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LDPC based differential unitary space-frequency coding for MIMO-OFDM systems*

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Abstract: This paper proposes a novel LDPC based differential unitary space-frequency coding (DUSFC) scheme for MIMO-OFDM systems when neither the transmitter nor the receiver has access to the channel state information (CSI). The new DUSFC strategy basically consists of coding across transmit antennas and OFDM tones simultaneously as well as differential modulation in the time-domain. It can fully exploit the inherent advantages provided by the multipath fading channels, resulting in a high degree of diversity. The state-of-the-art low-density parity-check (LDPC) codes are concatenated with our DUSFC as channel coding to improve the bit error rate (BER) performance considerably. Owing to the maximum multipath diversity and large coding advantages, LDPC-DUSFC strongly outperforms the differential unitary space-time coded OFDM techniques recently proposed in literature. The corresponding iterative decoding algorithm without channel estimation is finally provided to offer significant performance gain. Simulation results illustrate the merits of the proposed scheme.

Key words: Differential modulation, Unitary space-time codes, Space-frequency coding, MIMO, OFDM, LDPC
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INTRODUCTION

Unitary space-time modulation (USTM) (Hochwald and Marzetta, 2000) has been shown to be ideally suited for Rayleigh flat fading channels when there is no CSI. And for continuously changing fading channels, differential unitary space-time modulation (DUSTM) was proposed in Hochwald and Sweldens (2000). However, the performance of DUSTM degrades considerably in frequency-selective fading channels often encountered in broadband wireless communications. Hence concatenation of DUSTM with orthogonal frequency-division multiplexing (OFDM) has gained much interest recently. In some of these DUSTM-OFDM systems (Sun *et al*., 2002), the unitary space-time codes designed for flat fading channels is simply applied to each OFDM tone separately, which fails to exploit the embedded multipath diversity. Thus, coding across OFDM tones is required to exploit the inherent diversity in frequency-selective fading channels as pointed out in Boelcskei and Paulraj (2000). But, the scheme proposed by Boelcskei and Paulraj (2000) requires reliable estimation of the underlying multi-channels at the receiver. This is a challenging and costly task, especially when the channel experiences high mobility induced rapid channel fading.

This paper presents a new differential unitary space-frequency coding (DUSFC) scheme that achieves the maximum multipath diversity and significant coding gain for wideband systems without CSI. The coding scheme basically consists of employing existing unitary space-time codes across transmit antennas and OFDM tones simultaneously as

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well as differential modulation between two adjacent OFDM blocks, which will therefore be called differential unitary space-frequency coding (DUSFC).

From the perspective of information theory, it is necessary to apply channel coding to further approach the channel capacity limit. Recently, the state-of-the-art LDPC codes whose performance is close to the Shannon limit, have attracted much attention. In this paper, LDPC codes are concatenated with our proposed DUSFC-OFDM system as channel coding to improve the BER performance significantly. It outperforms existing differential unitary space-time coded OFDM techniques (Sun *et al.*, 2002) considerably and enjoys high coding advantages, which is confirmed by corroborating simulations.

NOTATION AND SYSTEM DESCRIPTION

We consider a communication link over frequency-selective multipath fading channels with M_T transmit antennas and M_R receive antennas. N_C denotes the number of OFDM tones. Let $x^n[k]$ denote the transmitted data symbol on the kth tone of the nth OFDM block. The MIMO-OFDM system's input-output relationship can be expressed as

$$y^n[k]=H^n[k]x^n[k]+N^n[k],\quad k=1, 2, \ldots, N_C \tag{1}$$

where $H^n[k]=\sum_{l=0}^{L-1} h^n[l]\mathrm{e}^{-j2\pi kl/N_C}$; $h^n[l]$ is the time-domain channel impulse response; L denotes the number of non-zero resolvable taps; and $N^n[k]$ is the complex-valued additive white Gaussian noise with mean zero and variance $N_0/2$ per dimension.

