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On stochastic optimal control of partially observable nonlinear quasi Hamiltonian systems*

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Abstract: A stochastic optimal control strategy for partially observable nonlinear quasi Hamiltonian systems is proposed. The optimal control forces consist of two parts. The first part is determined by the conditions under which the stochastic optimal control problem of a partially observable nonlinear system is converted into that of a completely observable linear system. The second part is determined by solving the dynamical programming equation derived by applying the stochastic averaging method and stochastic dynamical programming principle to the completely observable linear control system. The response of the optimally controlled quasi Hamiltonian system is predicted by solving the averaged Fokker-Planck-Kolmogorov equation associated with the optimally controlled completely observable linear system and solving the Riccati equation for the estimated error of system states. An example is given to illustrate the procedure and effectiveness of the proposed control strategy.

Key words: Nonlinear system, Partially observation, Stochastic optimal control, Separation principle, Stochastic averaging, Dynamical programming

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INTRODUCTION

Since many actual control systems such as those in structural engineering are subjected to random excitations and the system states are estimated from measurements with random noise, stochastic optimal control of partially observable systems is a research subject of much significance. One basic approach to this problem is to convert it into the stochastic optimal control of completely observable systems using separation theorem (Wonham, 1968; Fleming and Rishel, 1975; Bensoussan, 1992) and then to solve the later problem. For partially observable linear systems, the converted completely observable systems are of finite dimension and the later problem can be solved easily, e.g., by using the linear quadratic Gaussian (LQG) control strategy. A nonlinear stochastic optimal control strategy for partially observable linear systems was proposed recently by the present authors (Zhu and Ying, 2002). For partially observable nonlinear systems, the converted completely observable systems are usually of infinite dimension so that the problem can hardly be solved.

Charalambous and Elliott (1997; 1998) proved that if nonlinearities enter the dynamics of unobservable state with the observation being gradients of potential functions, then the partially observable

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nonlinear control system can be recast as completely observable linear system. The objective of the present letter is to present a summary account of the stochastic optimal control strategy for partially observable nonlinear quasi Hamiltonian systems proposed recently by us on the basis of separation theorem, Charalambous and Elliott theorem, and our previously proposed nonlinear stochastic optimal control strategy for completely observable quasi Hamiltonian systems (Zhu and Ying, 1999; Zhu *et al*., 2001). The full article will be submitted for publication elsewhere.

STOCHASTIC OPTIMAL CONTROL PROBLEM OF PARTIALLY OBSERVABLE STOCHASTICALLY EXCITED AND DISSIPATED HAMILTONIAN SYSTEMS

The stochastic optimal control problem of a partially observable nonlinear stochastically excited and dissipated Hamiltonian system can be formulated mathematically as

$$\mathrm{d}\boldsymbol{X}=\bar{\boldsymbol{A}}(\boldsymbol{X})\mathrm{d}t+\bar{\boldsymbol{U}}\mathrm{d}t+\boldsymbol{C}_1\mathrm{d}\boldsymbol{B}(t) \tag{1}$$

$$\mathrm{d}\boldsymbol{Y}=\bar{\boldsymbol{D}}(\boldsymbol{X})\mathrm{d}t+\boldsymbol{F}\bar{\boldsymbol{U}}\mathrm{d}t+\boldsymbol{C}_2\mathrm{d}\boldsymbol{B}(t)+\boldsymbol{C}_3\mathrm{d}\boldsymbol{B}_1(t) \tag{2}$$

$$J=E\{\int_0^T L(\boldsymbol{X},\boldsymbol{U})\mathrm{d}t+\Psi(\boldsymbol{X}(T))\} \tag{3a}$$

for finite time-interval control, or

$$J'=\lim_{T\to\infty}\frac{1}{T}\int_0^T L(\boldsymbol{X},\boldsymbol{U})\mathrm{d}t \tag{3b}$$

for semi-infinite time-interval ergodic control. In Eqs.(1)–(3),

$$\bar{\boldsymbol{A}}(\boldsymbol{X})=\begin{bmatrix}\partial H/\partial\boldsymbol{P}\\ -\partial H/\partial\boldsymbol{Q}-\boldsymbol{C}_0\partial H/\partial\boldsymbol{P}\end{bmatrix},$$

$$\bar{\boldsymbol{U}}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{U}\end{bmatrix},\ \boldsymbol{C}_1=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{K}_0\end{bmatrix} \tag{4}$$

