Journal of Zhejiang University SCIENCE
ISSN 1009-3095
http://www.zju.edu.cn/jzus
E-mail: jzus@zju.edu.cn

Diversification or splitting−an explanation based on Contract Theory

LUO Liang-zhong (罗良忠)†, SHI Zhan-zhong (史占中)

(*School of Management, Shanghai Jiaotong University, Shanghai 200052, China*)

†E-mail: mountaintopsnow@hotmail.com; hilltopnow@163.com

Received Feb. 2, 2004; revision accepted Aug. 1, 2004

Abstract: By inserting the variable of the exactness of corporate valuation into the classic model of Contract Theory, this paper, on the bases of the interaction of the variables of the veraciousness of corporate valuation, managerial incentives and operational risks, explores the deep-seated reasons for changes in corporate structures, and draws the conclusion that the divestment of the subsidiary is beneficial to shareholders when the parent corporate is undervalued and that the relation between the parent and the subsidiary is disordered, or vice versa. This conclusion is consistent with the motives of many divestiture cases in reality.

Key words: Diversification, Specialization, Divestment, Contract Theory
doi:10.1631/jzus.2005.A0329 **Document code:** A **CLC number:** F224.0

INTRODUCTION

Williamson (1975), Jensen and Meckling (1976), and Fama and Jensen (1983) held that incentive contracts and organizational form are closely tied, and that organizational form is an optimal response to agency problem. Stiglitz (1986) argued that the incentive structure of an organization ought to respond quickly to changes in the economic environment in order to provide appropriate managerial incentives. Corporate divestment is certainly one of the most drastic changes in the organizational form of a firm.

Holmstrom and Milgrom (1991) claimed that the manager could allocate effort optimally to different product lines in a multi-task analysis. Aron (1991) examined spin-offs in an agency framework and considered a spin-off as a contingent compensation for the subsidiary (product) manager.

Choi and Merville (1998) inserted synergistic effects into the agency model and argued that, to a parent company's shareholders, spin-offs and equity-carve-outs are the forms of changes in an organizational structure, and can improve the incentive structures of the parent company and its subsidiaries.

We (the authors) were inspired greatly by these scholars' insights. By inserting the variable of the veraciousness of corporate valuation into the classic model of contract theory, this paper explores the deep-seated reasons for changes in corporate structures on the bases of the interaction of the variables of the veraciousness of corporate valuation, managerial incentives and operational risks.

BASIC ASSUMPTIONS OF CONTRACT THEORY

Contract Theory is also called Principal-agent Theory and mainly deals with the issues of incentives and insurance between principal and agent. The optimal incentive contract must trade off between incentives and risks. Shareholders are the owners of the firm, and employ a manager to be in charge of the everyday operations to produce products. The principal and the agent write a compensation contract to induce the agent to do some actions (a), to produce output (y). The principal owns the output, but the agent may share it. The production is disturbed by some uncertain factors (ε), and the agent's output is also uncertain.

The general assumptions of contract theory are as follows: (1) Shareholders are risk-neutral, and their targets are to maximize their corporate expected profits; (2) Managers are risk-averse, as their incomes are tied to corporate performance; (3) Managers' actions are unobservable to the shareholders because of asymmetric information; (4) Corporate profit [$\pi(a, \varepsilon)$], managers' wages [$w(y)$] and utilities [$U(w, a)$] are common knowledge; (5) The principal-agent contract is static, i.e. only one period of contract is signed.

CONSTRUCTION OF THE MODEL

Assume that a diversified listing firm (firm 0) has only one subsidiary (firm 1). The manager of firm 0 controls the manager of firm 1. The managers are risk-averse while the shareholders are risk-neutral. The targets of the managers and shareholders are different. The managers' objectives are to maximize their incomes while the shareholders' objectives are to maximize their surplus.

