Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Research on the behavior of fiber orientation probability distribution function in the planar flows*

ZHOU Kun (周 锟), LIN Jian-zhong (林建忠)†

(*Department of Mechanics, State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, China*)

†E-mail: mecjzlin@public.zju.edu.cn

Received Dec. 25, 2003; revision accepted May 13, 2004

Abstract: The equation of two-dimensional fiber direction vector was solved theoretically to give the fiber orientation distribution in simple shear flow, flow with two direction shears, extensional flow and arbitrary planar incompressible flow. The Fokker-Planck equation was solved numerically to validify the theoretical solutions. The stable orientation and orientation period of fiber were obtained. The results showed that the fiber orientation distribution is dependent on the relative not absolute magnitude of the matrix rate-of-strain of flow. The effect of fiber aspect ratio on the orientation distribution of fiber is insignificant in most conditions except the simple shear case. It was proved that the results for a planar flow could be generalized to the case of 3-D fiber direction vector.

Key words: Fiber suspension flow, Orientation distribution, Probability distribution function
doi:10.1631/jzus.2005.A0257 **Document code:** A **CLC number:** O359

INTRODUCTION

The orientation of fibers plays an important role in determining the rheological properties of fiber suspensions and the behaviors of composite materials produced by various forming operations. A fiber can be treated as a rigid cylinder with length l and diameter d. It is convenient to define three regimes of concentration: dilute, semi-concentrated and concentrated. Dilute regime is defined when $c<(d/l)^2$, where c is the volume fraction of fibers. In this regime fibers are free to rotate, and interactions between fibers are rare. When the volume fraction of fibers falls in the range $(d/l)^2<c<(d/l)$, the suspension falls in the semi-concentrated regime where the interactions of fibers are frequent. The concentrated regime is defined as $c>(d/l)$, where the space between fibers is of the order of d. The semi-concentrated regime is of most importance in the applications.

Jeffery studied the motion of an ellipsoid immersed in simple shear Newtonian flow with neglecting the inertia and Brownian rotation. He found ellipsoid rotated periodically and the trace of one end of the ellipsoid was characterized by the known Jeffery orbits. Bretherton (1962) showed that the same equations could be used to describe the motion of any axisymmetric particle provided that one used an equivalent aspect ratio r_e that is equal to the actual aspect ratio r_p for ellipsoidal particles, and that for cylindrical particle $r_e \approx 0.7r_p$. Batchelor (1970) developed a general constitutive equation for suspensions of particles of any shape in Newtonian liquids at arbitrary concentrations, and gave the relationship between the microstructure of particles and the macroscopic property of solution. Dinh and Armstrong (1984) developed a rheological state equation for semi-concentrated suspension of stiff fibers in a Newtonian solvent by using Batchelor's "cell model" approach. It is necessary to get the information of fiber orientation state when solving the rheological equation.

*Project (No. 10372090) supported by the National Natural Science Foundation of China

There are two main ways to depict fiber orientation: the probability distribution function for orientation, ψ, is the most general description; orientation tensor is another efficient description (Leal and Hinch, 1971; Hinch and Leal, 1975a; 1975b; Advani and Tucker, 1987; 1989; Lin and Zhang, 2003). Especially, Szeri and Leal (1992; 1994) adopted a double Lagrangian model to solve the orientation evolving equation, and Chinesta *et al.*(2003) used this method for numerically solving the Fokker-Planck equations in steady recirculating flows. Despite the compactness and efficiency of the orientation tensor, it has great deficiencies in accuracy (Altan *et al.*, 1989). Folgar and Tuckers (1984) developed a mathematical model to predict the orientation distribution function of rigid fibers in concentrated suspensions containing a phenomenological term to account for interactions between fibers. Shaqfeh and Koch (1990) showed that in extensional flows, the orientational dispersion induced by hydrodynamic interactions of fibers increased as the concentration was increased from infinite dilution and then ultimately decreased in the semi-concentrated regime. Stover *et al.*(1992) showed experimentally that the measured φ-distributions were similar to Jeffery's solution in simple shear flow of semi-dilute suspensions, where, φ was the meridian angle in the flow-gradient plane. Chiba and Nakamura (1998) studied numerically a fiber suspension flow through a backward-facing step channel, and used a statistical scheme for calculation of fiber orientation and coupled the fiber orientation with flow by iterating solution.