$\boldsymbol{X}=[\boldsymbol{Q}^{\mathrm{T}},\boldsymbol{P}^{\mathrm{T}}]^{\mathrm{T}}$ is $2n$-dimensional system state vector; $\boldsymbol{Q}$ and $\boldsymbol{P}$ are n-dimensional generalized displacement and momentum vectors, respectively; $H=H(\boldsymbol{Q},\boldsymbol{P})$ is the Hamiltonian of the associated Hamiltonian system possibly modified by Wong-Zakai correction terms; $\boldsymbol{Y}$ is n_1-dimensional observation vector; $\bar{\boldsymbol{D}}(\boldsymbol{X})$ is n_1-dimensional function vector; $\boldsymbol{U}=\boldsymbol{U}(\boldsymbol{Q},\boldsymbol{P})$ is n-dimensional feedback control force vector; $\boldsymbol{B}(t)$ and $\boldsymbol{B}_1(t)$ are the m and m_1-dimensional independent Wiener process vectors, respectively; $\boldsymbol{C}_0=\boldsymbol{C}_0(\boldsymbol{Q},\boldsymbol{P})$ is $n\times n$-dimensional damping coefficient matrix possibly modified by Wong-Zakai correction terms; $\boldsymbol{K}_0$, $\boldsymbol{C}_1$, $\boldsymbol{F}$, $\boldsymbol{C}_2$, $\boldsymbol{C}_3$ are $n\times m$, $2n\times m$, $n_1\times 2n$, $n_1\times m$, and $n_1\times m_1$-dmimensional constant matrices, respectively. $E[\cdot]$ denotes expectation operator; $L(\boldsymbol{X},\boldsymbol{U})$ is continuous, differential and convex cost function; $\Psi(T)$ is terminal cost.

Let $\bar{\boldsymbol{U}}=\bar{\boldsymbol{U}}_1+\bar{\boldsymbol{U}}_2$, where $\bar{\boldsymbol{U}}_1=[\boldsymbol{0},\boldsymbol{U}_1^{\mathrm{T}}]^{\mathrm{T}}$ and $\bar{\boldsymbol{U}}_2=[\boldsymbol{0},\boldsymbol{U}_2^{\mathrm{T}}]^{\mathrm{T}}$, and combine $\bar{\boldsymbol{U}}_1$ with $\bar{\boldsymbol{A}}(\boldsymbol{X})$ and $\bar{\boldsymbol{D}}(\boldsymbol{X})$. Then Eqs.(1)–(3) become

$$\mathrm{d}\boldsymbol{X}=[\boldsymbol{A}\boldsymbol{X}+\boldsymbol{G}(\boldsymbol{X})]\mathrm{d}t+\bar{\boldsymbol{U}}_2\mathrm{d}t+\boldsymbol{C}_1\mathrm{d}\boldsymbol{B}(t) \tag{5}$$

$$\mathrm{d}\boldsymbol{Y}=[\boldsymbol{D}\boldsymbol{X}+\boldsymbol{E}(\boldsymbol{X})]\mathrm{d}t+\boldsymbol{F}\bar{\boldsymbol{U}}_2\mathrm{d}t+\boldsymbol{C}_2\mathrm{d}\boldsymbol{B}(t)+\boldsymbol{C}_3\mathrm{d}\boldsymbol{B}_1(t) \tag{6}$$

$$J_0=E\{\int_0^T L(\boldsymbol{X},\boldsymbol{U}_2)\mathrm{d}t+\Psi(\boldsymbol{X}(T))\} \tag{7a}$$

