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Recent development of vortex method in incompressible viscous bluff body flows*

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Abstract: Vortex methods have been alternative tools of finite element and finite difference methods for several decades. This paper presents a brief review of vortex method development in the last decades and introduces efficient vortex methods developed for high Reynolds number bluff body flows and suitable for running on parallel computer architectures. Included in this study are particle strength exchange methods, core-spreading method, deterministic particle method and hybrid vortex methods. Combined with conservative methods, vortex methods can comprise the most available tools for simulations of three-dimensional complex bluff body flows at high Reynolds numbers.

Key words: Vortex methods, Simulation of flows, Bluff body
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INTRODUCTION

A number of problems involve flows of gases or liquids over bluff bodies such as air flowing over cars and airplanes and wind blowing over bridges and buildings. Many experiments or simulations have focused on the above problems. In those phenomena, flows will not follow the solid bodies completely, but separate from them and create wakes. The generation and shedding of large coherent vortex structures due to flow separation make the bluff body flows complex and very difficult to predict. Study of bluff body aerodynamics has been carried out for more than two hundred years. Flow separation is one of the most important hydrodynamics problems investigated intensively. Due to its extreme complexity, the bluff body flows are traditionally researched by experiments with simple models. With the giant improvement of modern computing power, more and more numerical simulation methods have been applied to simulation of different bluff body flows. The conventional numerical methods such as finite difference methods and finite element methods are not suitable for dealing with problems of high Reynolds number gas flow over solid surface. So a new scheme named vortex method is founded. Vortex methods are based on the particle discretization of the Lagrangian form of the vorticity-velocity formulation of the Navier-Stokes equations. The vorticity field is discretized into a large number of interacting vortex “blobs”, whose position and strength determine the underlying velocity field (Fusen and Su, 1998).

Vortex methods have significant advantages for numerical simulations of separated flows:

1. In vortex methods, only small portion of the flow region where vorticity occurs need to be described.

2. Lacking fixed grid, vortex methods can easily handle flows around complicated geometries and can avoid significant numerical dissipation.

3. If the vorticity separates from the solid sur-

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faces, the vortices follow that motion, so that numerical resolution is maintained and small-scale structures can be recorded accurately.

4. In the computation of the velocity field from the vorticity, mass conservation can be satisfied exactly.

5. In external flows, vortex methods can treat boundary conditions without restricting the computational domain to a finite size.

At high Reynolds numbers, a critical concern is that the numerical dissipation should not overwhelm the natural viscous diffusion process and destroy the small-scale features. This makes vortex methods the natural choice. Recent advances in developing fast adaptive vortex methods and the parallel implementation of vortex algorithms on computers cluster have removed many limitations on the number of vortex elements that will be reasonably applied in complex and high Reynolds number flow simulations. In this paper we will mainly present the developments of the vortex methods for simulations in bluff body aerodynamics and try to probe into the prospect of the methods.

In order to discuss vortex methods conveniently, we sort vortex methods progress into two main stages: the first stage named classic vortex methods, which includes the investigation of vortex methods convergence and development in inviscous situations; the second is the advancement of vortex methods in the last decade, and we will introduce the discrete vortex methods and hybrid vortex methods in this stage.

DEVELOPMENT OF CLASSIC VORTEX METHODS

The emergence of the vortex method can be traced to the 1930s, with the Rosenhead calculations of the Kelvin-Helmholtz instabilities. The use of the fractional-step, Lagrangian vortex scheme for incompressible viscous flow was originally proposed by Chorin in 1973. The scheme was named random walk method and later, Chorin (1978) modified his method for wall boundary conditions. The method has been proved to converge for unbounded domains by Beale and Majda (1981; 1982).

During the following decades, different vortex methods were proposed to solve the Navier-Stokes equations. Van Dommelen and Rundensteiner developed the first ‘solution adaptive’ fast method that could efficiently handle the sparse and complex vorticity distributions of high Reynolds number separated flows. Similar fast algorithms were developed by Greengard and Rokhlin (1987). There are several detailed reviews vortex methods in incompressible flow (Greengard and Rokhlin, 1987; Leonard, 1980). The developments and applications of vortex methods were described by Cottet and Koumoutsakos (2000). We mainly discuss the extension of new vortex methods in recent years.