Suppose that the managers' actions (a) are their effort levels in work (e), and e is determined by the internal incentive structure. Let δ be the discount rate of a firm's stock price valued by outsiders (i.e. analysts). The outcome of a firm is assumed to be the firm's market value or stock price or earnings per share (EPS). The final outcome, y, in a whole organization, can be considered as a function of these two factors (e, δ) plus a random element (ε): $y=F(e, \varepsilon, \delta)$. For the sake of simplification, we assume that the outcome function is linear, that is, $y=e\delta+\varepsilon$, $\varepsilon_i \sim N(0, \sigma_i^2)$, where σ_i is the standard deviation of the synergetic risks of firm i. ρ measures the synergetic relations between firm 0 and firm 1. When $\rho>0$, it shows that there exist some conflicts between the two firms and the synergetic effects between the two firms' are negative (Hereafter, we use "disorder" to describe this status), or vice versa.

Given this production technology, δ can be interpreted as the marginal product of managerial effort that is determined by asset specificity in the operations of the parent and subsidiary, as the under-valuation of the corporate stock is due to the manager's insufficient effort to disclose the operations information to outsiders. The optimal managerial effort, e^*, is determined by the internal incentive structure. Therefore, the average outcome, $\bar{y}$, will be maximized when both δ and e are optimized.

PRODUCTION FUNCTION IN THE STATE OF DIVERSIFICATION

In the state of diversification, firm 1 is controlled by firm 0 completely. The parent company (firm 0) is often valued inexactly, either overvalued or undervalued. Therefore, let firm 0's production function be $y_0=e_0+\varepsilon_0$, and firm 1's be $y_1=\delta e_1+\varepsilon_1$, where δ, $e_i \geq 0$. It presents that the firm is valued exactly when $\delta=1$, the firm is undervalued when $\delta \in (0, 1)$, and the firm is overvalued when $\delta>1$. The firm's market value, $y(e, \varepsilon|\delta)$, can be symbolized by $y=y_0(e_0, \varepsilon_0)+y_1(e_1, \varepsilon_1|\delta)$.

PRODUCTION FUNCTION AFTER SPLITTING

Assuming that firm 1 is split off from firm 0. After splitting, firm 1 is independent of firm 0 completely, and its manager is in full charge of it. We further assume that the shareholders of firm 0 are still the shareholders of firm 1 after firm 1's being split off from firm 0. Here, shareholders can use the new variable observable, y_1, to obtained the effort level of firm 1's manager, and be able to design a more effective incentive contract. Suppose that, after splitting, firm 0 and firm 1 are both valued exactly, that is, $\delta=1$. The outcome function is $y_i=e_i+\varepsilon_i$, $i=0$, 1, where e_i represents the effort levels of the managers of firm 0 and firm 1.

UTILITY FUNCTIONS OF THE MANAGERS AND SHAREHOLDERS

Assume that shareholders are risk-neutral and managers are risk-averse. Consider the linear compensation contract, $s(y)=\alpha+\beta y$, where α represents managers' constant income (unrelated to y), β is the managers' fraction of shares in the output (β is also called incentive intension), that is, managers' compensation increase β units when the market value of the firm, y, increases a unit. β=0 means that managers do not bear any risk while β=1 means managers bear all risks. Because of the risk-neutral shareholders, given $s(y)=\alpha+\beta y$, the expected utility of shareholders equals the expected income:

$$\begin{aligned}Ev(y-s(y)) &= E(y-\alpha-\beta y)\\ &= -\alpha+E(1-\beta)y\\ &= -\alpha+E(1-\beta)y \qquad (1)\end{aligned}$$

Assume that each manager's utility function has the characteristics of the constant absolute risk aversion. Let the utility function of Manager 0 and Manager 1 at the end of the period be described by the exponential utility function: $U(s(y)-c(e))=-e^{[-r(s(y)-c(e))]}$, where r (>0) is the measurement of the constant absolute risk aversion, $s(y)-c(e)$ is the actual monetary income, $c(e)$ represents the cost of manager's effort in the operation (assume that it can be the monetary cost equivalent). For the sake of simplification, assume that $c(e)=be^2/2$, where b>0 represents the cost coefficient: The greater b is, the more negative effects the same effort level e brings about. The manager's real income is:

$$\begin{aligned}w &= s(y)-c(e)\\ &= \alpha+\beta(e+\varepsilon)-\frac{b}{2}e^2 \qquad (2)\end{aligned}$$