In this work we will give the analytical solution of the rotation equation for a fiber in planar flow, and then use a 4th-order Runge-Kutta method to solve the Fokker-Planck equations to confirm the analytical solution. Based on the solutions, the different behaviors of fiber orientation distribution in various planar flows are analyzed.

FIBER ORIENTATION IN PLANAR FLOW

Basic equations

When the inertia force is neglected, the rotational motion of a fiber in a Newtonian fluid can be described as (Advani and Tucker, 1987):

$$\dot{\boldsymbol{p}} = -\boldsymbol{\omega}\cdot\boldsymbol{p} + \lambda(\boldsymbol{\varepsilon}\cdot\boldsymbol{p} - \boldsymbol{\varepsilon}:\boldsymbol{ppp}) - \boldsymbol{D}_r\cdot\frac{1}{\psi}\frac{\partial\psi}{\partial\boldsymbol{p}} \tag{1}$$

where $\boldsymbol{p}$ is a unit direction vector parallel to the fiber axis; $\boldsymbol{\omega}=(\nabla\boldsymbol{u}^{\mathrm{T}}-\nabla\boldsymbol{u})/2$, is the vorticity tensor; $\boldsymbol{\varepsilon}=(\nabla\boldsymbol{u}^{\mathrm{T}}+\nabla\boldsymbol{u})/2$, is the deformation rate tensor; $\lambda=(r^2-1)/(r^2+1)$, r is aspect ratio of the fiber; $\boldsymbol{D}_r$ is the rotary diffusivity and equals to zero if the fibers interaction and Brownian rotation are neglected. Orientation probability distribution function ψ is described by the Fokker-Planck equation:

$$\frac{\partial\psi}{\partial t} = -\frac{\partial}{\partial\boldsymbol{p}}(\psi\dot{\boldsymbol{p}}) \tag{2}$$

In Cartesian coordinates moving with the velocity field, Eq.(2) can be deduced by considering the conservation of ψ.

Ignoring the fibers interaction and Brownian rotation, direction vector evolution equations of fiber moving in a planar flow are simplified as:

$$\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\lambda\left(p_x^3\frac{\partial u}{\partial x} + p_x^2 p_y\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) + p_x\left(p_y^2\frac{\partial v}{\partial y} - \frac{\partial u}{\partial x}\right) - \frac{1}{2}p_y\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right) + \frac{1}{2}p_y\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right) \tag{3a}$$

$$\frac{\mathrm{d}p_y}{\mathrm{d}t} = -\lambda\left(p_y^3\frac{\partial v}{\partial y} + p_x p_y^2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) + p_y\left(p_x^2\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - \frac{1}{2}p_x\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right) + \frac{1}{2}p_x\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{3b}$$

Simple shear flow

Simple shear flow is very important in rheological research and industrial processing, such as polymeric processing (Bird *et al.*, 1987).

For simple shear low, only one component of the velocity gradient is non-zero, presumed to be du/dy. Let $p_x=\sin\phi$, $p_y=\sin\phi$ (ϕ is the angle between the fiber axis and flow direction as shown in Fig.1), then Eqs.(3a) and (3b) are simplified by adding the square of Eqs.(3a) and (3b) as:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{2}(1-\lambda\cos 2\phi)\frac{\mathrm{d}u}{\mathrm{d}y} \tag{4}$$

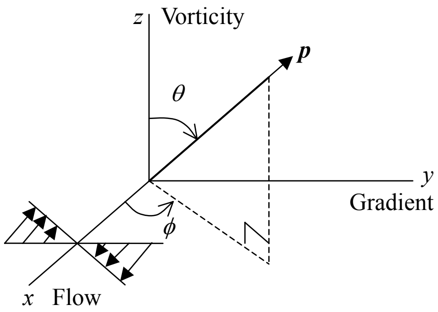

Fig.1 Coordinates and fiber

1. Stable orientation of fiber

If we let the right hand side of Eq.(4) be zero, we have

$$\cos 2\phi = \frac{1}{\lambda} \tag{5}$$

Only when λ=1 (infinite fiber) does a real solution exist (i.e., ordinary solution of differential Eq.(4)):