Our design of differential unitary space-frequency coding (DUSFC) involves designing the codeword $\boldsymbol{X}$ of size $N_C\times M_T$. However, N_C is typically large in practice. Thus we will adopt subcarrier grouping (SG) (Liu *et al.*, 2002) to reduce the dimensionality and facilitate the encoding and decoding process. First we assume the number of subcarriers satisfies that: $N_C=L\times Q$, where Q is a certain positive integer denoting the number of subgroups. Let $k_g=\{g, Q+g, 2Q+g, \ldots, (L-1)Q+g\}$ $(g=1, 2, \ldots, Q)$ represent the index set of L subcarriers within each subgroup g. Then we split the $N_C\times M_T$ codeword $\boldsymbol{X}$ into Q subgroups with equal size of $L\times M_T$ by defining the diagonal matrix

$$\boldsymbol{X}_g=\mathrm{diag}\left[x_1[1], x_2[2], \cdots, x_{M_T}[L]\right],\ g=1, 2, \ldots, Q \tag{2}$$

Accordingly, the MIMO-OFDM system described by Eq.(1) can be divided into Q subsystems

$$\boldsymbol{Y}_g=\boldsymbol{X}_g\boldsymbol{H}_g+\boldsymbol{N}_g,\qquad g=1, 2, \ldots, Q \tag{3}$$

where $\boldsymbol{H}_g=\begin{pmatrix} H_{11}[1] & \ldots & H_{1M_R}[1] \\ \vdots & \ddots & \vdots \\ H_{M_T1}[L] & \cdots & H_{M_TM_R}[L]\end{pmatrix}$; $\boldsymbol{Y}_g=\left[\boldsymbol{y}[1], \boldsymbol{y}[2], \ldots, \boldsymbol{y}[L]\right]^T$, $\boldsymbol{y}[k]=\left[y_1[k], \cdots, y_{M_R}[k]\right]^T$; $\boldsymbol{N}_g$ is defined similarly as in $\boldsymbol{Y}_g$; g denotes the gth subgroup. Since in the following we performing DUSFC only across a subgroup of tones defined by k_g, we will drop the subgroup index g for simplicity. Through subcarrier grouping, we split the set of generally correlated subchannels into subsets of independent subchannels. Thus, the information is distributed onto parallel subcarriers that are with independent fading, resulting in a high degree of diversity.

DIFFERENTIAL UNITARY SPACE-FREQUENCY CODING (DUSFC)

Scheme description

In conventional DUSTM systems, the unitary space-time modulator maps the information symbols into unitary constellation matrices $\boldsymbol{U}$ of size $T\times M_T$. The signals in $\boldsymbol{U}$ are transmitted from M_T transmit antennas during the T consecutive time slots. In our DUSFC scheme, however, we treat each time slot as an OFDM tone. Then these unitary matrices are transmitted through M_T transmit antennas over T different OFDM tones. Fig.1 depicts the block diagram of our proposed differential unitary space-frequency coding (DUSFC) scheme. Thus, our DUSFC can be viewed as a MIMO-OFDM system modeled in Eq.(3) which employs unitary space-frequency codes across L OFDM tones with differential modulation between two adjacent OFDM blocks. At the nth OFDM block, the coded signal is denoted as $\boldsymbol{X}_n$, which is a matrix of dimension $L\times M_T$. By differential transmission, we have

$$X_n = U_n X_{n-1} \quad (4)$$

with $X_0=I$. Specifically the unitary matrix U_n is space-frequency modulated, and forms a group of $L\times L$ unitary and diagonal matrices ψ with cardinality $|\psi|$, i.e.

$$\psi = \left\{ U_p \mid U_p^{\mathrm{H}} U_p = I, p = 1, 2, \cdots, |\psi| \right\}.$$

Superscript H denotes conjugate transpose.

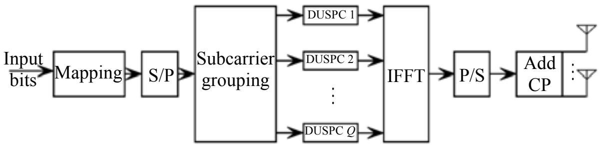

Fig.1 Block diagram of DUSFC scheme

The channel is assumed to be constant during two adjacent OFDM blocks. At the receiver, the received $L\times M_{\mathrm{R}}$ signal matrix Y_n at the nth OFDM block obeys

$$\begin{aligned} Y_n &= U_n X_{n-1} H + N_n \\ &= U_n Y_{n-1} - U_n N_{n-1} + N_n \\ &= U_n Y_{n-1} + \tilde{N}_n \end{aligned} \quad (5)$$

