c)\xi}{(\xi^2-s^2)G}$$

$$\begin{aligned}\overline{w}=&C_{41}A_1\mathrm{e}^{q\tilde{z}}-C_{41}B_1\mathrm{e}^{-q\tilde{z}}+C_{42}A_2\mathrm{e}^{\xi\tilde{z}}\\&-C_{42}B_2\mathrm{e}^{-\xi\tilde{z}}+A_4\mathrm{e}^{s\tilde{z}}+B_4\mathrm{e}^{-s\tilde{z}}\end{aligned} \quad (10d)$$

$$C_{41}=-\frac{q}{\xi}C_{31},\ C_{42}=-C_{32},\ A_3=-\frac{s}{\xi}A_4,\ B_3=\frac{s}{\xi}B_4$$

And for $\tau_{zr}=G(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}),\ \bar{\sigma}_z=\lambda\bar{e}+2G\frac{\mathrm{d}\bar{w}}{\mathrm{d}z}$ and $Q=k\frac{\partial\sigma}{\partial z}$, we can get:

$$\tilde{\tau}_{zr}=\frac{\partial\tilde{u}}{\partial\tilde{z}}+\frac{\partial\tilde{w}}{\partial\tilde{r}},\ \tilde{\sigma}_z=\tilde{\lambda}e+2\frac{\mathrm{d}\tilde{w}}{\mathrm{d}\tilde{z}},\ \tilde{Q}=\frac{\partial\tilde{\sigma}}{\partial\tilde{z}}$$

By means of Hankel transform, the equations above yield

$$\bar{\tau}_{zr}^1=2qC_{31}(A_1\mathrm{e}^{q\tilde{z}}-B_1\mathrm{e}^{-q\tilde{z}})+2\xi C_{32}(A_2\mathrm{e}^{\xi\tilde{z}}-B_2\mathrm{e}^{-\xi\tilde{z}})+\frac{s^2+\xi^2}{s}(A_3\mathrm{e}^{s\tilde{z}}-B_3\mathrm{e}^{-s\tilde{z}}) \quad (10e)$$

$$\bar{\sigma}_z=(\tilde{\lambda}-2\frac{q^2}{\xi}C_{31})(A_1\mathrm{e}^{q\tilde{z}}+B_1\mathrm{e}^{-q\tilde{z}})-2\xi C_{32}(A_2\mathrm{e}^{\xi\tilde{z}}+B_2\mathrm{e}^{-\xi\tilde{z}})-2\xi(A_3\mathrm{e}^{s\tilde{z}}+B_3\mathrm{e}^{-s\tilde{z}}) \quad (10f)$$

$$\bar{Q}=\xi C_{21}q(A_1\mathrm{e}^{q\tilde{z}}-B_1\mathrm{e}^{-q\tilde{z}})+\xi(A_2\mathrm{e}^{\xi\tilde{z}}-B_2\mathrm{e}^{-\xi\tilde{z}}) \quad (10g)$$

BOUNDARY CONDITIONS

u, w, σ_x, τ_{rx}, σ and Q are respectively the radial displacement of soil, the vertical displacement of soil, the vertical stress, the shear stress, the pore-water pressure and the quantity of the water flowing out of the soil, respectively. At the ground surface where the thickness is equal to zero, Eqs.(10b)–(10g) can be expressed as:

$$[\bar{u}^0\ \ \bar{w}^0\ \ \bar{\sigma}_z^0\ \ \bar{\tau}_{rz}^0\ \ \bar{\sigma}^0\ \ \bar{Q}^0]^{\mathrm{T}}=T^0{}_{6\times6}[A_1\ \ B_1\ \ A_2\ \ B_2\ \ A_3\ \ B_3]^{\mathrm{T}} \quad (11a)$$

Then we can get the same expression at the subjacent rock-stratum as:

$$[\bar{u}^h\ \ \bar{w}^h\ \ \bar{\sigma}_z^h\ \ \bar{\tau}_{rz}^h\ \ \bar{\sigma}^h\ \ \bar{Q}^h]^{\mathrm{T}}=T^h{}_{6\times6}[A_1\ \ B_1\ \ A_2\ \ B_2\ \ A_3\ \ B_3]^{\mathrm{T}} \quad (11b)$$

The figures/elements in $T^0{}_{6\times6}$ and $T^h{}_{6\times6}$ can be respectively obtained from Eqs.(7b)–(7g).

From Eqs.(8a) and (8b), we can get:

$$[\bar{u}^0\ \ \bar{w}^0\ \ \bar{\sigma}_z^0\ \ \bar{\tau}_{rz}^0\ \ \bar{\sigma}^0\ \ \bar{Q}^0]^{\mathrm{T}}=T_{6\times6}[\bar{u}^h\ \ \bar{w}^h\ \ \bar{\sigma}_z^h\ \ \bar{\tau}_{rz}^h\ \ \bar{\sigma}^h\ \ \bar{Q}^h]^{\mathrm{T}} \quad (12)$$
$$T_{6\times6}=T^0{}_{6\times6}(T^h{}_{6\times6})^{-1}$$

At the ground surface, we can get: $\sigma_x=P(0\le r\le r_0)$ and $\tau_{rx}=\sigma=0$, where P is the load applied to the subsoil. And at the subjacent rock-stratum u, w, Q are known as $u=w=Q=0$. When the boundary conditions are combined, we can get the other unknowns with the values deduced above.