The certainty equivalence (CE) for the manager is:

$$\begin{aligned}&E(s(y)-c(e))-\frac{1}{2}r\beta^2\sigma^2\\ &= \alpha+\beta e-\frac{1}{2}r\beta^2\sigma^2-\frac{b}{2}e^2 \qquad (3)\end{aligned}$$

where $E(s(y)-c(e))$ represents manager's expected income, σ is the standard deviation of the synergetic risks of the two subsidiary firms, $r\beta^2\sigma^2/2$ is the risk costs of agents; when β=0, the risk costs equal zero. Then, manager's maximum expected utility function $Eu=-Ee^{[-r(s(y)-c(e))]}$ is equivalent to the above maximum certainty equivalence (CE) (Zhang, 1996).

Let $\bar{w}$ be the manager's reservation income. If his certainty equivalence is smaller than $\bar{w}$, he will not accept the contract. And the manager's participation constraints can be defined as:

$$\alpha+\beta e-\frac{1}{2}r\beta^2\sigma^2-\frac{b}{2}e^2\geq\bar{w} \qquad (4)$$

OPTIMAL BEHAVIOR UNDER DIFFERENT PRODUCTION FUNCTIONS

State of diversification (DIV)

The firm 0's manager allocates his effort between firm 0 and firm 1 optimally. Assume that shareholders design for firm 0's manager a linear compensation plan, $s(y)=\alpha+\beta y$ (α and β are constants). The maximization behavior of shareholders is:

$$\text{Max}_{e_i,\alpha,\beta}\pi = e_0+\delta e_1-Es(\cdot) \qquad (5)$$

s.t.

$$\alpha+\beta(e_0+\delta e_1)-\frac{r\beta^2}{2}(\sigma_0^2+\sigma_1^2+2\sigma_{01})$$

$$(e_0,e_1)\in\arg\text{Max}\,U\left[s(Y)-\sum c_i(e_i)\right]$$

where $Es(\cdot)=\alpha+\beta(e_0+\delta e_1)$, $c(e_i)$ is firm i's cost function. Substitute cost function $\sum c_i(e_i)$ for $c(e_0+e_i)$, effects of some cross products can be eliminated. Take out β in the first constraint of the objective function Eq.(5), then the above maximization behavior changes to:

$$\text{Max}_{e_i,\beta}\pi = e_0+\delta e_1-\sum c_i(e_i)-(r/2)\beta^2T \qquad (6)$$

where $T=\sigma_0^2+\sigma_1^2+2\sigma_{01}$, and the constraint of incentive compatibility is $\beta=c_0'=c_1'/\delta$. The objective function can be defined as "expected surplus". Assume that b in $c_i=(b/2)e_i^2$ is equal to one (i.e. b=1). The constraint of incentive compatibility, $\beta=e_0=e_1/\delta$, can be obtained from $c_i=e_i^2/2$, which means that $e_0^*>e_1^*$ when δ<1, and that $e_0^*<e_1^*$ when δ>1. The optimal incentive is $\beta^*=(1+\delta^2+rT)^{-1}(1+\delta^2)$. Therefore, the optimal effort of the manager is:

$$e_0^*=\beta^*=((1+\delta^2+rT)^{-1}(1+\delta^2))$$

$$e_1^*=\delta\beta^*=(1+\delta^2+rT)^{-1}\delta(1+\delta^2)$$

Through the above commutation, shareholders' net surplus is:

$$\pi^*_{\mathrm{DIV}} = e_0^* + \delta e_1^* - \sum c(e_i^*) - r\beta^{*2}T/2 = \frac{(1+\delta^2)^2}{2(1+\delta^2+rT)} \quad (7)$$

Assume that there exist no synergistic effects between firm 0 and firm 1, i.e. $\sigma_0=\sigma_1=\sigma$.

Then, shareholders' net surplus is:

$$\pi^*_{\mathrm{DIV}} = \frac{(1+\delta^2)^2}{2[1+\delta^2+2r(1+\rho)\sigma^2]} \quad (8)$$

where ρ measures the synergetic relations between firm 0 and firm 1.