where $\tilde{N}_n = N_n - U_n N_{n-1}$ is an $L\times M_{\mathrm{R}}$ equivalent noise matrix. Hence, the differential demodulation of U_n can be done by the maximum-likelihood (ML) detector

$$\begin{aligned} \hat{U}_n &= \arg\max_{U\in\psi} \mathrm{Retr}\{Y_{n-1} Y_n^{\mathrm{H}} U\} \\ &= \arg\min_{U\in\psi} \left\| Y_n - U Y_{n-1} \right\|^2 \end{aligned} \quad (6)$$

where $\|\cdot\|$ denotes the Frobenius norm. Clearly, U_n can be decoded from the two-fold observations of Y_n and Y_{n-1} without CSI.

Performance analysis

In the following, we analyze the pairwise error probability (PEP) of this MIMO-OFDM system with DUSFC. Assuming equal transmitted power at all transmit antennas, using the Chernoff bound, the PEP of transmitting U and deciding in favor of another unitary constellation matrix U' at the receiver is upper bounded by

$$P(U \to U') \le \left\{ 1 \Big/ \prod_{i=1}^{r(G)} \left[1 + \frac{\gamma \lambda_i(G)}{8M_{\mathrm{T}}} \right] \right\}^{M_{\mathrm{R}}} \quad (7)$$

where γ is the signal-to-noise ratio (SNR) per receive antenna; $G=(U-U')^{\mathrm{H}}(U-U')$; $r(G)$ stands for the rank of G; $\lambda_i(G)$ is the nonzero eigenvalue of G. Assuming γ is high enough, the pairwise error probability (PEP) can be further upper bounded by

$$P(U \to U') \le \left[\frac{1}{2} \prod_{i=1}^{r(G)} \lambda_i(G) \right]^{-M_{\mathrm{R}}} \left[\frac{\gamma}{4M_{\mathrm{T}}} \right]^{-M_{\mathrm{R}} r(G)} \quad (8)$$

Following (Tarokh *et al.*, 1998) we define the diversity gain D_{g} and the corresponding coding gain C_{g} as

$$D_g = \min_{\forall U, U' \in \psi} r(G) \quad (9)$$

$$C_g = \frac{1}{2} \min_{\forall U, U' \in \psi} \left[\det(G) \right]^{1/r(G)} \quad (10)$$

where the minimization is taken over all possible pairwise errors. So far, based on the PEP analysis of our DUSFC system we have obtained two important performance metrics: the diversity gain and the coding gain. These results provide some implications on the design principle of U. First, $\forall U \ne U' \in \psi$, G should have full rank to maximize the diversity gain. Since ψ is a finite group of unitary and diagonal matrices, we only need to design $U-U'$ to have full rank for all possible pairwise errors. Second, $\forall U \ne U' \in \psi$, $\min_{\forall U, U' \in \psi} \left[\det(G) \right]$ should be maximized to achieve the maximum coding gain. Based on the design principle similar to this, the detailed construction of the unitary and diagonal matrix group ψ was well documented by Hochwald and Sweldens (2000), so we will simply use the constellations in that paper.

LDPC-DUSFC SCHEME

LDPC codes

Low-density parity-check (LDPC) codes were first proposed by Gallager (1962) and recently re-examined in (Mackay and Neal, 1996; Mackay, 1999), who showed that these codes achieved remarkable performance with iterative decoding that is very close to the Shannon limit (Mackay and Neal, 1996).

An LDPC code is a linear block code characterized by a very sparse parity-check matrix. The parity-check matrix $\boldsymbol{H}$ for an (n, k) LDPC code of rate $R=k/n$ is an $(n-k)\times n$ matrix. Both the number of 1's per column (column weight) and the number of 1's per row (row weight) are very small compared to the block length n. Apart from these constraints, the ones are placed randomly in $\boldsymbol{H}$. An LDPC code concatenated with our proposed DUSFC system can improve the BER performance significantly. However, it also loses bandwidth due to the code rate of k/n. For practical systems, we can tradeoff the desirable performance with the bandwidth efficiency.