for finite time-interval control, or

$$J_0'=\lim_{T\to\infty}\frac{1}{T}\int_0^T L(\boldsymbol{X},\boldsymbol{U}_2)\mathrm{d}t \tag{7b}$$

for semi-infinite time-interval ergodic control, where $\boldsymbol{A}$ and $\boldsymbol{D}$ are $2n\times 2n$ and $n_1\times 2n$-dimensional constant matrices, respectively,

$$\boldsymbol{A}=\begin{bmatrix}\dfrac{\partial^2\bar{H}(0)}{\partial\boldsymbol{Q}\partial\boldsymbol{P}} & \dfrac{\partial^2\bar{H}(0)}{\partial\boldsymbol{P}^2}\\ -\dfrac{\partial^2\bar{H}(0)}{\partial\boldsymbol{Q}^2}-\dfrac{\partial}{\partial\boldsymbol{Q}}\left(\boldsymbol{C}_0(0)\dfrac{\partial\bar{H}(0)}{\partial\boldsymbol{P}}\right) & -\dfrac{\partial^2\bar{H}(0)}{\partial\boldsymbol{P}\partial\boldsymbol{Q}}-\dfrac{\partial}{\partial\boldsymbol{P}}\left(\boldsymbol{C}_0(0)\dfrac{\partial\bar{H}(0)}{\partial\boldsymbol{P}}\right)\end{bmatrix}$$

$$\boldsymbol{G}(\boldsymbol{X})=\bar{\boldsymbol{A}}(\boldsymbol{X})+\bar{\boldsymbol{U}}_1-\boldsymbol{A}\boldsymbol{X},\ \boldsymbol{D}=\frac{\partial}{\partial\boldsymbol{X}}(\bar{\boldsymbol{D}}(0)+\boldsymbol{F}\bar{\boldsymbol{U}}_1(0)),$$

$$\boldsymbol{E}(\boldsymbol{X})=\bar{\boldsymbol{D}}(\boldsymbol{X})+\boldsymbol{F}\bar{\boldsymbol{U}}_1-\boldsymbol{D}\boldsymbol{X} \tag{8}$$

Here $\overline{H}$ is the new Hamiltonian modified by $\boldsymbol{U}_1$.

CONVERTED STOCHASTIC OPTIMAL CONTROL PROBLEM OF COMPLETELY OBSERVABLE LINEAR SYSTEMS

$\boldsymbol{G}(\boldsymbol{X})$ and $\boldsymbol{E}(\boldsymbol{X})$ are the nonlinear terms in system (5) and observation (6), which are the combinations of $\boldsymbol{U}_1$ with the nonlinear terms in original system (1) and observation (2). To convert the stochastic optimal control problem of partially observable nonlinear system governed by Eqs.(5)–(7) into one of completely observable linear system, according to Charalambous and Elliott (1997; 1998) theorem, $\boldsymbol{U}_1$ should be so selected that there is a potential function $\phi(\hat{\boldsymbol{X}},t)$ satisfying

$$\boldsymbol{G}(\hat{\boldsymbol{X}})=\boldsymbol{C}_1\boldsymbol{C}_1^{\mathrm{T}}\frac{\partial\phi(\hat{\boldsymbol{X}},t)}{\partial\hat{\boldsymbol{X}}},\quad \boldsymbol{E}(\hat{\boldsymbol{X}})=\boldsymbol{C}_2\boldsymbol{C}_1^{\mathrm{T}}\frac{\partial\phi(\hat{\boldsymbol{X}},t)}{\partial\hat{\boldsymbol{X}}} \quad (9)$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{2}\mathrm{tr}(\boldsymbol{C}_1\boldsymbol{C}_1^{\mathrm{T}}\frac{\partial^2\phi}{\partial\hat{\boldsymbol{X}}^2})+\frac{1}{2}\left|\boldsymbol{C}_1^{\mathrm{T}}\frac{\partial\phi}{\partial\hat{\boldsymbol{X}}}\right|^2 +(\boldsymbol{A}\hat{\boldsymbol{X}}+\overline{\boldsymbol{U}}_2)^{\mathrm{T}}\frac{\partial\phi}{\partial\hat{\boldsymbol{X}}}=0 \quad (10)$$