$$\phi = \frac{n\pi}{2}, \quad n \in Z$$

2. Orientation and rotation period of fiber

Integrating Eq.(4), we have

$$\phi = -\arctan\left\{\frac{\Delta \tanh\left[\frac{(t+C_i)\Delta}{2}\frac{\mathrm{d}u}{\mathrm{d}y}\right]}{\lambda+1}\right\}, \Delta = \sqrt{\lambda^2 - 1} \tag{6}$$

where C_i is an integration constant decided by the initial position of fiber. Notice that Δ is an imaginary number for λ<1 and hyperbolic function has period of 2πi (i is unit imaginary number), we obtain the fiber rotation period:

$$T = \frac{4\pi}{\sqrt{1-\lambda^2}\frac{\mathrm{d}u}{\mathrm{d}y}} = \frac{2\pi(r+1/r)}{\frac{\mathrm{d}u}{\mathrm{d}y}} \tag{7}$$

Eq.(7) is the same as Jeffery's result. We can see that the finite fiber will rotate periodically, while infinite fiber will align with the flow direction.

Flow with two direction shears

This case appears mostly when there is circumfluence in the flow.

In the flow with two direction shears, we keep du/dy as the basic variable, and express dv/dx as kdu/dy, where k is a proportional constant. Similar to subsection "Simple shear flow", we have

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{1}{2}[\lambda(k+1)\cos 2\phi + (k-1)]\frac{\mathrm{d}u}{\mathrm{d}y} \tag{8}$$

1. Stable orientation of fiber

If we set the right hand side of Eq.(8) to be zero, we have

$$\cos 2\phi = \frac{1-k}{\lambda(k+1)} \tag{9}$$

To obtain a real solution of ϕ, $\left|\frac{1-k}{\lambda(k+1)}\right| \le 1$ i.e.

$$\frac{1-\lambda}{1+\lambda} \le k \le \frac{1+\lambda}{1-\lambda} \tag{10}$$

should be satisfied.

2. Orientation and rotation period of fiber

Solution of Eq.(8) is:

$$\phi = \arctan\left\{\frac{\Delta \tanh\left[\frac{(t+C_i)\Delta}{2}\frac{\mathrm{d}u}{\mathrm{d}y}\right]}{k(\lambda-1)+\lambda+1}\right\} \tag{11}$$

$$\Delta = \sqrt{\lambda^2(k+1)^2 - (k-1)^2} \tag{12}$$

Notice that when k is out of the range, Δ will be an imaginary number, then we can obtain the fiber rotation period:

$$T = \frac{4\pi}{\sqrt{(k-1)^2 - \lambda^2(k+1)^2}\frac{\partial u}{\partial y}} \tag{13}$$

Under the condition of Eq.(10), the fiber will reach an asymptotic orientation, otherwise, it will rotate periodically.

Extensional flow

Many extrusion processes accord with this topic. The continuity in incompressible planar flow gives

$\partial v/\partial y=-\partial u/\partial x$, so we have

$$\frac{\mathrm{d}\phi}{\mathrm{d}t}=-\lambda\sin 2\phi\frac{\mathrm{d}u}{\mathrm{d}x} \tag{14}$$

The fiber stable orientation is $\phi=n\pi/2$, and the fiber orientation is:

$$\phi=-\frac{\mathrm{i}}{2}\ln\left(\frac{1-\Delta^2}{1+\Delta^2}+\mathrm{i}\frac{2\Delta}{1+\Delta^2}\right),\ \Delta=\mathrm{e}^{-2\lambda(t+C_i)\frac{\mathrm{d}u}{\mathrm{d}x}}$$

In the extensional flow, the fiber will align with the extensional direction.

Arbitrary planar incompressible flow

Mixing, dispersion, rotational molding etc. are all examples of arbitrary planar incompressible non-Newtonian flow.