The algorithm used for LDPC decoding is a probability propagation algorithm known as the sum-product algorithm (SPA) or belief propagation (Mackay, 1999). It determines a posteriori probabilities for bit values based on a priori information, improving the accuracy of these calculations at each iteration. The initialization of SPA is important for LDPC decoding, whose task is to compute the likelihood ratio (LLR) of the received signal.

Decoding algorithm for LDPC-DUSFC scheme

In the transmitter, we employ finite-geometry LDPC codes (Kou *et al.*, 2001) using the parameters shown in Table 1. In the receiver, the received signal can be expressed by $\boldsymbol{Y}_n$ and $\boldsymbol{Y}_{n-1}$ as shown in Eq.(5). In order to perform the decoding, the LLR of code bits "1" and "0" for all the code bits corresponding to received signals should be computed. And then, SPA is used to decode iteratively. The algorithm for computation of LLR is as follows.

Table 1 Parameters of finite-geometry LDPC code

Code	Type-I EG-LDPC
n	255
k	175
$D_{\min}$	17
Column weight	16
Row weight	16
Decoding algorithm	SPA

The received signals corresponds to $\log_2|\psi|$ bits, where $|\psi|$ denotes the cardinality of group ψ. We denote the transmitted bits as $\boldsymbol{b}=\left(b_1,...,b_l,...,b_{\log_2|\psi|}\right)$. The log-likelihood for the lth bit in $\boldsymbol{b}$ is given as

$$L(b_l)=\log\frac{\Pr[b_l=1\,|\,\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n]}{\Pr[b_l=0\,|\,\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n]} \tag{11}$$

which can be further written as

$$L(b_l)=\frac{\sum_{\boldsymbol{b}:b_l=1}\Pr[\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n,\boldsymbol{b}\text{ is transmitted}]}{\sum_{\boldsymbol{b}:b_l=0}\Pr[\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n,\boldsymbol{b}\text{ is transmitted}]} \tag{12}$$

Because $\boldsymbol{b}$ is one-to-one mapped into $\boldsymbol{U}$, which can be denoted as $\boldsymbol{U}=F(\boldsymbol{b})$, we get

$$L(b_l)=\frac{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=1}\Pr[\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n,\boldsymbol{U}\text{ is transmitted}]}{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=0}\Pr[\boldsymbol{Y}_{n-1},\boldsymbol{Y}_n,\boldsymbol{U}\text{ is transmitted}]} \tag{13}$$

Assuming all the constellation points are equiprobable, Eq.(13) can be rewritten as

$$L(b_l)=\frac{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=1}\Pr[\boldsymbol{Y}_n\,|\,\boldsymbol{Y}_{n-1},\boldsymbol{U}]}{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=0}\Pr[\boldsymbol{Y}_n\,|\,\boldsymbol{Y}_{n-1},\boldsymbol{U}]} \tag{14}$$

Since

$$\Pr[\boldsymbol{Y}_n\,|\,\boldsymbol{Y}_{n-1},\boldsymbol{U}]=\frac{\exp\left\{-\mathrm{tr}[\Sigma^{-1}(\boldsymbol{Y}_n-\boldsymbol{U}\boldsymbol{Y}_{n-1})(\boldsymbol{Y}_n-\boldsymbol{U}\boldsymbol{Y}_{n-1})^{\mathrm{H}}]\right\}}{\pi^{L\times M_{\mathrm{R}}}\det^{M_{\mathrm{R}}}(\Sigma)} \tag{15}$$

where Σ is the variance matrix of $\boldsymbol{Y}_n$ with $\Sigma=2N_0\boldsymbol{I}$, $L(b_l)$ can be derived as Eq.(16):

$$L(b_l)=\log\frac{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=1}\dfrac{E}{\pi^{L\times M_{\mathrm{R}}}\det^{M_{\mathrm{R}}}(\Sigma)}}{\sum_{\boldsymbol{U}:\boldsymbol{U}=F(\boldsymbol{b}),b_l=0}\dfrac{E}{\pi^{L\times M_{\mathrm{R}}}\det^{M_{\mathrm{R}}}(\Sigma)}} \tag{16}$$

where $E=\exp\left\{-\mathrm{tr}[\Sigma^{-1}(\boldsymbol{Y}_n-\boldsymbol{U}\boldsymbol{Y}_{n-1})(\boldsymbol{Y}_n-\boldsymbol{U}\boldsymbol{Y}_{n-1})^{\mathrm{H}}]\right\}$

Because $\boldsymbol{Y}_n$, $\boldsymbol{Y}_{n-1}$, $\boldsymbol{U}$ and Σ are all diagonal matrices, the computation of $L(b_l)$ has low complexity.