EXAMPLE AND DISCUSSION

With the equations listed above, we can calculate the dynamic response of the soils. The integrants are very oscillatory and time-consuming to obtain, and MATHEMATICA was applied to reduce the truncation error in the calculation.

The conditions of the examples are listed as follows:

Maximum pressure of the force: $\bar{p}=10$ kPa

Radius of the pressure: r_0=1 m

Boundary condition: the pore pressure of the surface is equal to zero; the subjacent boundary is unpenetrable.

Fig.2 shows the vertical displacements of the ground surface under different loading frequency and permeability. The solid line curves in the figures are the maximum displacements of the soil, and the broken line curves are the displacements when the loading force attains maximum pressure.

The figures obviously show that the fluctuation does not occur under static conditions. Because there is a rigid subjacent boundary, the wave motion created by the vibration of the force may interfere with its reversed echo in the soil. Thus, with the increase of the radius, the swing of the vertical displacement at the surface ground will not attenuate monotonously, but decreases with fluctuation. This phenomenon is also related with the freq-

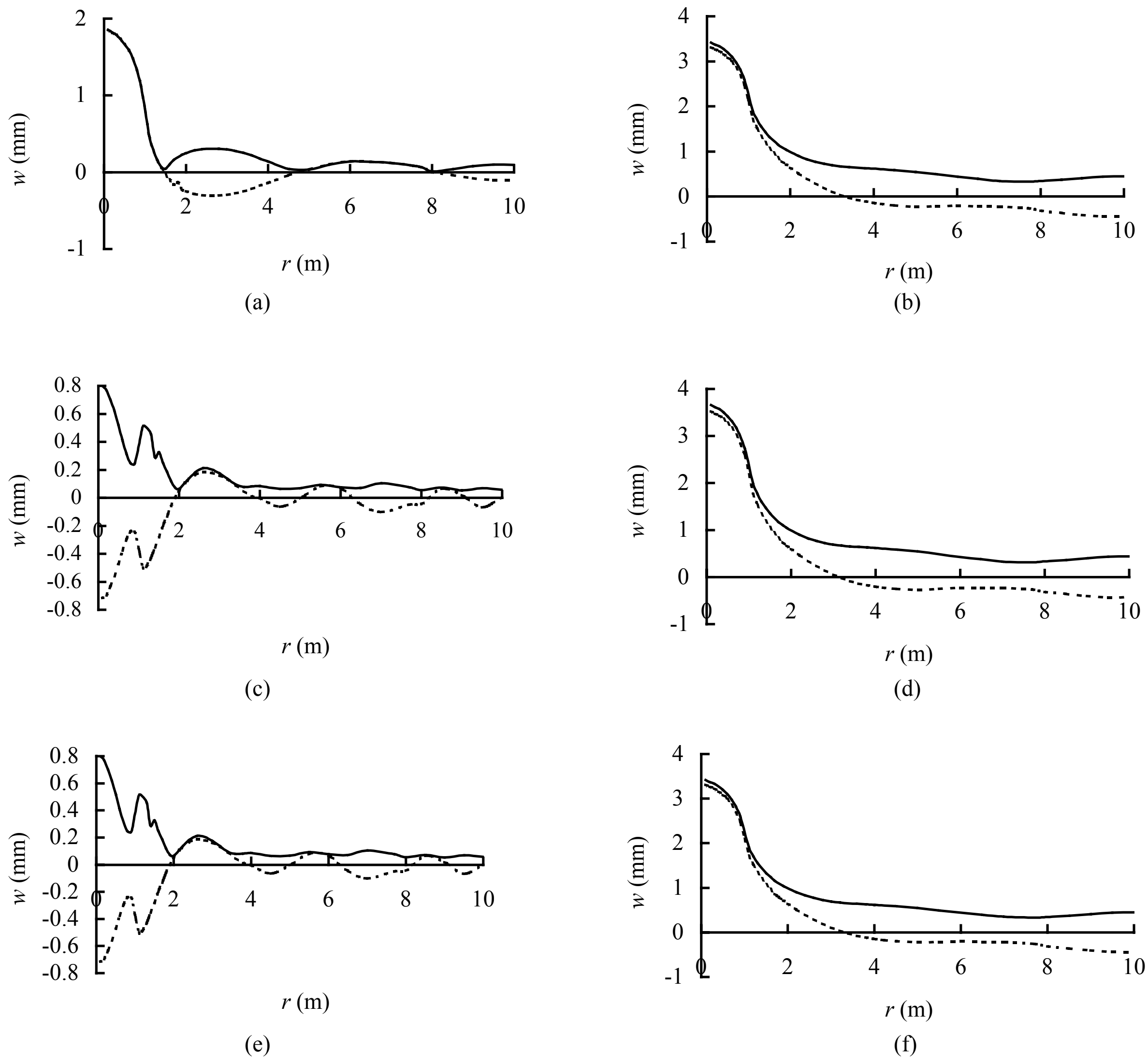

Fig.2 Curve of one-layered soils

(a) $a_0 = 3.46$, $\tilde{b} = 42.5$; (b) $a_0 = 0.346$, $\tilde{b} = 42.5$; (c) $a_0 = 3.46$, $\tilde{b} = 425$; (d) $a_0 = 0.346$, $\tilde{b} = 425$; (e) $a_0 = 3.46$, $\tilde{b} = 4250$; (f) $a_0 = 0.346$, $\tilde{b} = 4250$;