State of splitting (SPF)

After splitting, the parent still controls the subsidiary. The shareholders must design a new compensation plan to satisfy two managers (the manager of firm 0 and the manager of firm 1). The output function of firm 0 is $y_0=w_0+\varepsilon_0$ while the output function of firm 1 is $y_1=w_1+\varepsilon_1$. The marginal product of the effort of firm 1's manager is equal to 1, for the effort of firm 1's manager is independent of that of firm 0. Let each manager's compensation be $s_i(y_0,y_1)=\alpha_i+\beta_{0i}y_0+\beta_{1i}y_1$, i=0,1. The optimal behavior of shareholders is as follows:

$$\mathrm{Max}_{e_i,\alpha_i,\beta_{ii}}\,\pi = e_0+e_1-Es_0(\cdot)-Es_1(\cdot) \quad (9)$$

s.t.

$$\alpha_i+\beta_{0i}e_0+\beta_{1i}e_1-(r/2)(\beta_{0i}^2\sigma_0^2+\beta_{1i}^2\sigma_1^2+2\beta_{0i}\beta_{1i}\sigma_{01}) = c_i(e_i),\ i=0,1$$

$$(e_0,e_1)\in \arg\mathrm{Max}\,U(s_i(y)-c_i(e_i)),\ i=0,1$$

where $Es_i(\cdot)=\beta_{0i}e_0+\beta_{1i}e_1+\alpha_i$.

By deploying the resolution method in the diversified state, we can have the optimal incentive structure: $\beta_{00}^*=e_0^*$, $\beta_{11}^*=e_1^*$, $\beta_{10}^*=-\rho(\sigma_0/\sigma_1)\beta_{00}^*$, $\beta_{01}^*=-\rho(\sigma_1/\sigma_0)\beta_{11}^*$. The managers' optimal effort is $e_i^*=(1+r\sigma_i^2(1-\rho^2))^{-1}$. The maximum income of shareholders is:

$$\pi^*_{\mathrm{SPF}} = [1+r\sigma^2(1-\rho^2)]^{-1} \quad (10)$$

EXPLANATIONS OF CHANGES IN A CORPORATE STRUCTURE

This paper puts forward a general proposition to determine the optimal structure of a firm. The proposition is based on the optimal net payoff to shareholders, π^*: Splitting is preferred when shareholders' surplus in splitting form is greater than that in diversified form, or vice versa. In order to focus on the effect of the interaction between the exactness of corporate valuation, managerial incentive, and operational risks on the optimal structure choice of a firm, we assume equal risks in firm 0 and firm 1, i.e., $\sigma_0^2=\sigma_1^2$.

Proposition The splitting form is preferred to the diversification form, or vice versa, when

$$r\sigma^2\{(1+\delta^2)^2(1-\rho^2)-4(1+\rho)\}+\delta^4-1<0$$

Proof of the proposition

The difference

$$\pi^*_{\mathrm{DIV}}-\pi^*_{\mathrm{SPF}} = \frac{(1+\delta^2)^2}{2[1+\delta^2+2r(1+\rho)\sigma^2]} - \frac{1}{1+r\sigma^2(1-\rho^2)} = \frac{\delta^2-1+r\sigma^2\{(1+\delta^2)^2(1-\rho^2)-4(1+\rho)\}}{2\{1+\delta^2+2r(1+\rho)\sigma^2\}\{1+r\sigma^2(1-\rho^2)\}} \quad (11)$$

Apparently, the denominator of Eq.(11) is greater than zero. If the numerator is greater than zero, then $\pi^*_{\mathrm{DIV}}-\pi^*_{\mathrm{SPF}}>0$, that is, $\pi^*_{\mathrm{DIV}}>\pi^*_{\mathrm{SPF}}$; if the numerator is smaller than zero, then $\pi^*_{\mathrm{DIV}}-\pi^*_{\mathrm{SPF}}<0$, that is $\pi^*_{\mathrm{DIV}}<\pi^*_{\mathrm{SPF}}$.