After the derivation of $L(b_l)$, the iterative decoding is executed via SPA.

SIMULATION RESULTS

We compared our DUSFC-OFDM to the existing DUSTM-OFDM scheme (Sun *et al.*, 2002) over frequency-selective fading channels with parameters: N_C=48 and L=2, 4, whose performance is shown in Fig.2. BPSK modulation was adopted for both test cases. The optimal (4;1;1) group codes (Liu *et al.*, 2001) are employed for all the systems. For both frequency-selective fading channels (i.e. L=2, 4), DUSFC-OFDM outperformed DUSTM-OFDM considerably, especially at high SNR. The DUSFC-OFDM provided a gain of about 2 dB at a BER of 10^{-3}, because the DUSFC-OFDM employs unitary space-frequency coding across OFDM tones, which can fully exploit the inherent frequency diversity. As we can also see, DUSFC-OFDM achieves higher diversity gain as the channel order L increases.

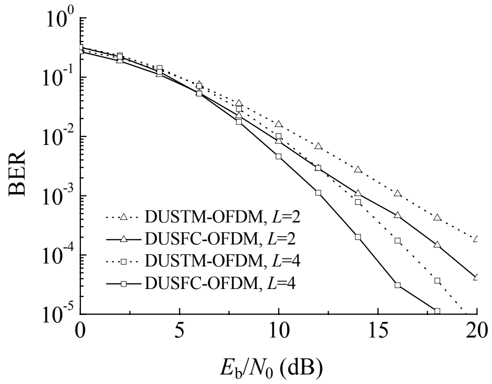

Fig.2 Performance comparison for frequency-selective fading channels

Fig.3 shows the performance comparison of our LDPC-DUSFC-OFDM and DUSTM-OFDM scheme (Sun *et al.*, 2002) over time- and frequency-double selective fading channels with the normalized Doppler frequency $F_d N_C T_s$=0.01, 0.03, 0.05 and L=2, where F_d denotes the maximum Doppler frequency and T_s denotes the period of one OFDM time block. When $F_d N_C T_s$=0.01, the LDPC-DUSFC-OFDM outperforms DUSTM-OFDM and provides a superior performance, at gain of about 6.5 dB at a BER of 10^{-3}. When $F_d N_C T_s$=0.03, the improvement is up to 9 dB. With $F_d N_C T_s$ increasing, DUSTM-OFDM is largely affected by the Doppler spread and its performance degrades significantly, while our LDPC-DUSFC-OFDM is more tolerant to the Doppler spread and still has good performance. Therefore, the LDPC-DUSFC-OFDM scheme can achieve better BER performance than conventional DUSTM-OFDM, especially for high SNR, which indicates its robustness for the time- and frequency-double selective fading channels.

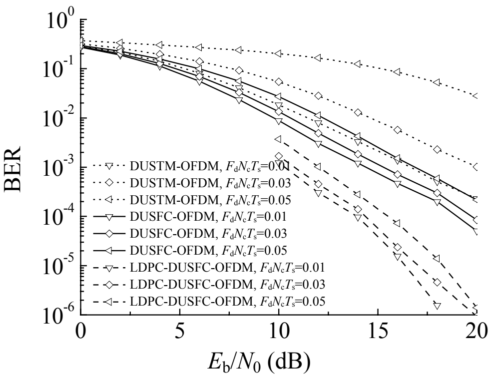

Fig.3 Performance comparison for time- and frequency-double selective fading channels

CONCLUSION

The novel differential unitary space-frequency coded OFDM system proposed in this paper, when compared with the existing DUSTM-OFDM, significantly improves the BER performance by efficiently exploiting both the inherent frequency diversity and coding advantages. We also studied the concatenation scheme of low-density parity-check (LDPC) codes with our DUSFC-OFDM system and proposed a low-complexity non-coherent decoding algorithm. LDPC-DUSFC-OFDM system is robust for the time- and frequency-double selective fading channels, which was confirmed by corresponding simulation analysis.

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