NEW ADVANCES OF VORTEX METHODS IN THE LAST DECADES

The modern vortex methods used to simulate viscous fluid flow use a number of Lagrangian discrete vortex elements (whose position and strength determine the velocity field of the flow), to simulate the evolution of a continuously distributed vorticity field. In those vortex methods, the vorticity equation is divided into two steps: convection and diffusion. In the bluff body flows, no-slip boundary condition was used.

Governing equations

Vortex methods are based on the Lagrangian description and use particles as vorticity carrying computational elements. The method amounts to tracking these particles and their circulations. Incompressible viscous flows are governed by the vorticity equation:

$$\frac{\partial \omega}{\partial t} + (u \cdot \nabla)\omega = \nu \nabla^2 \omega \tag{1}$$

$$\omega = \nabla \times u \tag{2}$$

where u is the velocity field, v is the kinematic viscosity. The equations can be expressed in Lagrangian formulation as

$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = u(x_p, t) \tag{3}$$

$$\frac{\mathrm{d}\omega_p}{\mathrm{d}t} = [\nabla u(x_p, t)]\omega_p + \nu \Delta \omega(x_p) \tag{4}$$

where x_p, ω_p denote the locations and the vorticity carried by the fluid elements.

For viscous fluid vortex methods, the equations are always split into two steps: one without viscidity and one with diffusivity.

$$\frac{\partial \omega}{\partial t}+(u\cdot\nabla)\omega=0 \tag{5}$$

$$\frac{\partial \omega}{\partial t}=\nu\nabla^2\omega \tag{6}$$

A general expression of the fluid velocity by the Biot-Savart law is:

$$u(x)=U_\infty-\frac{1}{A_d}\iint_\Omega \frac{\omega(x_0)\times(x_0-x)}{|x_0-x|^d}\mathrm{d}\Omega \tag{7}$$

where A_d=4π, d=3 in three dimensions and A_d=2π, d=2 in two dimensions. We assume that the circulation is concentrated at N_p discrete points:

$$\omega(x,t)=\sum_{p=1}^{N_p}\delta(x-x_p(t))\Gamma_p \tag{8}$$

where $\Gamma_p=\int_{\Omega_p}\omega\mathrm{d}\Omega_p$ and δ is the Dirac delta function. Using position x_p characterizes the fluid, Eq.(3) in two dimensions becomes:

$$\frac{\partial x_p}{\partial t}=U_\infty-\frac{1}{2\pi}\sum_{p=1}^{N_p}\frac{\Gamma_p\times(x_p-x)}{|x_p-x|^2} \tag{9}$$

For viscid step, random walk method is used usually. The solution of Eq.(4) can be expressed as:

$$\omega(x,t)=\int_\Omega G(x-x_0,t)\omega_0(x_0,t')\mathrm{d}\Omega_0 \tag{10}$$

where $\omega_0(x_0,t')$ is the initial vorticity distribution and $G(r,t)$ is the Green function described by the diffusion equation

$$G(r,t)=\frac{1}{4\pi\nu t}\exp\left(-\frac{|r|^2}{4\nu t}\right) \tag{11}$$

On solid body, the no-slip boundary condition is used. The pressure on the solid body can be obtained along the surface. Expressed in the vorticity form:

$$\nabla p\cdot \boldsymbol{t}_0=-\mu\nabla\omega\cdot n_0 \tag{12}$$

where $\boldsymbol{t}_0$ is the surface unit tangential vector.

The evolution of viscous vortex methods in the last decade mainly includes the method developed by Degond and Mas-Gallic (1989), now named particle strength exchange method (PSE), the core-spreading method of Leonard (1980), a deterministic particle method proposed by Fishelov (1990) and several hybrid vortex methods. In the following parts, the progress and applications of the methods in bluff body flows will be recounted.