and the initial system state $\hat{\boldsymbol{X}}(0)=\boldsymbol{X}(0)$ should have the following probability density

$$p_0(\hat{\boldsymbol{X}})=\frac{1}{\sqrt{(2\pi)^n|\boldsymbol{\sigma}_0|}}\mathrm{e}^{-(\hat{\boldsymbol{X}}-\boldsymbol{m}_0)^{\mathrm{T}}\boldsymbol{\sigma}_0^{-1}(\hat{\boldsymbol{X}}-\boldsymbol{m}_0)/2}\times\mathrm{e}^{\phi(\hat{\boldsymbol{X}},0)} \quad (11)$$

If these requirements are satisfied, then the control problem is recast as stochastic optimal control of completely observable linear system governed by

$$\mathrm{d}\hat{\boldsymbol{X}}=(\boldsymbol{A}\hat{\boldsymbol{X}}+\overline{\boldsymbol{U}}_2)\mathrm{d}t+(\boldsymbol{R}_C\boldsymbol{D}^{\mathrm{T}}+\boldsymbol{C}_1\boldsymbol{C}_2^{\mathrm{T}})\boldsymbol{C}^{-1}\mathrm{d}\boldsymbol{V}_I \quad (12)$$

$$\mathrm{d}\boldsymbol{V}_I=\mathrm{d}\boldsymbol{Y}-\boldsymbol{D}\hat{\boldsymbol{X}}\mathrm{d}t \quad (13)$$

$$J_2=E\{\int_0^T L_2(\hat{\boldsymbol{X}},\boldsymbol{U}_2)\mathrm{d}t+\Psi_2(\hat{\boldsymbol{X}}(T))\} \quad (14a)$$

for finite time-interval control, or

$$J_2'=\lim_{T\to\infty}\frac{1}{T}\int_0^T L_2(\hat{\boldsymbol{X}},\boldsymbol{U}_2)\mathrm{d}t \quad (14b)$$

for semi-infinite time-interval ergodic control. In Eqs.(12)–(14), $\boldsymbol{V}_I$ is n_1-dimensional innovation process vector; $\boldsymbol{R}_C$ is the covariance matrix of state estimation error $\tilde{\boldsymbol{X}}=\boldsymbol{X}-\hat{\boldsymbol{X}}$, with Gaussian probability density

$$p(\tilde{\boldsymbol{X}})=\frac{1}{\sqrt{(2\pi)^n|\boldsymbol{R}_C|}}\mathrm{e}^{-\tilde{\boldsymbol{X}}^{\mathrm{T}}\boldsymbol{R}_C^{-1}\tilde{\boldsymbol{X}}/2} \quad (15)$$

$\boldsymbol{R}_C$ satisfies the following differential Riccati equation

$$\dot{\boldsymbol{R}}_C=\boldsymbol{A}\boldsymbol{R}_C+\boldsymbol{R}_C\boldsymbol{A}^{\mathrm{T}}-(\boldsymbol{R}_C\boldsymbol{D}^{\mathrm{T}}+\boldsymbol{C}_1\boldsymbol{C}_2^{\mathrm{T}})\boldsymbol{C}^{-1}(\boldsymbol{D}\boldsymbol{R}_C +\boldsymbol{C}_2\boldsymbol{C}_1^{\mathrm{T}})+\boldsymbol{C}_1\boldsymbol{C}_1^{\mathrm{T}} \quad (16a)$$

for finite time-interval control, or algebraic Riccati equation

$$\boldsymbol{A}\boldsymbol{R}_C+\boldsymbol{R}_C\boldsymbol{A}^{\mathrm{T}}-(\boldsymbol{R}_C\boldsymbol{D}^{\mathrm{T}}+\boldsymbol{C}_1\boldsymbol{C}_2^{\mathrm{T}})\boldsymbol{C}^{-1}(\boldsymbol{D}\boldsymbol{R}_C +\boldsymbol{C}_2\boldsymbol{C}_1^{\mathrm{T}})+\boldsymbol{C}_1\boldsymbol{C}_1^{\mathrm{T}}=\boldsymbol{0} \quad (16b)$$

for semi-infinite time-interval ergodic control.

