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Research of relationship between uncertainty and investment*

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Abstract: This study focuses on revealing the relationship between uncertainty and investment probability through real option model involving investment critical trigger and project earning. Use of Matlab software on the experimental results showing that project earning volatility influences investment probability, led the authors to conclude that this notion is not always correct, as increasing uncertainty should have an inhibiting effect on investment, and that in certain situation, increasing uncertainty actually increases the investment probability and so, should have positive impact on investment.

Key words: Investment, Uncertainty, Real option, Stochastic process
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INTRODUCTION

Real option involves application of finance option in material deal. "When evaluating the value of investment opportunities and optimizing strategies, rational investors will be wiser to find a method to obtain the maximal value of projects based on market information than those who simply utilize subjective possibility or the function of utility" (Everstinefc, 1981; Dixit and Pindyck, 1995; Cox and Rubinstein, 1979; Amram and Kulatilaka, 2001; Galitz, 1998; Meng *et al*., 2003; Caballero, 1991; Song, 1999). The relationship between uncertainty and investment probability has been of interest to economists around world for a long time. The issue was addressed in various ways as discussed by Caballero (1991). This article is focused on the "real option" approach to investment decision pioneered by McDonald and Siegel (1986), Dixit (1989), Pindyck (1988), Dixit and Pindyck (1994).

In this "real option", the investment opportunity is viewed as an option to invest, which has to be exercised optimally. The investment in new project is determined by the exercise policy, which is frequently (for an infinite-horizon setting) of the form: invest if the level of earnings (or NPV of project), say x, exceeds some critical value x^*. This critical value x^*, of course, depends on the parameters of the economy, particularly important being the level of uncertainty or the volatility of the project being considered. According to option theory, the investment rule can be equivalently stated as follows: invest when the value of the project exceeds its cost by an amount equal to the option value of waiting to invest.

Financial options literature showed that higher level of uncertainty increases option value, and that this leads to a more critical value for option exercise (for American options). Consistent with this intuition, real options literature also predict a negative relationship between uncertainty and investment, because greater uncertainty increases the value of the option to wait.

It is necessary to gauge the overall effect of uncertainty on investment. One can look at the probability investment will occur (i.e. the critical trigger value will be reached) within a specified time period. An increase (decrease) in this probability implies a

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positive (negative) effect on investment.

Based on pioneering study, therefore this article focused on uncertainty-investment relationship research by examining the effect of higher volatility on this probability measure, and using Matlab software to draw a curve for expressing that. The influences of all parameters of the real option value model to the investment policy are also analyzed.

MODEL

We use a canonical real options model of investment, along the lines of McDonald and Siegel (1986) or Dixit and Pindyck (1994; 1995), with two differences:

1. The state variable is earnings rather than firm value;
2. Systematic risk is explicitly taken into account.

The firm is considering an infinite-horizon investment project which generates a random net cash flow (or earnings) stream of \$ X per unit time. Because the earnings stream is influenced by many factors such as market condition, management system reform, people's psychology, and so on, it follows the stochastic lognormal process below:

$$dx_t = \mu x_t dt + \sigma x_t dz_t \quad (1)$$

where, μ is the expected growth rate of the cash flow stream; σ the standard deviation of the growth rate and dz the increment of a standard Weiner process. The level of uncertainty of the project (or of the earnings process) is measured by the volatility term σ.

The project can be accepted at any time, when it is accepted, the firm can implement the project instantaneously at a cost of \$1 (this is just a normalization; there is no loss of generality in assuming a unit investment cost). The risk-free interest rate is a constant r. The correlation of the project with the market portfolio is ρ (i.e., $dz dz_m=\rho dt$), and the market price of risk is λ [defined in Merton (1973)].

In the above setting, the postponable project can be viewed as an (American) option to invest, which should be exercised optimally, i.e., exercising the option generates a higher payoff than holding it. The firm's investment decision is therefore equivalent to an optimal stopping problem: at what point is it optimal to implement the project? Alternatively, what is the optimal exercise policy for the option to invest? In an infinite-horizon setting, this translates into some critical value of earnings (say, x^*) such that the firm should implement the project as soon as x reaches or exceeds this critical trigger level.