Selecting $\partial u/\partial y$ as basic variable, then $\partial v/\partial x=k\partial u/\partial y$, $\partial u/\partial x=j\partial u/\partial y$, $\partial v/\partial y=-j\partial u/\partial y$ (k, $j\neq 0$ are different proportional constants), so that the fiber rotation can be described by

$$\frac{\mathrm{d}\phi}{\mathrm{d}t}=\frac{1}{2}[\lambda(k+1)\cos 2\phi+k-1-2\lambda j\sin 2\phi]\frac{\partial u}{\partial y} \tag{15}$$

When fiber orientation stabilizes, $\lambda(k+1)\cos 2\phi+k-1-2\lambda j\sin 2\phi=0$, then

$$\tan\phi=\frac{-2\lambda j\pm\Delta}{\lambda(k+1)+1-k}$$
$$\Delta=\sqrt{\lambda^2((k+1)^2+4j^2)-(k-1)^2} \tag{16}$$

If $4j^2\geq(k-1)^2/\lambda^2-(k+1)^2$, Δ is a real number, we have

$$\phi_1=\arctan\frac{-2\lambda j+\Delta}{\lambda(k+1)+1-k}$$
$$\phi_2=\arctan\frac{-2\lambda j-\Delta}{\lambda(k+1)+1-k} \tag{17}$$

If $4j^2<(k-1)^2/\lambda^2-(k+1)^2$, Δ is an imaginary number, we have

$$\phi=\mathrm{Re}\left(\arctan\frac{-2\lambda j+\Delta}{\lambda(k+1)+1-k}\right)$$

where arc tangent is defined in the complex plane, and Re( ) means to take the real part.

Solution of Eq.(15) is:

$$\phi=-\arctan\left\{\frac{2\lambda j-\Delta\tanh\left[\frac{(t+C_i)\Delta}{2}\frac{\partial u}{\partial y}\right]}{k(\lambda-1)+\lambda+1}\right\}$$

When $4j^2<(k-1)^2/\lambda^2-(k+1)^2$, the fiber rotation period is:

$$T=\frac{4\pi}{\sqrt{(k-1)^2-\lambda^2((k+1)^2+4j^2)}\dfrac{\partial u}{\partial y}}$$

Under the condition of Δ in Eq.(16) being a real number, the fiber will reach an asymptotic orientation, otherwise, it will rotate periodically.

FIBER ORIENTATION PROBABILITY DISTRIBUTION IN PLANAR FLOW

Basic equations and computational method

Orientation probability distribution function $\psi(\boldsymbol{r}_c, \boldsymbol{p}, t)$ is the most precise description of fiber orientation. It accounts for the probability that the test fiber selected has a specific location $\boldsymbol{r}_c$ and orientation $\boldsymbol{p}$ at time t. Considering homogeneous flow and steady state, ψ is dependent only on $\boldsymbol{p}$. From Eq.(2), we have

$$\frac{\partial}{\partial\boldsymbol{p}}(\psi\dot{\boldsymbol{p}})=0$$

then

$$\psi=\frac{C\mathrm{d}t}{\mathrm{d}\boldsymbol{p}}$$

where constant C can be determined by the normalization condition of ψ.

When a fiber rotates with period T in a planar flow, we use ϕ to depict the fiber direction. The probability to find fiber in the range $d\phi$ is $\psi d\phi$, which is the same as dt/T (dt is the time the fiber spent rotating through angle $d\phi$, so we have:

$$\psi(\theta_i) = \frac{dt}{Td\theta_i} \approx \frac{1}{N\,|\,\theta_{i+1} - \theta_i\,|} \tag{18}$$

where N=dt/T is a discrete number. It is obvious that ψ defined by Eq.(18) satisfies the normalization condition. After solving the evolution of ϕ, Eq.(18) can be used to efficiently compute the fiber orientation distribution.

In order to confirm the analytical solution and give visual results, we use a 4th-order Runge-Kutta method to compute the fiber rotation by using Eqs. (3a), (3b), then use the relation $\tan\phi = p_1/p_2$ to obtain ϕ, and finally obtain the fiber orientation distribution ψ.

Simple shear flow

Fig.2 gives the fiber orientation distribution with different shear rate. The total superposition of two curves shows that the shear rate has no effect on the orientation distribution. It can be proved by substituting Eqs.(4) and (7) into Eq.(18), that the shear rate will not appear in the final expression. In fact, it is true for all flows since the increasing magnitude of the velocity gradient will accelerate the fiber rotation proportionally and shorten the rotation period in inverse proportion. Then the fiber orientation will be affected not by the absolute but the relative magnitude of velocity gradient. Dinh and Armstrong (1984) showed that it was true for infinite fiber in arbitrary 3-D flows.