Core-spreading methods

The core-spreading method (core expansion method) was first put forward by Leonard (1980). It is a deterministic method accounting for diffusion by allowing each discrete vortex core to grow at a rate proportional to the kinematic viscosity, and the solution is not convergent for Navier-stokes equations (Leonard, 1980). Rossi (1996) modified the basic core-spreading method to a convergence scheme by allowing the cores to grow but setting a maximum core size. When vortices grow beyond limit, they are split into a number of smaller vortices in a conservative manner. In this method, the size of the problem will be expanded exponentially, so it is only suitable for low Reynolds numbers flow (Rossi, 1996). Zhu and Kyoji (2003) developed a Lagrangian vortex method for solving the moving bluff body flow. This method joints an adaptive grid-free splitting and merging algorithm to the core spreading model for simulating the viscous diffusion, and describes the evolving vorticity field by Lagrangian representation. In order to elucidate the diffusion of matter in free turbulent flow, Uchiyama and Okita (2003) used a particle method to predict the plume diffusion field around a circular cylinder. In this method, the velocity field is calculated by a core-spreading vortex method, while the concentration field is calculated by a particle method analogous to the vortex method. The numerical resolution exhibits the meandering behavior observed in the experiment.

The corrected core-spread method can yield an efficient grid-free method for viscous flow. It is believed that the method offers greatest potential for high-Reynolds number flow calculations. The method can be used for long-time, high-resolution simulation of moving or complex geometries. But it has a low convergence rate and second order accuracy due to convection error limits.

Particle strength exchange methods

The particle strength exchange method was introduced by Degond and Mas-Gallic (1989). In 1990, they extended this deterministic particle method to problems with boundary conditions. Rouvreau (2001) used a deterministic method for simulating vortex flow around a circular cylinder. The solid wall is discretised into wall-segments with a control point in the middle of each one and the association of one vortex blob and one source-segment is applied to satisfy the no-slip boundary conditions. The vorticity redistribution method developed by Shankar and van Dommenlen (1996) was similar to the particle strength exchange method but without the necessity of remeshing. Circulation is exchanged between nearby particles to simulate the diffusion process. If the distribution of vortices in adjacent area does not yield a solution to this set of equations, extra vortices are added. Cottet *et al.*(2000) developed variable size vortex methods based on the PSE algorithm to simulate the flow around a bluff body. In this method, small size particles are used to resolve the vorticity gradients near the surface of the body, with larger blobs being suitable for discretizing the smoothly varying vortices in the wake of the flow. It was shown that the method offers significant improvement on the computational efficiency while maintaining the adaptive character of the method. The PSE has been successfully used for a number of high-resolution studies (Koumoutsakos and Leonard, 1995; Koumoutsakos and Shiels, 1996). Ploumhans and Winckelmans (2000) used modified PSE scheme near the solid boundaries; and used image particles to guarantee a zero vorticity flux at the boundary, with the no-slip boundary condition being enforced in two steps. The method is a high-resolution tool for general geometry flows (Ploumhans and Winckelmans, 2000). Ploumhans *et al.*(2002) extended this method to three-dimensional bluff body flow and obtained superior computational efficiency by the use of parallel tree codes based on multipole expansions of vortex particles and of vortex panels.

In summary, the main feature of the PSE method is the replacement of differential operators by integral operators more suited for the representation of data. Although the PSE method is grid-free, it can be an even more useful simulation method when it is combined with remeshing schemes. PSE method has been amply developed and applied in high-resolution simulation of moderate to high-Reynolds number bluff-body flows.