CRITICAL INVESTMENT TRIGGER

With the above specifications, it can be shown that the project value (in capital budgeting terms, the NPV of the project when accepted) is given by

$$Project.value = \frac{X}{\gamma + \lambda\rho\sigma - \mu} - 1 \quad (2)$$

The value of the option to invest (i.e., value of project prior to acceptance), $F(x)$ follows an ordinary differential equation of the form specified in McDonald and Siegel (1986) or Dixit and Pindyck (1994; 1995). Along with the appropriate boundary conditions (value matching and smooth pasting), the solution is given by

$$F(x)=AX^{a} \quad (3)$$

where

$$\alpha = \frac{1}{2} - \frac{\mu - \lambda\rho\sigma}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu - \lambda\rho\sigma}{\sigma^2}\right)^2 + \frac{2\gamma}{\sigma^2}} \quad (4)$$

$$A = \frac{(x^*)^{1-\alpha}}{(\gamma + \lambda\rho\sigma - \mu)\alpha} \quad (5)$$

In Eq.(5), x^* is the optimal stopping boundary; that is, the optimal investment rule is to invest when x rises to x^*. The boundary x^* is given by:

$$x^* = \frac{\alpha(\gamma + \lambda\rho\sigma - \mu)}{\alpha - 1} \quad (6)$$

For the investment decision, the important result is Eq.(6), which gives a closed-form expression for the optimal or critical investment trigger x^*.

Intuitively, it is obvious that a higher level of uncertainty will increase the critical trigger level x^* (as can be verified by differentiating Eq.(6) with respect to σ), and thereby have a negative effect on investment. However, there is an additional effect of the higher volatility: because of higher volatility, the variable x is now more likely to reach the critical level x^*, which was discussed by Metcalf and Hassett (1995) and will have a positive effect on investment. Thus, there are two effects of higher volatility on investment, one negative and the other positive. In order to get an idea of the overall effect, we examine the probability of investment next.

PROBABILITY OF INVESTING

The probability of reaching the critical level x^* (i.e. probability of investing) within some time period T is given by Harrison (1985) as

$$Prob(Inv) = \Phi\left(\frac{\ln(x_0/x^*) + (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}\right) + \left(\frac{x^*}{x_0}\right)^{2\mu/\sigma^2-1} \Phi\left(\frac{\ln(x_0/x^*) - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}}\right) \quad (7)$$

where x_0 is the starting (or time 0) value of x, and $\Phi(\cdot)$ the area under the standard normal distribution. Substituting for x^* from Eq.(6), we get

$$Prob(Inv) = \Phi\left(\frac{\ln[x_0(1-1/\alpha)/(\gamma+\lambda\rho\sigma-\mu)] + (\mu-0.5\sigma^2)T}{\sigma\sqrt{T}}\right) + \left(\frac{\rho+\lambda\rho\sigma-\mu}{x_0(1-1/\alpha)}\right)^{2\mu/\sigma^2-1} \times\Phi\left(\frac{\ln[x_0(1-1/\alpha)/(\gamma+\lambda\rho\sigma-\mu)] - (\mu-0.5\sigma^2)T}{\sigma\sqrt{T}}\right) \quad (8)$$

Eq.(8) gives the probability of investment occurring within time T, in terms of the parameters of the economy and the project. A higher (lower) probability implies a greater (smaller) chance of project acceptance, hence a positive (negative) effect on investment. Since the sign of the derivative $[d(Prob)/d\sigma]$ cannot be determined unambiguously, it is not clear how a higher σ will affect the probability of investing. We therefore have to use numerical results to illustrate the uncertainty-investment relationship.

NUMERICAL ANALYSIS

We start with the following base case parameter values: μ=0, r=10%, ρ=0.7, λ=0.4, x_0=0.1 and T=5 yr. We use μ=0 because we wish to focus on volatility effects and not growth effects; ρ=0.7 reflects a project is imperfectly (but positively) correlated with the market, which is a good description of the majority of projects; and λ=0.4 is the approximate historical average. Note that using a different value of x_0 will result in different $Prob(Inv)$, but will make no difference to the relationship between σ and $Prob(Inv)$, which is what we are interested in. With the above parameter values, we computed $Prob(Inv)$ for different values of σ by using Matlab software. The results are displayed in Fig.1.

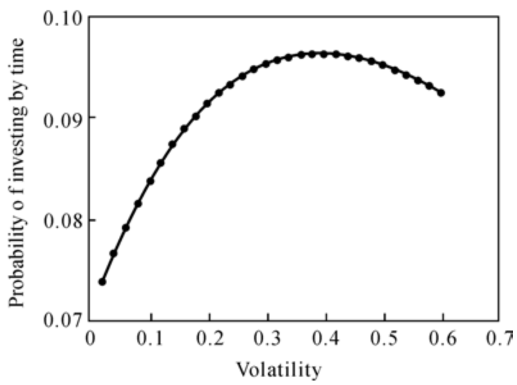

Fig.1 Probability of investing as a function of volatility. Parameters: μ=0, r=10%, ρ=0.7, λ=0.4, x_0=0.1, and T=5 yr

Fig.1 shows that the probability of investing is initially an increasing function of volatility, but after a certain point (about σ=0.39 for the base case), it becomes a decreasing function of volatility. Therefore, for low levels of uncertainty, increase in uncertainty increases the probability of investing and thereby has a positive effect on the expected rate of investment. With the base case parameters, the direction of the overall effect of volatility on investment is thus ambiguous. This result is also robust to the exact choice of parameter values, as was confirmed by repeating the computations for a wide range of parameter values around the base case. This illustrates our main result: An increase in uncertainty might actually speed up investment, contrary to what the literature generally predicts.