The curves are drawn within the time range [0, T/2], where T is computed by Eq.(7). The results showed that the fiber rotates exactly through 180°, so the analytical and computational results coincide completely. The same treatment is used in the following subsections to confirm the period.

Figs.2 and 3 show that a fiber has great opportunity to align with the flow direction and that the tendency of aligning is increased with the increase of the fiber length.

Flow with two direction shears

Analytical results showed that a fiber will rotate periodically when Eq.(10) is satisfied and reach an asymptotic direction when Eq.(10) is not satisfied.

Fig.4 shows the asymptotic cases, for three asymptotic directions with values of −0.7, −0.3, 0.3 which almost coincide with the analytical values −0.6959, −0.3041, 0.3041. Numerical results showed that ϕ=arccos[(1−k)/λ(k+1)]/2 is a stable equilibrium direction while ϕ=−arccos[(1−k)/ λ(k+1)]/2 is an unstable equilibrium direction.

Fig.5 shows the periodic case with different shear rate. It can be seen that the fiber tends to align with bigger shear rate direction and that the tendency is to strengthen with increasing ratio of two direction shear rate.

Extensional flow

In both extension and compression directions, the orientation distribution becomes infinite, while

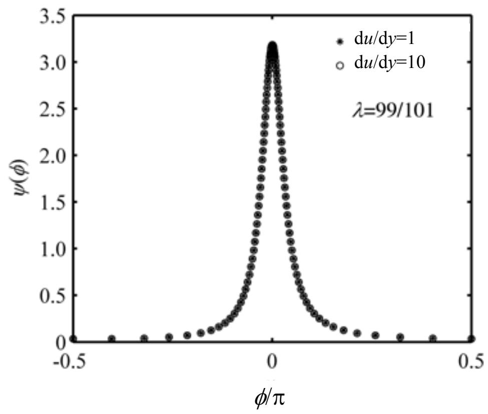

Fig.2 Orientation probability distribution of fiber with different shear rate

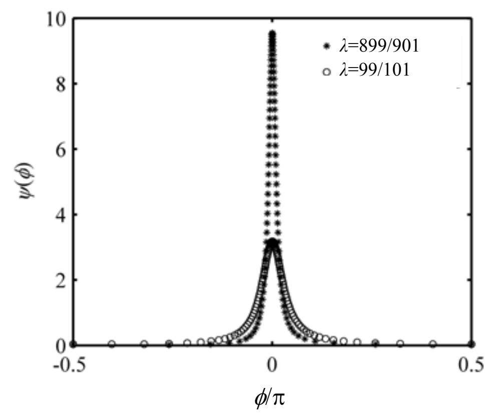

Fig.3 Orientation probability distribution of fiber with different aspect ratio

only the compression direction has asymptotic stability (Fig.6), i.e., the fiber will align with the compression direction finally.

Arbitrary planar incompressible flow

Since planar flow is the combination of shear and extensional flow, the property of simple flow will appear more or less.

On condition Δ of Eq.(16) is a real number, the fiber reaches the asymptotic direction defined by Eq.(17) finally (Fig.7).

On condition Δ of Eq.(16) is an imaginary number, the fiber rotates periodically. The summit shifts between the two shear directions with different shear rate (Fig.8), while the summit value is peaked with increasing extensional rate (Fig.9). The effect of aspect ratio is also considered in the case (Fig.10), when there is only slight difference between infinite fiber (λ=1) and very short fiber (λ=99/101), so the infinite fiber supposition is a very good approximation.

GENERALIZATION

In the above sections, the fiber is assumed to lie in a plane. Next we will show that the foregoing results can be extended to the case of fiber in an arbitrary space. Under this circumstance, an additional Eq.(19) aside from Eqs.(3a) and (3b) will be added to describe fiber orientation.

$$\frac{\mathrm{d}p_z}{\mathrm{d}t} = -\lambda p_z \left(p_x^2 \frac{\partial u}{\partial x} + p_x p_y \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + p_y^2 \frac{\partial v}{\partial y} \right) \quad (19)$$

Three components of unit direction vector are $p_x=\sin\theta\cos\phi$, $p_y=\sin\theta\sin\phi$, $p_z=\cos\theta$, based on Fig.1.