OPTIMAL CONTROL LAW

The stochastic optimal control problem of completely observable linear system governed by Eqs.(12)–(14) can be solved by using LQG strategy. However, it will be more effective and efficient if the nonlinear stochastic optimal control strategy previously proposed by us (Zhu and Ying, 1999; 2002; Zhu *et al*., 2001) can be applied. Assume that the dissipation, stochastic excitation and control in Eq.(12) are of the same small order. Then applying the stochastic averaging method for quasi integrable Hamiltonian systems (Zhu *et al*., 1997) to Eq.(12) yields the controlled diffusion process $\hat{\boldsymbol{H}}$ governed by

$$\mathrm{d}\hat{\boldsymbol{H}}=\left[\boldsymbol{m}(\hat{\boldsymbol{H}})+ <(\frac{\partial\hat{\boldsymbol{H}}}{\partial\hat{\boldsymbol{P}}})^{\mathrm{T}}\boldsymbol{U}_2>\right]\mathrm{d}t+\boldsymbol{\sigma}(\hat{\boldsymbol{H}})\mathrm{d}\boldsymbol{B}_3(t) \quad (17)$$

where $\hat{\boldsymbol{H}}=[\hat{H}_1,\ \hat{H}_2,\ \cdots,\hat{H}_n]^{\mathrm{T}}$, $\hat{H}_i$ is the ith modal energy; <·> denotes averaging operation; $\boldsymbol{B}_3(t)$ is Wiener process vector;

$$m_i(\hat{\boldsymbol{H}}) =<-\sum_{j,k=1}^{n}\bar{c}_{jk}\ \frac{\partial \hat{H}_i}{\partial \hat{P}_j}\ \frac{\partial \hat{H}}{\partial \hat{P}_k} +\int_{-\infty}^{0}\sum_{k,l=1}^{n_1}\sum_{j=1}^{n}\sum_{r,s=1}^{2n}\left[\left(\frac{\partial \hat{H}_j}{\partial \hat{X}_s}f_{sl}\right)_{t+\tau}\frac{\partial}{\partial \hat{H}_j}\left(\frac{\partial \hat{H}_i}{\partial \hat{X}_r}f_{rk}\right)_t +\left(\frac{\partial \hat{\theta}_j}{\partial \hat{X}_s}f_{sl}\right)_{t+\tau}\frac{\partial}{\partial \hat{\theta}_j}\left(\frac{\partial \hat{H}_i}{\partial \hat{X}_r}f_{rk}\right)_t\right]R_{kl}(\tau)\mathrm{d}\tau > \quad (18)$$

$$\sum_{k=1}^{m_1}\sigma_{ik}(\hat{\boldsymbol{H}})\ \sigma_{jk}(\hat{\boldsymbol{H}}) = <\int_{-\infty}^{\infty}\sum_{k,l=1}^{n_1}\ \sum_{r,s=1}^{2n}\left(\frac{\partial \hat{H}_j}{\partial \hat{X}_s}f_{sl}\right)_{t+\tau} \times\left(\frac{\partial \hat{H}_i}{\partial \hat{X}_r}f_{rk}\right)_t R_{kl}(\tau)\ \mathrm{d}\tau > \quad (19)$$