From the above proposition, it can be seen that splitting the firm is better than keeping it in diversification when the smaller δ is (the more the firm is undervalued) and the relation between firm 0 and firm 1 is positive ($\rho>0$). When the relation between firm 0 and firm 1 is negative ($\rho<0$) and $\delta>1$ (the more the firm is overvalued), to keep the firm in diversification is more beneficial. When $\delta=1$ (the firm is exactly valued), the numerator of Eq.(11) converts into $-4r\sigma^2(\rho+\rho^2)$. Here, whether splitting the firm is very difficult to decide and it does not just depend on the symbols of ρ. Generally, splitting can be done when the relation between firm 0 and firm 1 is positive.

A NUMERICAL EXAMPLE

In this section, a simple numerical example is provided to highlight the above proposition. Assume that r=1, and $\sigma_1=\sigma_2=\sigma^2=1$. That is, the risk aversion parameter and firm risks are fixed. $\pi^*_{\mathrm{SPF}}=\dfrac{1}{2-\rho^2}$ and the difference, $\pi^*_{\mathrm{SPF}}-\pi^*_{\mathrm{DIV}}=1/(2-\rho^2)-\dfrac{(1+\delta^2)^2}{2[1+\delta^2=2(1+\rho)]}$.

Table 1 shows that a spin-off option becomes optimal only when the parent firm's value is undervalued (δ<1). However, as the correlation between the two firms increases, a positive synergy begins to accelerate. When the synergy parameter (ρ) is greater than 1.745, the diversified status dominates a spin-off choice even for a perfect correlation.

Table 1 Optimal organizational structure for combinations of δ and ρ

δ	ρ	δ	ρ
1	−1.0	1.033	0.1
0.932	−0.9	1.076	0.2
0.891	−0.8	1.121	0.3
0.878	−0.7	1.172	0.4
0.876	−0.6	1.238	0.5
0.884	−0.5	1.289	0.6
0.898	−0.4	1.381	0.7
0.916	−0.3	1.562	0.8
0.942	−0.2	1.594	0.9
0.968	−0.1	1.745	1.0
1	0.0		

CONCLUSION

This paper examines the mechanism determining a firm's optimal structure through the trade-off between the optimal incentive contract and insurance. The interaction of the variables of the exactness of corporate valuation, managerial incentives and operational risks determining a firm's being diversified or being split off. The divestment of the subsidiary is beneficial to shareholders when the parent corporation is undervalued and the relation between the parent and the subsidiary is disordered, or vice versa. This conclusion is consistent with the motives of many divestiture cases in reality.

In reality, many diversified firms' stock prices often have discounts, and the firms are undervalued. Therefore, in order to raise stock prices, firms usually split off some subsidiaries. There exist a large number of cases like this. Of course, it is preferred to split off the subsidiary when the relation between the parent and the subsidiary is tense and the conflicts between them are difficult to mitigate. What is more, it is difficult for us to test the above proposition with domestic data because only Tongrentang T&S Inc., Ltd. and Lenovo Group Limited in mainland China carved out equity in Hongkong, and there have been no cases of American spin-offs yet.

References

Aron, D.J., 1991. Using the capital market as a monitor: corporate spin-offs in an agency framework. *RAND Journal of Economics*, **22**:505-518.

Choi, Y.K., Merville, L.J., 1998. A unified model of corporate acquisitions and divestitures: an incentive perspective. *Review of Quantitative Finance and Accounting*, **10**:127-154.

Fama, E., Jensen, M., 1983. Agency problems and residual claims. *Journal of Law and Economics*, **26**:327-349.

Holmstrom, B., Milgrom, P., 1991. Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics and Organization*, **7**:24-52.

Jensen, M., Meckling, W., 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, **3**:305-360.

Stiglitz, J., 1986. Theory of Competition, Incentive and Risk. *In*: Stiglitz, J., Mathewson, G.F.(Eds.), New Developments in the Analysis of Market Structure. MIT Press, Cambridge.

Williamson, O., 1975. Markets and Hierarchies: Analysis and Antitrust Implications. Free Press, New York.

Zhang, W.Y., 1996. Game Theory and Information Economics. Shanghai Sanlian Publishing House and Shanghai People's Press, Shanghai, China (in Chinese).