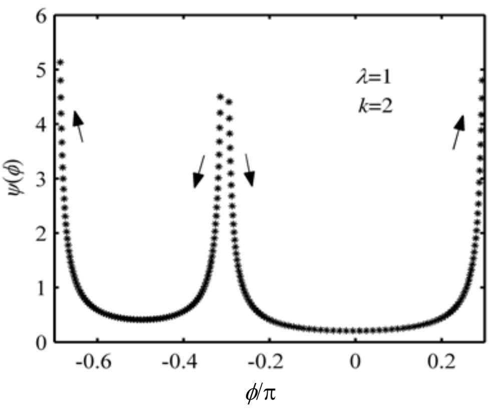

Fig.4 Orientation probability distribution of fiber for *k*=2

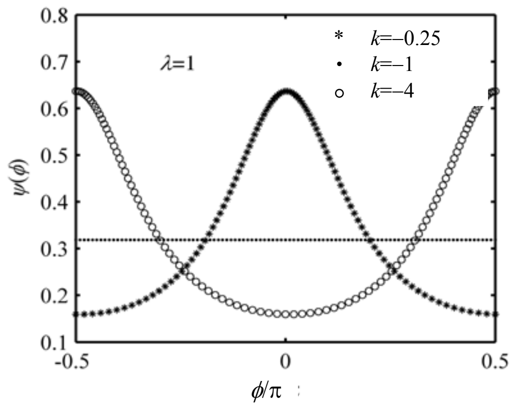

Fig.5 Orientation probability distribution of fiber for *k* = −0.25, −1, −4

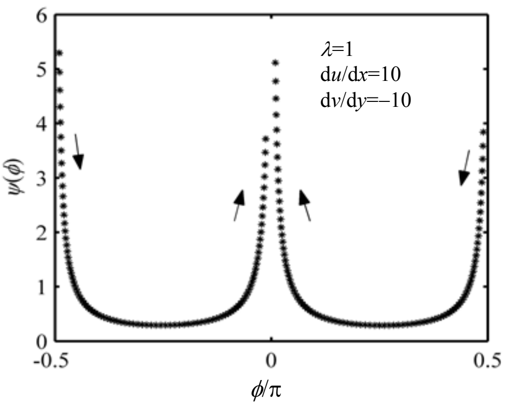

Fig.6 Orientation probability distribution in extensional flow

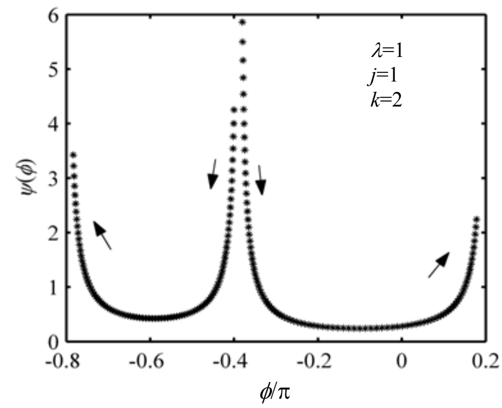

Fig.7 Orientation probability distribution for *k*=2, *j*=1

Substituting these components into Eqs.(3a), (3b) and (19), we have

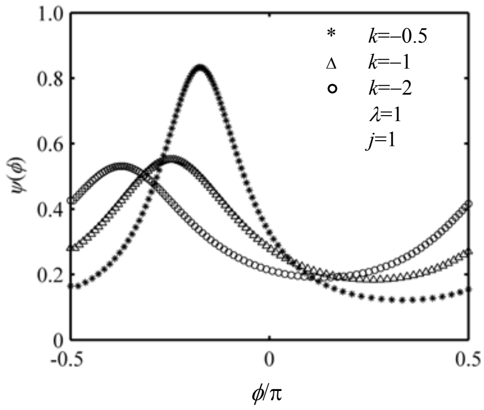

Fig.8 Orientation probability distribution at different shear rate with extension