in which $\bar{c}_{jk}$ is damping coefficient dependent on $\boldsymbol{A}$ in Eq.(8); $\hat{\theta}_j$ is generalized phase process; f_{rk} is the element of matrix $(\boldsymbol{R}_C\boldsymbol{D}^{\mathrm{T}}+\boldsymbol{C}_1\boldsymbol{C}_2^{\mathrm{T}})\boldsymbol{C}^{-1}$; $R_{kl}(\tau)$ is the correlation function of $\boldsymbol{V}_l(t)$. Performance index (14) is recast as

$$J_3 = E\{\int_0^T < L_3(\hat{\boldsymbol{H}},\ \boldsymbol{U}_2) > \mathrm{d}t + \Psi_3(\hat{\boldsymbol{H}}(T))\} \quad (20)$$

for finite time-interval control, or

$$J_4 = \lim_{T\to\infty}\frac{1}{T}\int_0^T < L_3(\hat{\boldsymbol{H}},\ \boldsymbol{U}_2) > \mathrm{d}t \quad (21)$$

for semi-infinite time-interval ergodic control.

In Eq.(20) or Eq.(21), let $L_3(\hat{\boldsymbol{H}},\boldsymbol{U}_2) = g(\hat{\boldsymbol{H}}) + \boldsymbol{U}_2^{\mathrm{T}}\boldsymbol{R}\boldsymbol{U}_2$, then the optimal control force $\boldsymbol{U}_2^*$ is obtained from minimizing the stochastic dynamical programming equation as

$$\boldsymbol{U}_2^* = -\frac{1}{2}\boldsymbol{R}^{-1}\ \frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}} \quad (22)$$

where V is the solution to the final dynamical programming equation

$$\frac{\partial V}{\partial t}+\frac{1}{2}\mathrm{tr}\left(\boldsymbol{\sigma\sigma}^{\mathrm{T}}\ \frac{\partial^2 V}{\partial \hat{\boldsymbol{H}}^2}\right)+\boldsymbol{m}^{\mathrm{T}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}} - \frac{1}{4}<\left(\frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}}\right)^{\mathrm{T}} R^{-1}\ \frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}} > + g(\hat{\boldsymbol{H}}) = 0 \quad (23)$$

for finite time-interval control, or

$$\frac{1}{2}\mathrm{tr}\left(\boldsymbol{\sigma\sigma}^{\mathrm{T}}\ \frac{\partial^2 V}{\partial \hat{\boldsymbol{H}}^2}\right)+\boldsymbol{m}^{\mathrm{T}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}} - \frac{1}{4}<\left(\frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}}\right)^{\mathrm{T}} R^{-1}\ \frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}}\ \frac{\partial V}{\partial \hat{\boldsymbol{H}}} > + g(\hat{\boldsymbol{H}}) = \lambda \quad (24)$$

for semi-infinite time-interval ergodic control. The total optimal control force is then $\boldsymbol{U}^*=\boldsymbol{U}_1+\boldsymbol{U}_2^*$.

PERFORMANCE

Substituting $\boldsymbol{U}_2^*$ in Eq.(22) into Eq.(17) and averaging the terms involving $\boldsymbol{U}_2^*$ yield

$$\mathrm{d}\hat{\boldsymbol{H}} = \bar{\boldsymbol{m}}(\hat{\boldsymbol{H}})\mathrm{d}t + \boldsymbol{\sigma}(\hat{\boldsymbol{H}})\mathrm{d}\boldsymbol{B}_3(t) \quad (25)$$

where

$$\bar{\boldsymbol{m}}(\hat{\boldsymbol{H}}) = \boldsymbol{m}(\hat{\boldsymbol{H}}) + <(\frac{\partial \hat{\boldsymbol{H}}}{\partial \hat{\boldsymbol{P}}})^{\mathrm{T}}\boldsymbol{U}_2^* > \quad (26)$$