Deterministic particle methods

The deterministic particle method was originally proposed by Fishelov (1990), who added spatial derivatives with a smoothing function into the vorticity equation and then transferred the derivatives to that function. This scheme is stable and it readily extends to a higher order of accuracy. Nordmark (1996) agreed with Fishelov's idea, but used a moving grid, dispensing with the needs to approximate the gradient of the vorticity. Bernard (1995) used this method in boundary layer flows and developed a new deterministic vortex sheet method, which can reliably account for viscous diffusion in turbulent flow and is suitable for simulating three-dimensional vorticity structures. The diffusion velocity method (Ogami and Akamatsu, 1991) is a deterministic method which splits the velocities of the discrete vortices into convective and diffusive components. The diffusion velocities are proportional to the vorticity gradient and kinematic viscosity, and inversely proportional to the vorticity. This method has the advantage in that it can be used in higher Reynolds number flow (Clarke and Tutty, 1994; Chang and Chen, 1991), but is relatively more sensitive to disturbance in the particle field. This must be resolved before it is used to simulate bluff-body flows.

Hybrid vortex methods

With the development of computers and cluster works in recent years, direct numerical simulation of three-dimensional high Reynolds number flows is possible. Many researchers presented methods that combined grid-based and vortex scheme. In those methods, high-order interpolation formulas are necessary for establishing a low cost and minimal numerical dissipation scheme. In what follows, we discuss several hybrid vortex methods.

The first is blending finite-difference and vortex methods, Chen *et al.*(2002) used it to simulate flow around multiple circular cylinders and found that the flow differed from that around single cylinder. Ould-Salihi *et al.*(2000) described two class numerical procedures that combined grid and vortex method for solving incompressible flow Navier-Stokes equations. Those methods were based on the principle of vortex-in-cell and take advantage of fast FFT-based Poisson solvers and provide a flexible, high-order schemes for three-dimensional high Reynolds number flow involving minimal computation time and cost. The HODIE method based on the hybrid scheme of discrete vortex method and fourth-order difference method was presented by Cheng *et al.*(2001a) for simulating flow past a rotating circular cylinder. By this approach, some basic behaviors of vortex shedding and the lock-on range for vortex shedding were revealed. It is well known that three-dimensional effects are important for high Reynolds numbers flow, so Cheng used the hybrid vortex method for dealing with high Reynolds number (=1000) flow around a rotationally oscillating cylinder. The method is very efficient computationally and allows computation over a long period for investigating the development of vortex wake and the global behavior of lift and drag force Cheng *et al.*(2001b).

A novel immersed interface method in the framework of vortex-in-cell algorithm was proposed by Walther and Morgenthal (2002) to capture important flow features of separate flows. This hybrid mesh method was based on algorithms including the particle-mesh and particle-particle-particle-mesh algorithm with exact particle-particle correction (Walther, 2003). It enables efficient solution of the Poisson equation using an FFT solver and exactly describes free-space boundary conditions using a minimal number of grid points and is an appropriate method for dealing with complex bluff body flows.

In order to study the dynamics of vorticity in a bluff-body geometry for high Reynolds number incompressible unsteady flow, Mortazavi and Giovannini (2001) used a hybrid of the random vortex method and finite element method. The scheme depicted quite well the vortex shear events, the process of the vorticity generation, and the evolution dynamics.

Akbari and Price (2003) presented a proven grid-based vortex method for flow over elliptic airfoils. The convection and diffusion parts of Navier-Stokes equation are calculated sequentially at each time step. The convection equation is solved using the vortex-in-cell method, and the diffusion equation is solved using a second-order ADI finite difference scheme. The detail presented in this paper includes the derivation of equations, accuracy of the numerical schemes and the computational grid. They used this scheme to simulate flow over an NACA 0012 airfoil and obtained satisfactory solution (Akbari and Price, 2000).

SUMMARY AND CONCLUSION

This paper presents an overview of the vortex methods introduced in the last few decades. In the future, we seek for new schemes enabling accurate and computationally efficient simulations of bluff body wake behaviors and are suitable for running on scalable, parallel computer architectures.

In three-dimensional high Reynolds number situation, we need several millions mesh points for vorticities, so we must find new schemes to decrease computing time. In vortex methods, the computing time needed for diffusion is less than the time needed for convection, and it is very important to reduce the vorticity redistribution time. An optimal parallel scheme executed on powerful computing clusters can eliminate limits in spatial and temporal resolution.

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