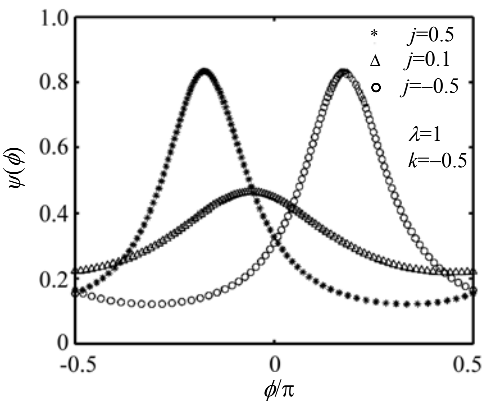

Fig.9 Orientation probability distribution in different extension with shear

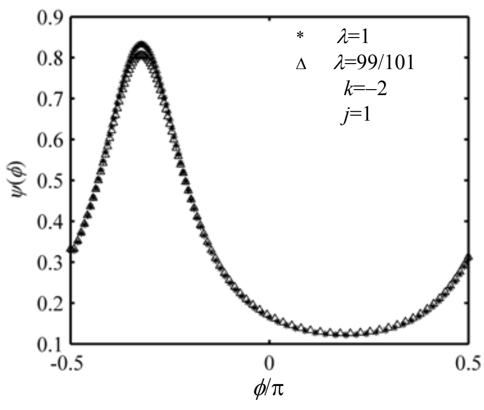

Fig.10 Orientation probability distribution with different aspect ratio in planar flow

$$\cos\theta\cos\phi\frac{\mathrm{d}\theta}{\mathrm{d}t}-\sin\theta\sin\phi\frac{\mathrm{d}\phi}{\mathrm{d}t}$$
$$=-\lambda\sin^3\theta\left(\sin^2\phi\cos\phi\frac{\partial v}{\partial y}+\cos^3\phi\frac{\partial u}{\partial x}\right.$$
$$+\cos^2\phi\sin\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\frac{1}{2}\sin\theta\left(\sin\phi\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)\right.$$
$$\left.\left.+\lambda\left(\sin\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\cos\phi\frac{\partial u}{\partial x}\right)\right)\right) \quad (20)$$

$$\cos\theta\sin\phi\frac{\mathrm{d}\theta}{\mathrm{d}t}+\sin\theta\cos\phi\frac{\mathrm{d}\phi}{\mathrm{d}t}$$
$$=-\lambda\sin^3\theta\left(\cos^2\phi\sin\phi\frac{\partial u}{\partial x}+\sin^3\phi\frac{\partial v}{\partial y}\right.$$
$$+\sin^2\phi\cos\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\frac{1}{2}\sin\theta\left(\cos\phi\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right.$$
$$\left.\left.+\lambda\left(\cos\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\sin\phi\frac{\partial v}{\partial y}\right)\right)\right) \quad (21)$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\lambda\sin\theta\cos\theta\left(\cos^2\phi\frac{\partial u}{\partial x}+\sin^2\phi\frac{\partial v}{\partial y}\right.$$
$$\left.+\sin\phi\cos\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right) \quad (22)$$

Substituting Eq.(22) into Eqs.(20) and (21), we have

$$-\sin\phi\frac{\mathrm{d}\phi}{\mathrm{d}t}=-\lambda(\sin^2\theta+\cos^2\theta)$$
$$\times\left(\sin^2\phi\cos\phi\frac{\partial v}{\partial y}+\cos^3\phi\frac{\partial u}{\partial x}+\cos^2\phi\sin\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)$$
$$+\frac{1}{2}\left(\lambda\left(\sin\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\cos\phi\frac{\partial u}{\partial x}\right)+\sin\phi\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)\right) \quad (23)$$

$$\cos\phi\frac{\mathrm{d}\phi}{\mathrm{d}t}=-\lambda(\sin^2\theta+\cos^2\theta)$$
$$\times\left(\cos^2\phi\sin\phi\frac{\partial u}{\partial x}+\sin^3\phi\frac{\partial v}{\partial y}+\sin^2\phi\cos\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)$$
$$+\frac{1}{2}\left(\lambda\left(\cos\phi\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\sin\phi\frac{\partial v}{\partial y}\right)+\cos\phi\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\right) \quad (24)$$

Since $\sin^2\theta+\cos^2\theta=1$, θ in Eqs.(23) and (24) will not appear, i.e., ϕ is separable from θ. In addition, the results given in Sections 2 and 3 are some special cases of Eqs.(23) and (24), so that all the forgoing results can be applicable to the case of fiber in an arbitrary space when considering the azimuthal motion of the fiber. If the meridian motion of the fiber needs consideration, Eq.(22) can be solved without coupling to other equations.