PARAMETERS DISCUSSION

The effects of the various parameters, i.e. uncertainty σ; the correlation of the project with the market portfolio ρ; the market price of risk λ; risk-free interest rate r; the expected growth rate μ and exercises option time T can be summarized as follows:

The uncertainty-investment relationship is more likely to be positive when

i) The current level of uncertainty σ is low;

ii) ρ is high; iii) λ is high;

iv) r is high; v) μ is low;

vi) T is short.

We also find that the trigger x^* is always an increasing function of σ, as predicted by Fig.2.

The critical investment trigger x^* as function of volatility σ and other parameters r, ρ, λ are shown in Figs.3, 4, 5.

CONCLUSION

To summarize, this study focuses on revealing the relationship between uncertainty and investment probability through real option model involving investment critical trigger and project earning. Based on an experiment which showed the influence of project earning volatility on investment by using Matlab software, the author concludes that this notion is not always correct, that increase in uncertainty should

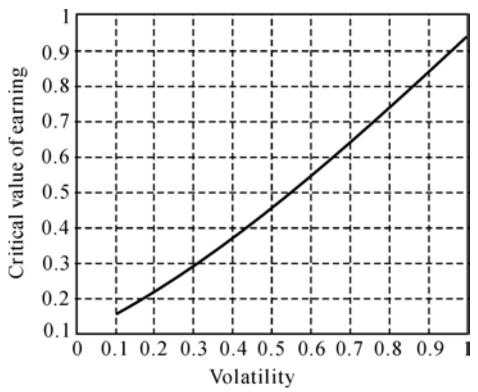

Fig.2 Critical value of earning as a function of volatility

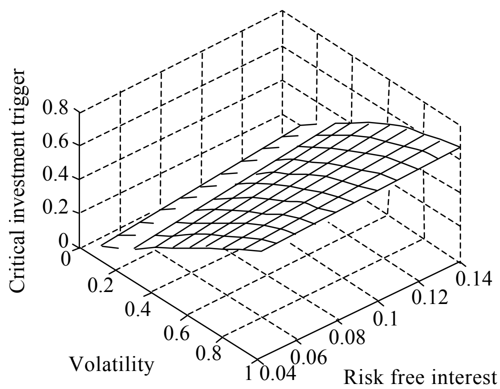

Fig.3 Critical investment trigger as a function of volatility and risk free interest

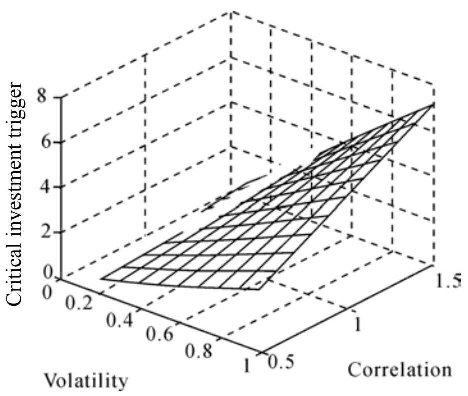

Fig.4 Critical investment trigger as a function of volatility and correlation

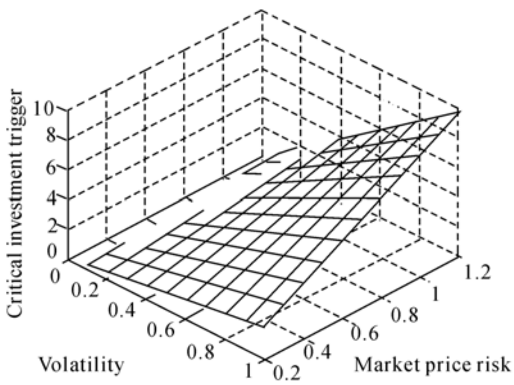

Fig.5 Critical investment trigger as a function of volatility and market price risk

have an inhibiting effect on investment; that in certain situation, increase in uncertainty actually increases the investment probability and so, should have positive impact on investment.

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