uency of the force. By comparing (a), (c) and (e) respectively with (b), (d) and (f) in Fig.2, we can see that the fluctuation of the soil is more obvious when ω equals 100 rad/s. We can also see from the broken line in the figures that the phase difference of the force and the displacement increases with the frequency of the force. These phenomena accord with the results of similar low-frequency situation presented by Zhang Yu-hong.

The permeability of the soil influences not only the seepage velocity but also the deformation of the soil skeleton. The figures show that with the decrease of the coefficient of permeability, the swings of the surface displacement also attenuate. This conclusion agrees with the results of Yang and Song (1999), who studied the dynamic problems of soil foundations by finite-element method. But what is remarkable here is the combined effect of the soil permeability and the loading frequency. At low frequency the effect is not obvious. When a_0 equals

0.346, the swing of the soils in which $\tilde{b}$ equals 4250 differs only by about 10% from that of the soils in which $\tilde{b}$ equals 42.5. But at a higher frequency the difference becomes more obvious. When a_0 equals 3.46, the swing of the soils in which $\tilde{b}$ equals 4250 is two-fifth of that of the soils in which $\tilde{b}$ equals 42.5. We can also see from the figures that if $\tilde{b}$ goes beyond certain ranges, its change will have little influence on the result. For example, the swing of (e) differs only by about 20% of that of (c), and their curves are quite similar. These phenomena accord with the result of soils of the half-space model studied by Chen *et al*.(2002).

Fig.3 shows the curve of the vertical swing at the center point of the load under different loading frequencies. Generally speaking, when the swing of the dynamic load is invariable, the work done by the dynamic load in a half cycle decreases with the increase of the frequency. Thus the vertical swing, which is mainly determined by the work, may also generally attenuate. For example, the swing of the center point shown in Fig.2a is obviously less than that shown in Fig.2b. But from the curve we can see that it does not decrease monotonously. Within the range of the frequency shown in the figure, the peak value of the swing occurs at about 18.5 Hz.

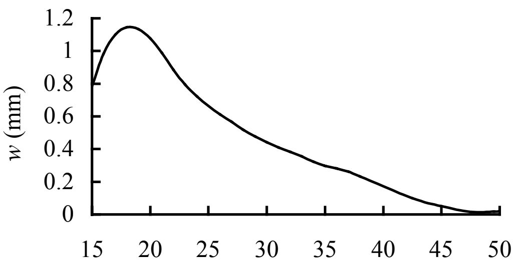

Fig.3 Curve of vertical displacement of the loading center for different frequency

CONCLUSION

The dynamic interaction problems of harmonic vibration of a circular flexible foundation were studied for the first time by analysis method, with the displacement of water relative to soil and the influence of a subjacent rock stratum considered. From the numerical results, conclusions can be drawn that the response of the soils strongly depends on both the properties of the saturated soil-foundation system and the load acting on the foundation, and other factors such as the frequency of the load and Darcy's permeability coefficient of the medium. These should not be neglected in determining the response of structures to dynamic loadings. Because some integrants for computing problems cannot be calculated at the present time, in this paper we only presented the vertical displacements of the soils at the ground surface, and other results will come out with further studies.

References

Biot, M.A., 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid. *The Journal of the Acoustical Society of America*, **28**(2):168-191.

Bougacha, S., Tassoulas, J.L., Roesset, J.M., 1986. Analysis of foundations on fluid-filled poroelastic stratum. *J. Eng. Mech, ASCE*, **119**:1632-1648.

Halpern, M.R., Christiano, P., 1986. Response of poroelastic half-space to steady state harmonic surface tractions. *Int. J. Numer. Anal. Mech. Geomech*, **10**:609-632.

Jin, B., 1999. The vertical vibrations of an elastic circular plate on a fluid-saturated porous half-space. *Int. J. Eng. Sci*., **37**:379-393.

Lamb, H., 1904. On the propagation of tremors over the surface of an elastic solid. *Phil Trans Roy Soc*, **203**:1-42.

Yang, J., Song, E.X., 1999. Dynamic analysis of infinite saturated soil foundation by finite elements. *J. of Tsinghua University*, **39**(12):82-85 (in Chinese).

Zhang, Y.H., Huang, Y., Wang, Z.J., 2002. Analysis procedures of dynamic response of 3-D non-axisymmetric layered saturated soil. *Earthquake Engineering and Engineering Vibration*, **22**(4):47-52.