Solving the FPK equation associated with Itô Eq.(25) yields the probability density $p(\hat{\boldsymbol{H}},t)$ and mean square values $E[\hat{Q}_i^2]$ and $E[\hat{P}_i^2]$, while solving Riccati Eq.(16) yields mean square values $E[\tilde{Q}_i^2]=(\boldsymbol{R}_C)_{ii}$ and $E[\tilde{P}_i^2]=(\boldsymbol{R}_C)_{n+i,n+i}$. Then the total mean square values of generalized displacements and momenta are

$$E[Q_i^2]=E[\hat{Q}_i^2]+E[\tilde{Q}_i^2]\ ,\ E[P_i^2]=E[\hat{P}_i^2]+E[\tilde{P}_i^2] \quad (27)$$

from which the mean Hamiltonian $E[H_C]$ of optimally controlled system can be obtained. The mean Hamiltonian $E[H_{UC}]$ of uncontrolled system can be obtained by directly applying the stochastic averaging method to Eq.(1) with $\boldsymbol{U}$=0. Thus the control effectiveness is indicated by

$$K_1 = \frac{E[H_{UC}] - E[H_C]}{E[H_{UC}]} \times 100\% \tag{28}$$

EXAMPLE

The proposed control strategy has been applied to the ergodic control of Duffing oscillator subjected to Gaussian white noise excitation with system equation

$$\ddot{X}_1 + c\dot{X}_1 + aX_1 + bX_1^3 = e\xi(t) + u \tag{29}$$

and observation equation

$$\dot{Y} = \dot{X}_1 + e_1\xi_1(t) \tag{30}$$

Some numerical results are shown in Fig.1 showing that the proposed control strategy is very effective even for large observation noise.

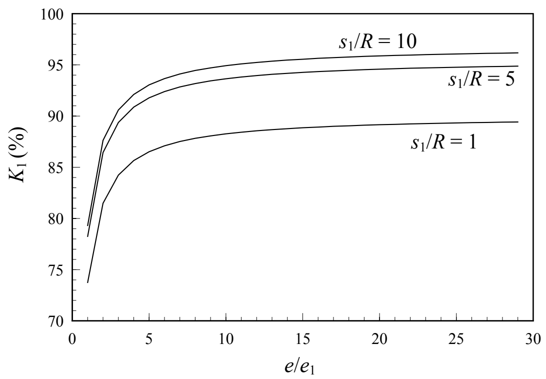

Fig.1 The percentage reduction K_1 of mean Hamiltonian versus ratio e/e_1 for different control parameter ratio s_1/R (c=0.1 and b/a=0.16)

References

Bensoussan, A., 1992. Stochastic Control of Partially Observable Systems. Cambridge University Press, Cambridge.

Charalambous, C.D., Elliott, R.J., 1997. Certain nonlinear partially observable stochastic optimal control problems with explicit control laws equivalent to LEQG/LQG problems. *IEEE Trans. Automatic Control*, **42**:482-497.

Charalambous, C.D., Elliott, R.J., 1998. Classes of nonlinear partially observable stochastic optimal control problems with explicit optimal control laws. *SIAM J. Control Optim.*, **36**:542-578.

Fleming, W.H., Rishel, R.W., 1975. Deterministic and Stochastic Optimal Control. Springer-Verlag, Berlin.

Wonham, W.M., 1968. On the separation theorem of stochastic control. *SIAM J. Control*, **6**:312-326.

Zhu, W.Q., Ying, Z.G., 1999. Optimal nonlinear feedback control of quasi-Hamiltonian systems. *Sci. China, Ser. A*, **42**:1213-1219.

Zhu, W.Q., Ying, Z.G., 2002. Nonlinear stochastic optimal control of partially observable linear structures. *Eng. Struct.*, **24**:333-342.

Zhu, W.Q., Huang, Z.L., Yang, Y.Q., 1997. Stochastic averaging of quasi-integrable Hamiltonian systems. *J. Appl. Mech. ASME*, **64**:975-984.

Zhu, W.Q., Ying, Z.G., Soong, T.T., 2001. An optimal nonlinear feedback control strategy for randomly excited structural systems. *Nonlinear Dyn.*, **24**:31-51.