CONCLUSION

The stable orientation and orientation period of fiber in simple shear flow, flow with two direction shears, extensional flow and arbitrary planar incompressible flow are given. Steady orientation distribution of fiber is not affected by the absolute magnitude of the rate-of-strain of flow but by the ratio of components. The temporal property of fiber rotation depends closely on the magnitude of the rate-of-strain. For arbitrary planar incompressible flow, when Δ in Eq.(16) is real number, the fiber will reach an asymptotic orientation, otherwise, the fiber will rotate periodically. The effect of the fiber aspect ratio on the orientation distribution of fiber is insignificant under most conditions except the simple shear case, so the infinitely long fiber assumption is proper in most cases. The results can also be used to analyze the rationality of different fiber suspension flows such as parallel flow, converging flow, stepping flow, etc. if only the fiber orientation distribution is involved.

Please note carefully that the foregoing researches are limited to a fixed point of the flow, although this limitation is trivial concerning computer calculation.

References

Advani, S.G., Tucker, C.L., 1987. The use of tensors to describe and predict fiber orientation in short fiber composites. *J Rheol*, **31**(8):751-784.

Advani, S.G., Tucker, C.L., 1989. Closure approximations for three-dimensional structure tensors. *J Rheol*, **34**(3):367-386.

Altan, M.C., Advani, S.G., Guceri, S.I., Pipes, R.B., 1989. On the description of the orientation state for fiber suspensions in homogeneous flows. *J Rheol*, **33**(7):1129-1155.

Batchelor, G.K., 1970. The stress system in a suspension of force-free particles. *J Fluid Mech*, **41**:545-570.

Bird, R.B., Armstrong, R.C., Hassager, O., 1987. Dynamics of Polymeric Liquids. John Willey & Sons, New York.

Bretherton, F.P., 1962. The motion of rigid particles in a shear flow at low Reynolds number. *J Fluid Mech*, **14**:284-304.

Chiba, K., Nakamura, K., 1998. Numerical solution of fiber suspension flow through a complex channel. *J Non-Newt Fluid Mech*, **78**:167-185.

Chinesta, F., Chaidron, G., Poitou, A., 2003. On the solution of Fokker-Planck equations in steady recirculating flows involving short fiber suspensions. *J Non-Newt Fluid Mech*, **113**:97-125.

Dinh, S.M., Armstrong, R.C., 1984. Rheological equation of state for semiconcentrated fiber suspensions. *J Rheol*, **28**(3):207-227.

Folgar, F., Tucker, C.L., 1984. Orientation Behavior of Fibers in Concentrated suspensions. *J Reinf Plast Compos*, **3**:98-119.

Hinch, E.J., Leal, L.G., 1975a. Constitutive equations in suspension mechanics. Part 1. General formation. *J Fluid Mech*, **71**:481-495.

Hinch, E.J., Leal, L.G., 1975b. Constitutive equations in suspension mechanics. Part 2. Approximate forms for a suspension of rigid particles affected by Brownian rotations. *J Fluid Mech*, **76**:187-208.

Leal, L.G., Hinch, E.J., 1971. The effect of weak Brownian rotations on particles in shear flow. *J Fluid Mech*, **46**:685-703.

Lin, J.Z., Zhang, L.X., 2003. On the structural features of fiber suspensions in converging channel flow. *J Zhejiang University SCIENCE*, **4**(4):400-406.

Shaqfeh, E.S.G., Koch, D.L., 1990. Orientational dispersion of fibers in extensional flows. *Phys Fluids A*, **2**(7):1077-1093.

Stover, C.A., Koch, D.L., Cohen, C., 1992. Observations of fibre orientation in simple shear flow of semi-dilute suspensions. *J Fluid Mech*, **238**:277-296.

Szeri, A.J., Leal, L.G, 1992. A new computational method for the solution of flow problems of microstructured fluids. Part 1. Theory. *J Fluid Mech*, **242**:549-576.

Szeri, A.J., Leal, L.G, 1994. A new computational method for the solution of flow problems of microstructured fluids. Part 2. Inhomogeneous shear flow of a suspension. *J Fluid Mech*, **262**:171-204.